Lecture 3:

PROPAGATION OF GALACTIC COSMIC RAYS

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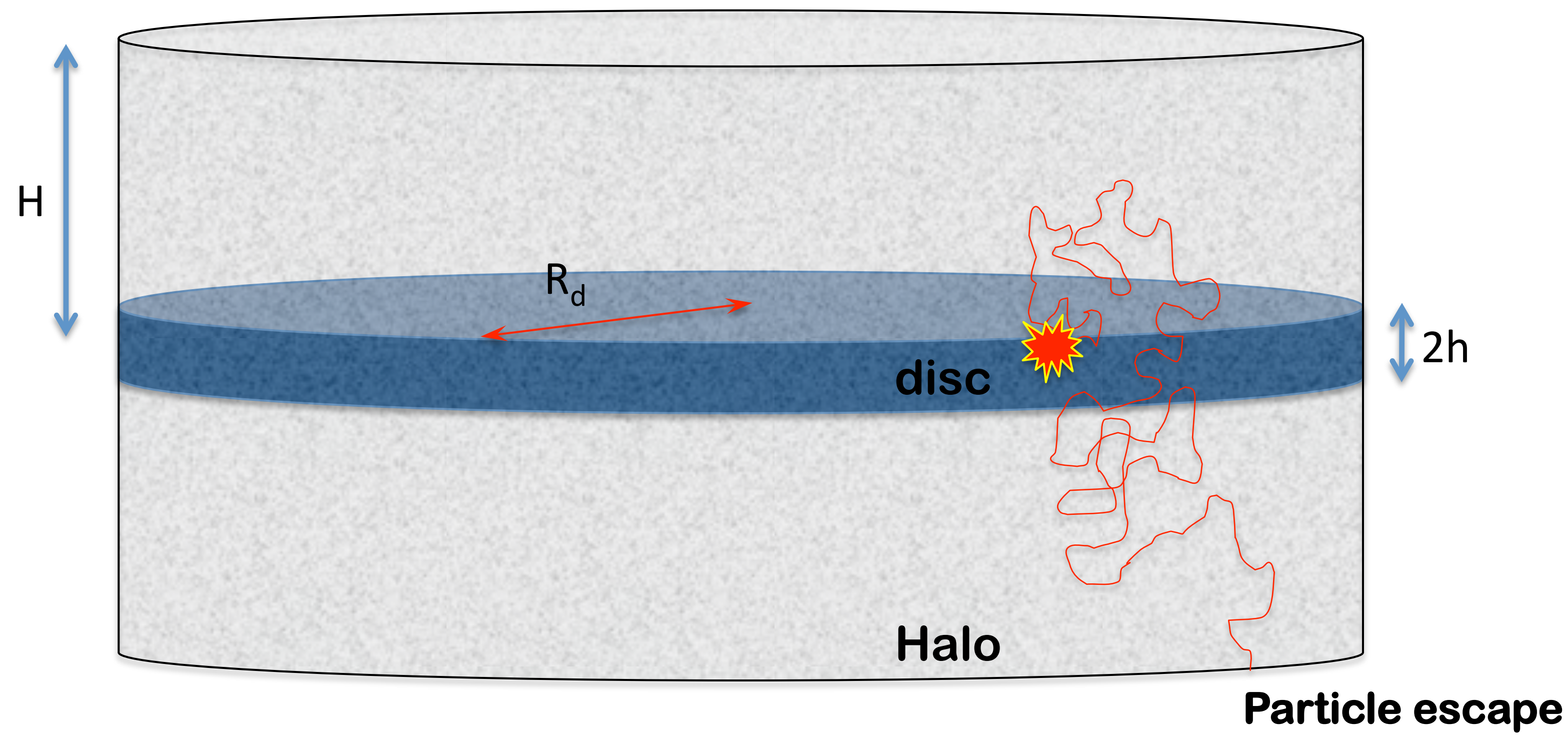
- *DIFFUSIVE PROPAGATION OF CR IN THE GALAXY*
- *SPALLATION AND ROLE OF SECONDARY/PRIMARY RATIOS*
- *GREEN FUNCTION AND ROLE OF STOCHASTIC SOURCES*
- *ORIGIN OF SCATTERING CENTERS*
- *ELECTRONS*
- *SOME NEW PUZZLES*



All sources are assumed to be in the disc and are assumed to be SNRs which explode in the Galaxy at a rate $\mathscr{R}$ per unit time

Diffusive propagation of CR in the Galaxy

The propagation of nuclei of type k is described by the transport equation:

$$\frac{\partial n_k(E,\vec{r},t)}{\partial t} = \nabla\left[D_k(E)\nabla n_k(E,\vec{r},t)\right] - \Gamma_k^{sp}(E) n_k(E,\vec{r},t) + N_k(E)\delta(t-t_s)\delta^3(\vec{r}-\vec{r}_s)$$

DIFFUSION SPALLATION INJECTION

H $(k=1)$, He $(k=2)$, CNO $(k=3)$, Mg-Al-Si $(k=4)$ and Fe $(k=5)$

Injection is taken in the form:

$$N_k(E) \propto E^{-\gamma} \exp\left[-\left(\frac{E}{E_{max,k}}\right)\right]$$

THE TRANSPORT EQUATION IS SOLVED WITH THE BOUNDARY CONDITION THAT

$$n_k(E, z=\pm H) = 0$$

The Green function of the transport equation without boundary conditions (namely without imposing n(H)=0) is easily found to be:

$$\mathcal{G}_k^{free}(\vec{r}, t; \vec{r}_s, t_s) = \frac{N_k(E)}{[4\pi D_k \tau]^{3/2}} \exp\left[-\Gamma_k^{sp}(E)\tau\right] \exp\left[-\frac{(\vec{r}-\vec{r}_s)^2}{4D_k\tau}\right]$$

therefore one can use the method of image charges to obtain the Green function that satisfies the boundary condition:

$$\mathcal{G}_k(\vec{r}, t; \vec{r}_s, t_s) = \frac{N_k(E)}{[4\pi D_k \tau]^{3/2}} \exp\left[-\Gamma_k^{sp}(E)\tau\right] \exp\left[-\frac{(x-x_s)^2+(y-y_s)^2}{4D_k\tau}\right] \times$$

$$\sum_{n=-\infty}^{+\infty} (-1)^n \exp\left[-\frac{(z-z'_n)^2}{4D_k\tau}\right],$$

$z'_n = (-1)^n z_s + 2nH$ are the z coordinates of the image sources

Let us consider the simple case in which we are sitting in the center of the disc and we limit ourselves with protons, namely there are no nuclei and no spallation. In this case the density of particles at Earth is:

$$n_{CR}(E) = \int_0^\infty d\tau \int_0^{R_d} dr \frac{2\pi r}{\pi R_d^2} \frac{N(E)\,\mathcal{R}}{[4\pi D(E)\tau]^{3/2}} \exp\left[-\frac{r^2}{4D(E)\tau}\right] \sum_{n=-\infty}^{+\infty} (-1)^n \exp\left[-\frac{(2nH)^2}{4D(E)\tau}\right]$$

Carrying out the integration on τ first and then on r, one easily obtains

$$n_{CR}(E) = \frac{N(E)\,\mathcal{R}}{2\pi D(E) R_d} \sum_{n=-\infty}^{+\infty} (-1)^n \left[\sqrt{1+\left(\frac{2nH}{R_d}\right)^2} - \sqrt{\left(\frac{2nH}{R_d}\right)^2}\right]$$

If $H \ll R_d$ then one can easily see that the sum over n tends to H/R_d therefore:

$$n_{CR}(E) = \frac{N(E)\,\mathcal{R}}{2\pi R_d^2} \frac{H}{D(E)} \equiv \frac{N(E)\,\mathcal{R}}{2H\pi R_d^2} \frac{H^2}{D(E)}$$

The case of nuclei is more complicated but not too much…
Keeping the spallation term and introducing the spallation time $\tau_{sp,k} = 1/\Gamma_{sp,k}$

$$n_k(E) = \int_0^{\infty} d\tau \int_0^{R_d} dr \frac{2\pi r}{\pi R_d^2} \frac{N_k(E)\,\mathcal{R}}{[4\pi D_k(E)\tau]^{3/2}} \exp\left[-\frac{\tau}{\tau_{sp,k}}\right] \times$$

$$\exp\left[-\frac{r^2}{4D_k(E)\tau}\right] \sum_{n=-\infty}^{+\infty} (-1)^n \exp\left[-\frac{(2nH)^2}{4D_k(E)\tau}\right],$$

The integrals over tau and r are both analytical and lead to:

$$n_k(E) = \frac{N_k(E)\,\mathcal{R}}{2\pi D_k(E) R_d^2} \sqrt{D_k(E)\tau_{sp,k}} \times$$

$$\sum_{n=-\infty}^{+\infty} (-1)^n \left\{ \exp\left[-\left(4n^2 \frac{\tau_{esc,k}}{\tau_{sp,k}}\right)^{1/2}\right] - \exp\left[-\left(\frac{\tau_{esc,k}}{\tau_{sp,k}}\right)^{1/2} \left(4n^2 + \frac{R_d^2}{H^2}\right)^{1/2}\right] \right\}$$

$$\tau_{esc,k}(E) = H^2/D_k(E)$$

The asymptotic behavior is:

$$n_k(E) = \frac{N_k(E)\,\mathcal{R}}{2\pi R_d^2}\frac{H}{D_k(E)}, \qquad \frac{\tau_{esc,k}}{\tau_{sp,k}} \ll 1$$

In the opposite limit one has to be careful defining the average density over the propagation volume:

$$n_{gas}(E) = n_{disc}\frac{h}{\sqrt{D(E)\tau_{sp}}} = \frac{n_{disc}^2 h^2 c\sigma_{sp}}{D(E)}$$

and in this condition:

$$n_k(E) = \frac{N_k(E)\,\mathcal{R}}{2H\pi R_d^2}\sqrt{\tau_{sp,k}\tau_{esc,k}}\ , \qquad \frac{\tau_{esc,k}}{\tau_{sp,k}} \gg 1$$

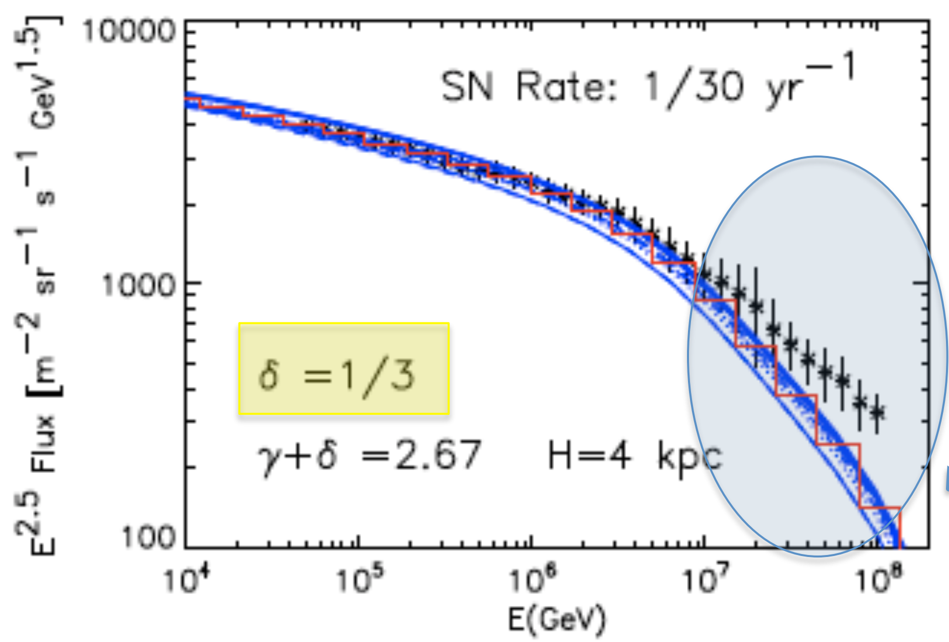
which leads to

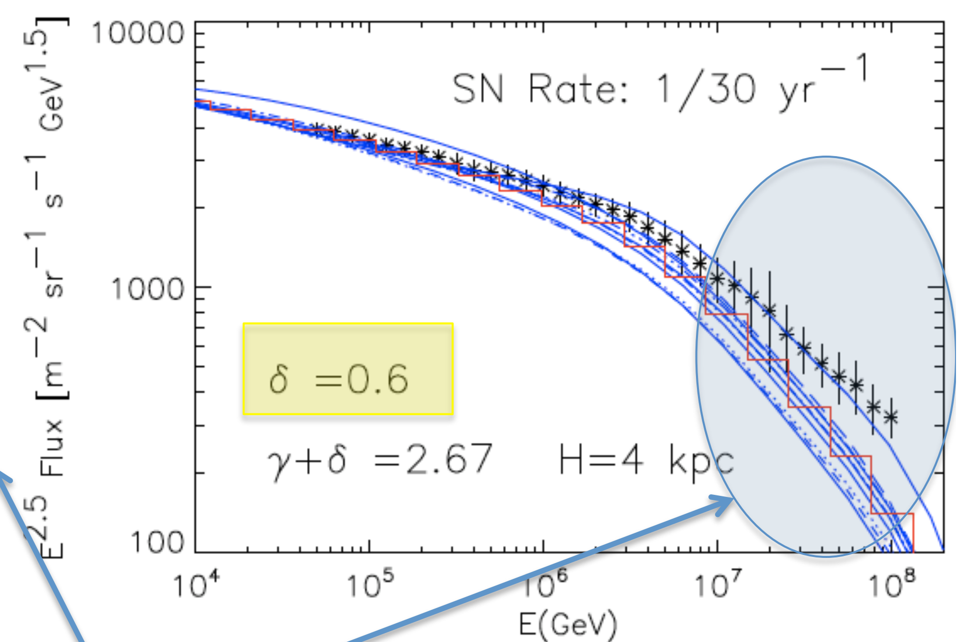
$$n_k(E) \propto E^{-\gamma}$$

namely in the regime of strong spallation the equilibrium spectrum reproduces the injection spectrum

EFFECT OF RANDOMNESS IN SNRs

Blasi & Amato 2012





DEFICIT POSSIBLY INDICATING THAT HERE IS WHERE EXTRAGALACTIC CR KICK IN

SIMPLE BUT EFFECTIVE MODEL

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right]+(v_W+v_A)\frac{\partial f}{\partial z}-\frac{d(v_W+v_A)}{dz}\frac{p}{3}\frac{\partial f}{\partial p}=q_{cr}(z,p)$$

v_W is the velocity of possible wind from the Galaxy, while v_A is the speed of the Alfven waves on which particles scatter.

Notice that while it is plausible to build models in which there is no wind (v_W=0), it is hard to eliminate the wave velocity. To do so one has to assume that waves propagate isotropically so that their local mean vanishes.

Let us assume a simple model in which v_W=0. Moreover let us assume that injection and interactions occur only in the disc at z=0.

$$q_{cr}(p,z)=\frac{\xi_{CR}E_{SN}\mathcal{R}_{SN}}{\pi R_d^2\mathcal{I}(\alpha)c(mc)^4}\left(\frac{p}{mc}\right)^{-\alpha}\delta(z)\equiv Q_0(p)\delta(z)$$

For $z>0$ and $z<0$ the equation reduces to

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right]+(v_W+v_A)\frac{\partial f}{\partial z}=0$$

That has a simple solution:

$$f(z,p)=f_0(p)\frac{1-e^{-\zeta(1-|z|/H)}}{1-e^{-\zeta}}\,,\qquad \zeta(p)\equiv\frac{v_A H}{D(p)}$$

Now recall that the Alfven speed simply changes sign in crossing the disc, so that

$$dv_A/dz=2v_A\delta(z)$$

Therefore integrating the transport equation across the disc one gets:

$$-2D(p)\left[\frac{\partial f}{\partial z}\right]_{z=0^+}-\frac{2}{3}v_A p\frac{df_0}{dp}=q_0(p)$$

And calculating the derivative from the expression for f(z,p):

$$\left[\frac{\partial f}{\partial z}\right]_{z=0^+}=\frac{v_A f_0}{D(p)}\frac{1}{\lambda(p)},\qquad \lambda(p)=1-\exp\left[\zeta(p)\right]$$

Finally one can solve for the spectrum in the disc $f_0(p)$:

$$f_0(p) = \frac{3}{2v_A} \int_p^\infty \frac{dp'}{p'} q_0(p) \exp\left[\int_p^{p'} \frac{dp''}{p''} \frac{3}{\lambda(p'')}\right]$$

As an exercise you can show that in the limit $v_A H/D(p) \ll 1$ this reduces to the well known diffusive solution that we have already obtained:

$$f_0^{\rm diff}(p) = Q_0(p) H/(2D(p))$$

On the other hand, in the opposite limit (advection dominated regime) one has $f_0(p)=Q_0(p)/2v_A$.

As another exercise you can determine the grammage for this simple model:

$$X(E) = \frac{n_d m_p h v}{2v_A}\left[1-\exp\left(-\frac{v_A H}{D(E)}\right)\right]$$

AT HIGH ENERGY THE GRAMMAGE DOES INDEED SCALE AS 1/D(E) WHILE AT LOW ENERGY BECOMES ENERGY INDEPENDENT

AT HIGH ENERGY ONE HAS:

$$X(E) = \frac{n_d m_p h v}{2 v_A} \frac{v_A H}{D(E)} = \frac{n_d m_p h v H}{2 D(E)}$$

BUT THIS MEANS THAT:

PROPAGATION TIME

$$X(E) = \rho_d \left(\frac{h}{H} \right) \frac{H^2}{2D(E)} v$$

Average density experienced by CR during their diffusive journey

PHYSICS QUESTION: WHY ARE THERE WAVES THAT CR CAN SCATTER UPON?

POSSIBILITY N. 1: WAVES HAVE BEEN INJECTED BY SNR EXPLOSIONS THROUGHOUT THE GALAXY. THEIR SPECTRUM IS $W(k) \sim k^{-s}$

$$\int_{k_0}^{\infty} dk\, W(k) = \eta_B = \frac{\delta B^2}{B_0^2} \Rightarrow W(k) = \eta_B (s-1) L_0 \left(\frac{k}{k_0}\right)^{-s} \qquad L_0 = 1/k_0$$

FOR A KOLMOGOROV SPECTRUM s=5/3 AND L_0=50 pc. THE DIFFUSION COEFFICIENT READS THEN (see Lecture 1):

$$D(p) = \frac{1}{3} r_L(p) v(p) \frac{1}{k_{res} W(k_{res})} \qquad k_{res} = \frac{1}{r_L(p)}$$

$$D(p)=\frac{1}{3}r_L v(p)\frac{1}{\frac{1}{r_L}\eta_B L_0(s-1)\left(\frac{L_0}{r_L}\right)^{-s}}=\frac{1}{3}r_L^{2-s}\frac{v(p)}{\eta_B L_0^{1-s}(s-1)}$$

THE TYPE OF CASCADE IN K SPACE FROM K_0 TO SMALLER SPATIAL SCALES IS NOT WELL UNDERSTOOD (NON LINEAR LANDAU DAMPING), BUT TWO PHENOMENOLOGIES ARE OFTEN STUDIED:

KOLMOGOROV PHENOMENOLOGY → S=5/3

KRAICHNAN PHENOMENOLOGY → S=3/2

FOR S=5/3 ONE HAS:

$$D(p)=4.2\times 10^{27}\frac{\beta}{\eta_B}(ZB_\mu)^{-1/3}\left(\frac{p}{m_p c}\right)^{1/3}\ cm^2/s$$

FOR s=3/2:

$$D(p)=4.5\times 10^{26}\frac{\beta}{\eta_B}(ZB_\mu)^{-1/2}\left(\frac{p}{m_p c}\right)^{1/2}\ cm^2/s$$

POSSIBILITY N. 2

THE DIFFUSION PROPERTIES ARE DETERMINED BY THE STREAMING OF COSMIC RAYS THEMSELVES.

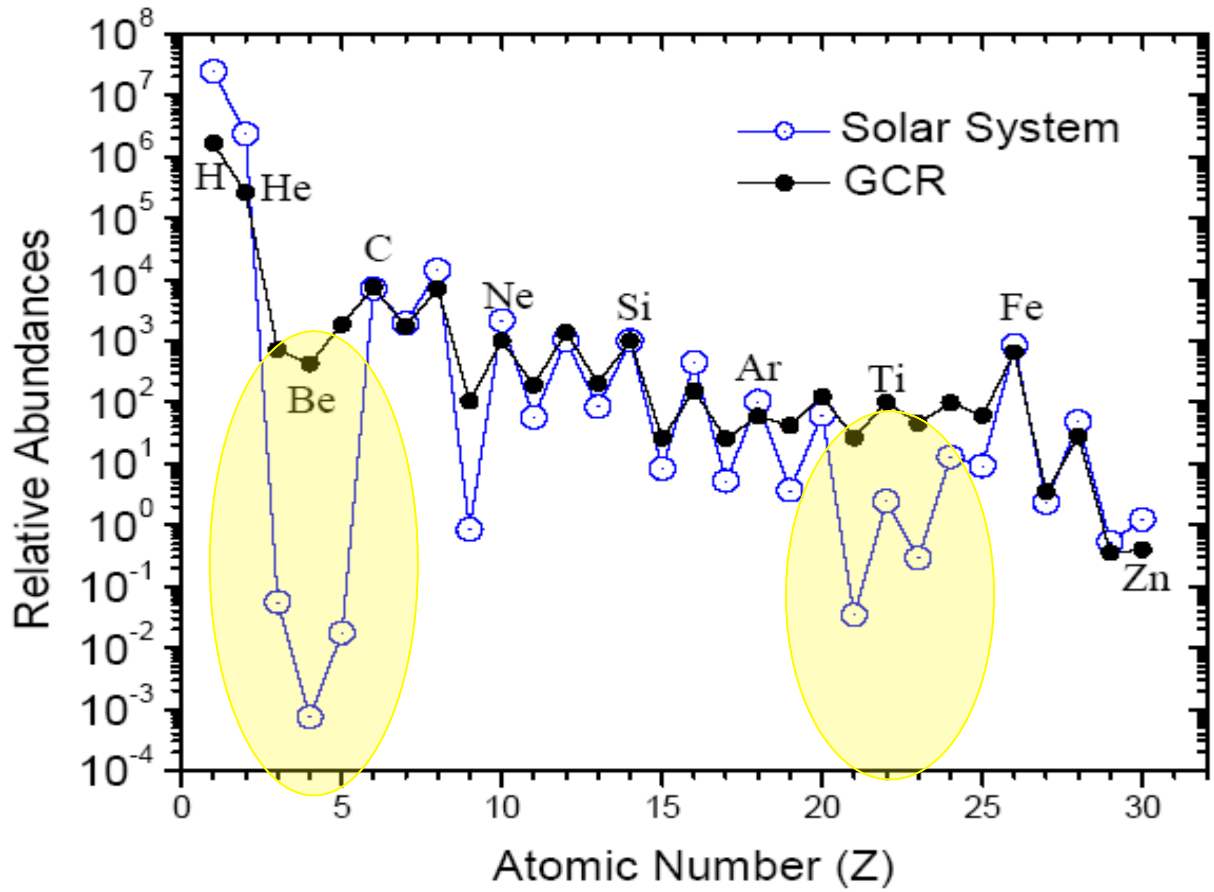
WAVES ARE GENERATED AT A (GROWTH RATE) OF:

$$\Gamma_{cr}(k) = \frac{16\pi^2}{3}\frac{v_A}{\mathcal{F}(k)B_0^2}\left[p^4 v(p)\frac{\partial f}{\partial z}\right]_{p=qB_0/kc}$$

AND DAMPED WITH A RATE THAT DEPENDS ON WHETHER THE PLASMA IS APPRECIABLY NEUTRAL (ION-NEUTRAL DAMPING) OR NOT (NON LINEAR LANDAU DAMPING).

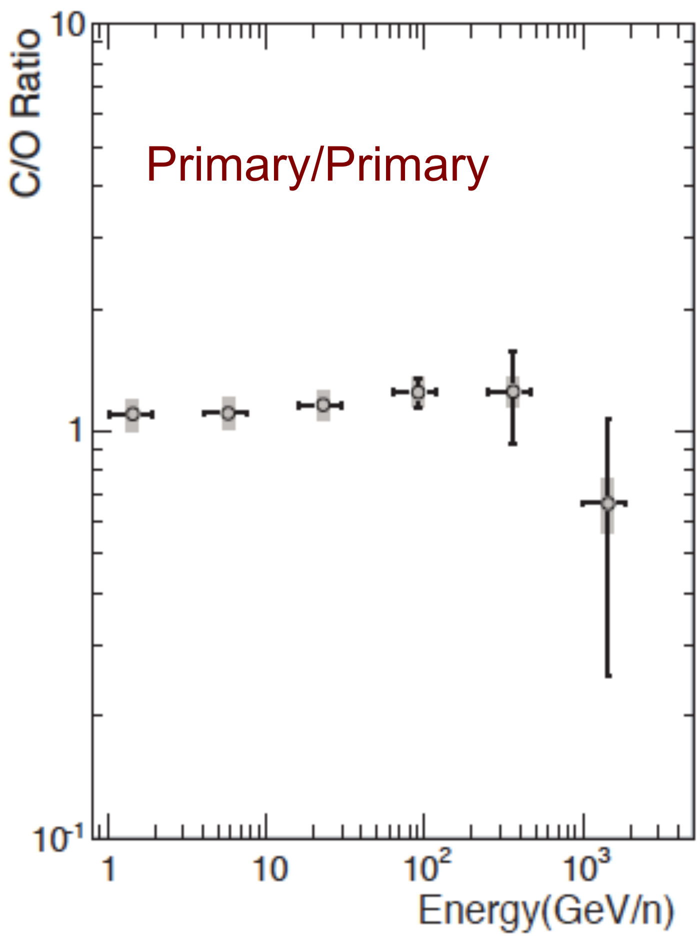
ALREADY IN THE 70s THIS POSSIBILITY WAS INVESTIGATED (Skilling, Cesarsky, Wentzel, Holmes). IT MAY BE RESPONSIBLE FOR DIFFUSION OF COSMIC RAYS UP TO SEVERAL HUNDRED GeV.

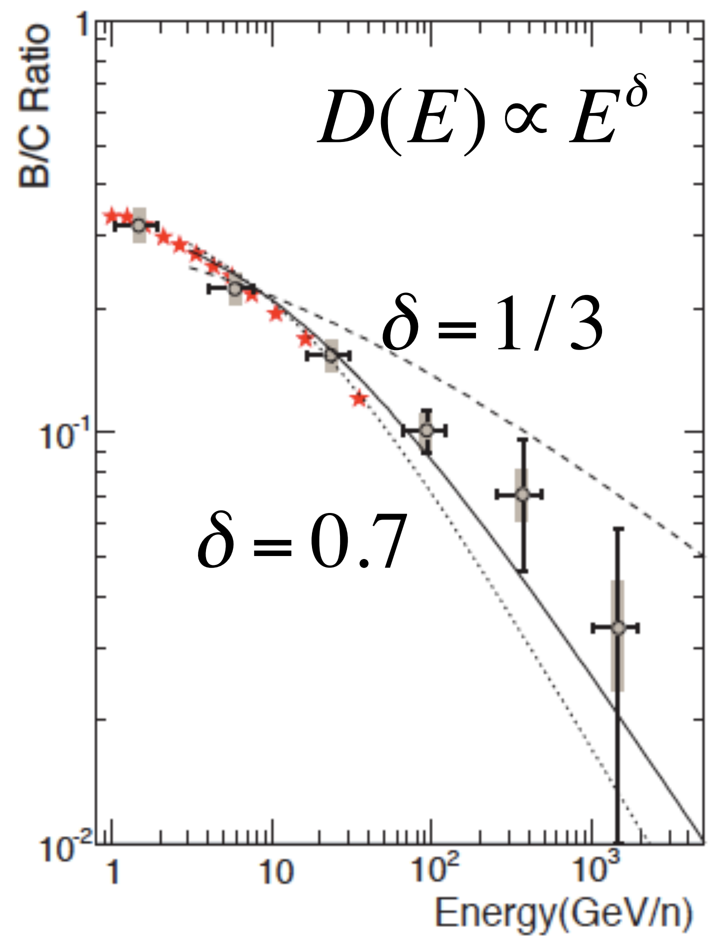
SPALLATION



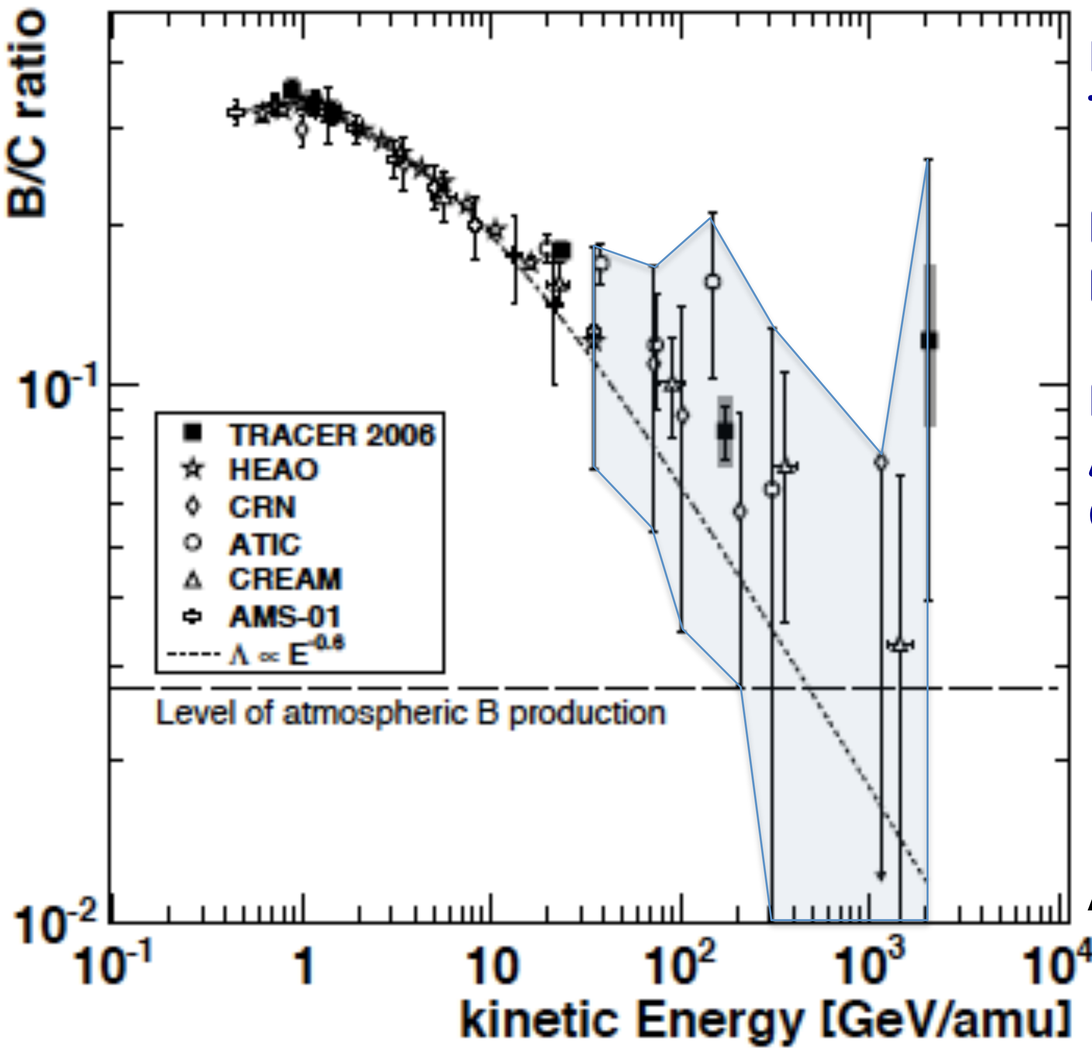
Solar System
GCR
H
He
Be
C
Ne
Si
Ar
Ti
Fe
Zn
Relative Abundances
Atomic Number (Z)
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30

Secondary/Primary ratios





THE B/C RATIO AS A DIFFUSION INDICATOR



IN PRINCIPLE B/C ~ 1/D(E) IN THE HIGH RIGIDITY REGIME

BUT UNCERTAINTIES ARE STILL LARGE

NOT EASY TO DISCRIMINATE AMONG DIFFERENT DIFFUSION COEFFICIENTS

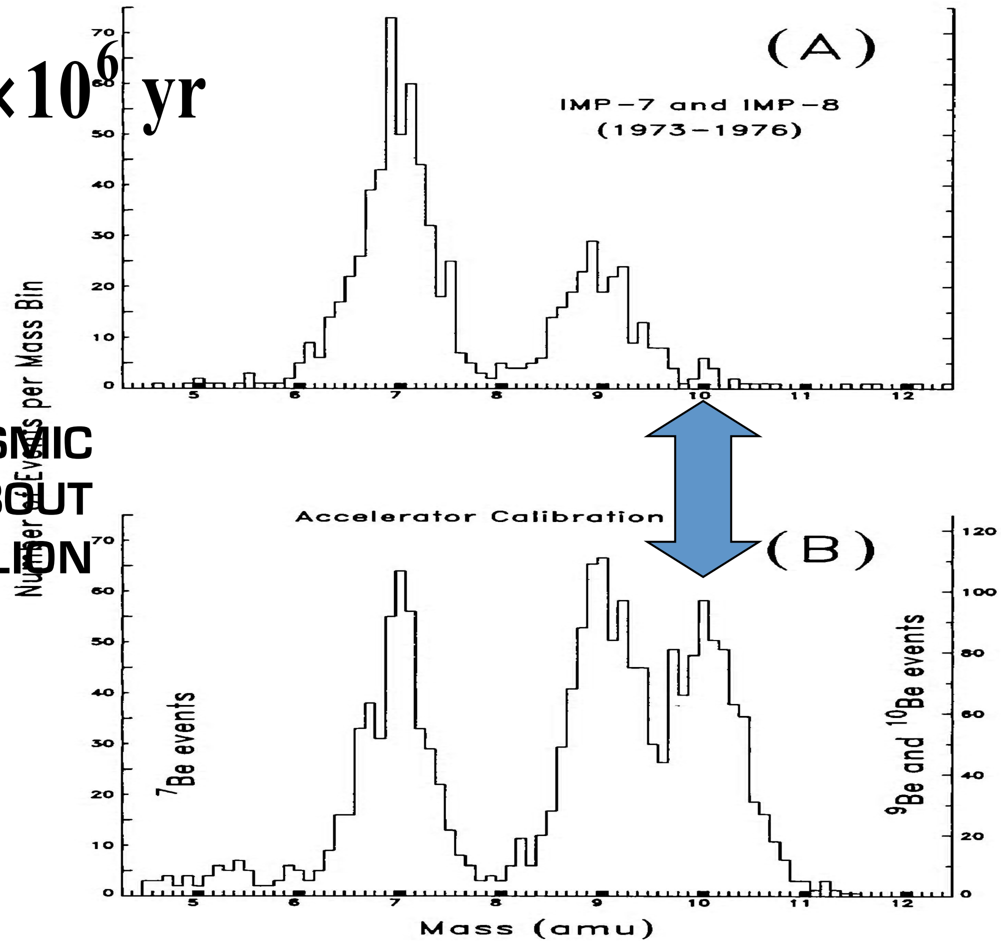
Adapted from Obermeier et al. 2011

Unstable Elements

Simpson and Garcia-Munoz 1988

$$\tau_{^{10}Be} = 1.5 \times 10^6 \text{ yr}$$

AGE OF COSMIC RAYS ABOUT 10-15 MILLION YEARS



Balloon flights
Cosmic Rays

Laboratory
Experiment

Simple scaling relations

IN ORDER FOR INJECTION AND ESCAPE TO LEAD TO A STATIONARY SITUATION ONE HAS TO REQUIRE:

$$n(E) \approx q(E)\tau_{esc}(E) \sim E^{-\gamma-\delta}$$

DURING PROPAGATION CR PRODUCE SECONDARY NUCLEI AT A RATE

$$q_{\sec}(E) \approx Y_{sp} n(E) \sigma_{sp} n_{gas} c$$

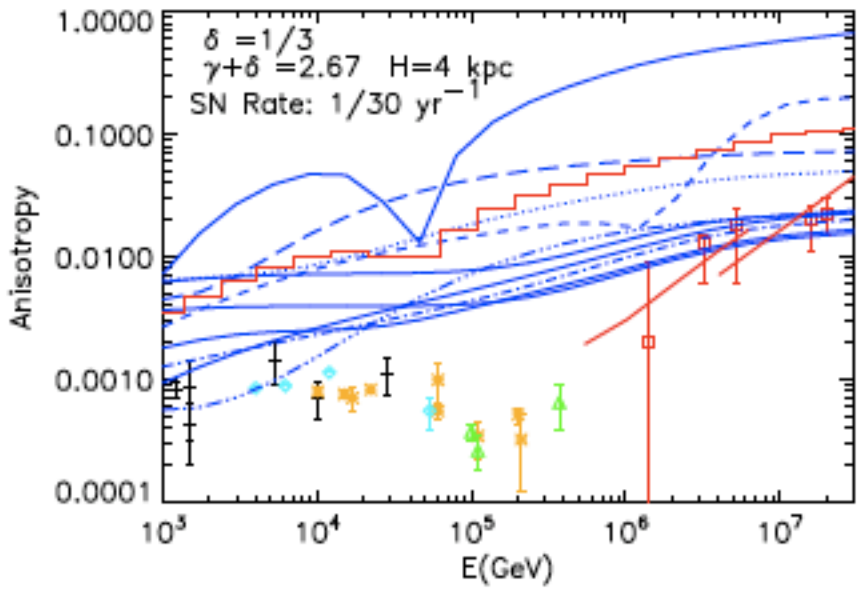
AND AGAIN IN A STATIONARY REGIME:

$$n_{\sec}(E) \approx q_{esc}(E)\tau_{esc}(E) \quad \Rightarrow \quad \frac{n_{\sec}(E)}{n(E)} \sim x(E) \sim \frac{1}{D(E)}$$

BASED ON PRESENT DATA ON B/C IT IS STILL HARD TO DISCRIMINATE BETWEEN DIFFERENT VALUES OF THE SLOPE OF D(E)

LARGE SCALE CR ANISOTROPY

ANISOTROPY DOMINATED BY NEARBY AND MOST RECENT SOURCES.



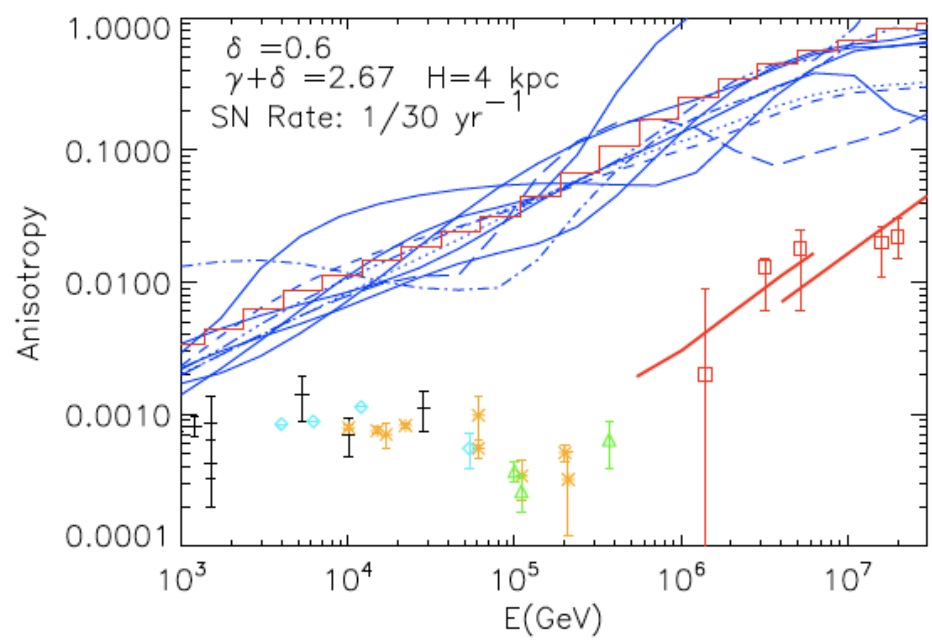
Naïve expectation:

$$\delta_A = \frac{3}{2^{3/2}} \frac{1}{\pi^{1/2}} \frac{D(E)}{Hc}$$

proportional to E^d

d=1/3

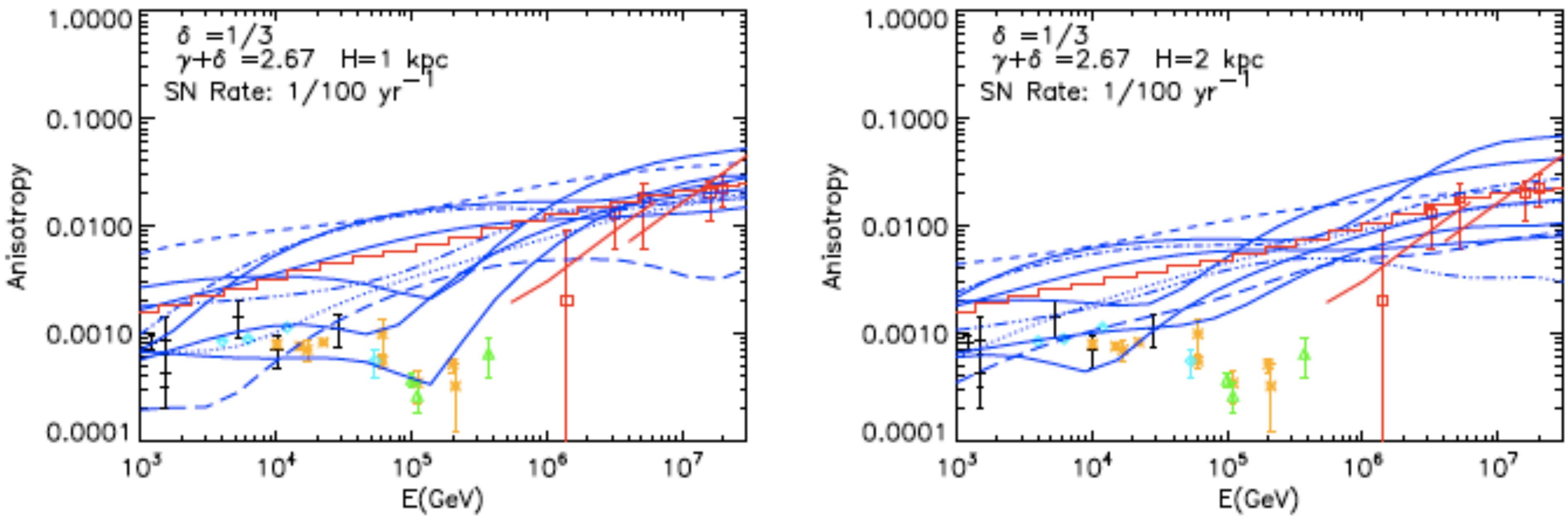
d=0.6



Blasi & Amato 2011

Dependence of Halo Size

PB and Amato 2011



PROPAGATION OF ELECTRONS

ELECTRONS' PROPAGATION IS ESSENTIALLY DIFFERENT FROM THAT OF NUCLEI IN THAT ENERGY LOSSES ARE VERY IMPORTANT. LOSSES ARE DOMINATED BY SYNCHROTRON AND ICS.

THE GREEN FUNCTION FOR ELECTRONS IS:

$$\mathcal{G}(x,y,z,t,E;x',y',z',t',E') =$$

$$\frac{1}{|b(E)|(4\pi\lambda)^{3/2}} \exp\left[-\frac{(x-x')^2+(y-y')^2}{4\lambda}\right] \delta(t-t'-\tau) \sum_{n=-\infty}^{+\infty} \exp\left[-\frac{(z-z'_n)^2}{4\lambda}\right]$$

$$b(E) = -AE^2 \qquad \tau(E,E') = -\int_E^{E'} \frac{d\epsilon}{b(\epsilon)} \qquad \lambda(E,E') = \int_{E'}^{E} d\epsilon \frac{D(\epsilon)}{b(\epsilon)}$$

AND AUTOMATICALLY SATISFIES THE FREE ESCAPE BOUNDARY CONDITION AT |z| =H. HERE WE HAVE ASSUMED THAT ESCAPE TAKES PLACE ONLY AT |z|=H.

THE TOTAL FLUX AT EARTH IS THE SUM OVER ALL SNR EXPLODED AT DIFFERENT TIMES IN THE GALAXY.

SIMPLE SCALINGS

THE TRANSITION FROM A DIFFUSION DOMINATED TO A LOSS DOMINATED REGIME OCCURS WHERE:

$$\frac{H^2}{D(E)} = \frac{E}{|b(E)|}$$

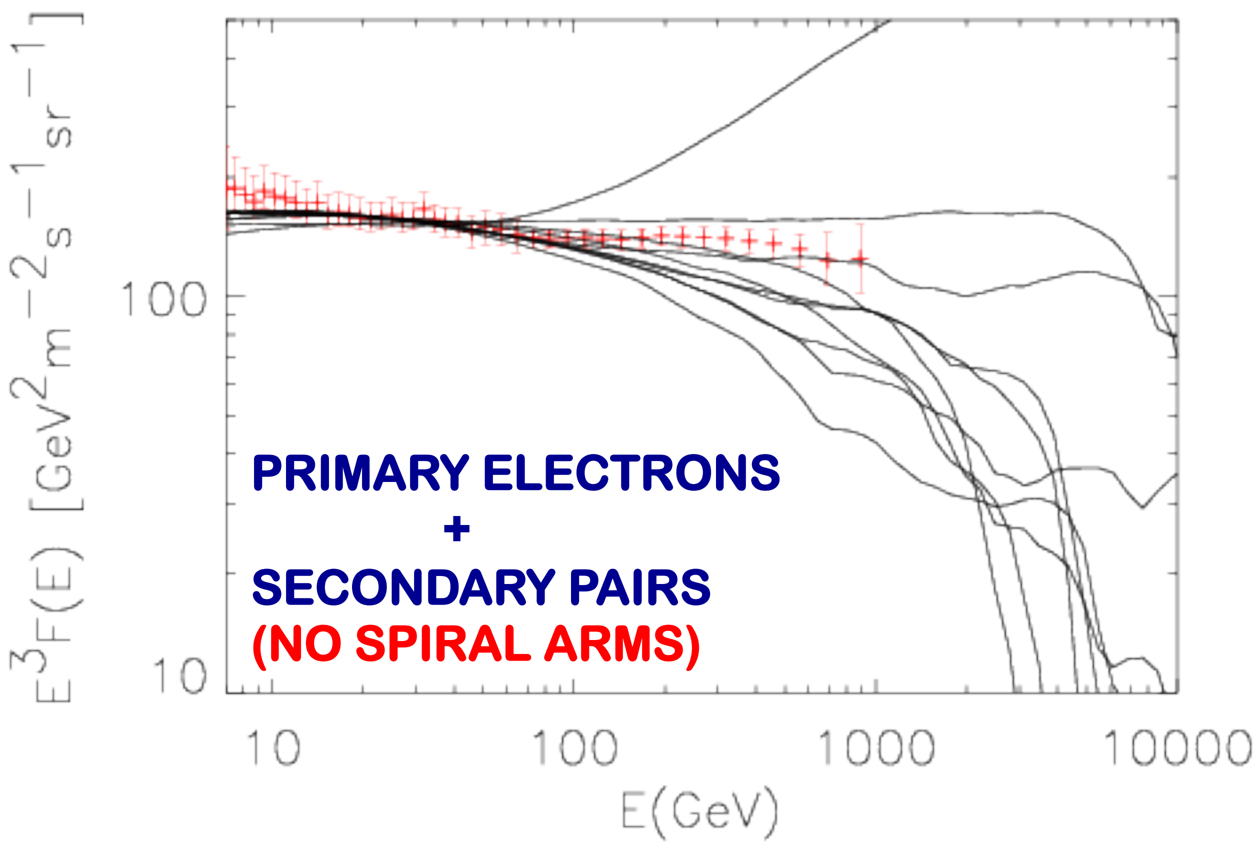
FOR TYPICAL VALUES OF THE PARAMETERS THIS OCCURS AROUND FEW GeV.

IN THE LOSS DOMINATED REGIME:

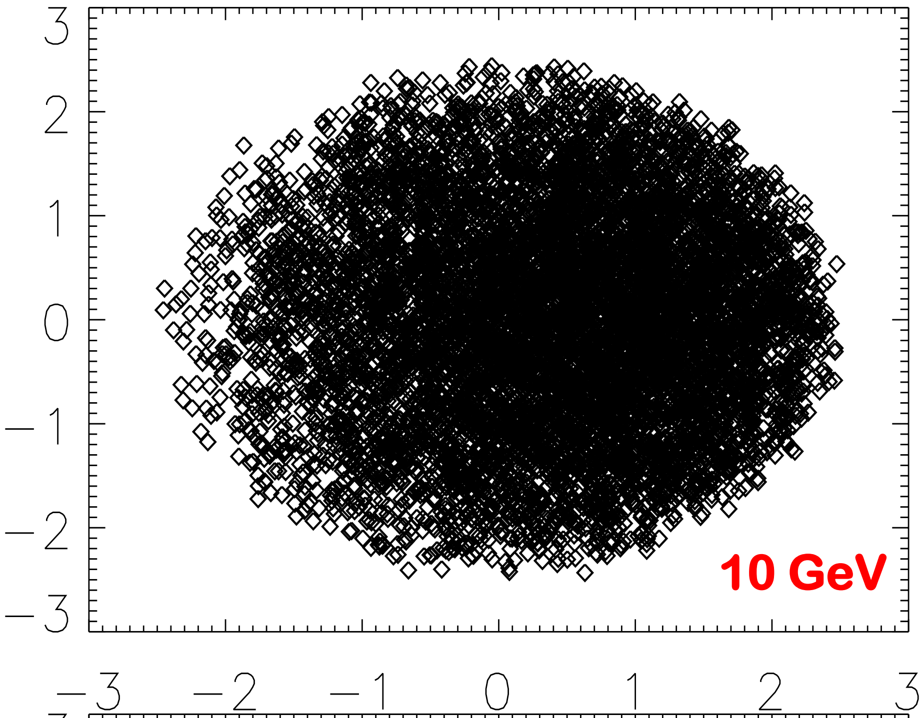
$$n(E) \sim \frac{Q(E)\tau_{loss}(E)}{\left[4D(E)\tau_{loss}(E)\right]^{1/2}} \sim E^{-\gamma-1-\delta/2+1/2} \sim E^{-\gamma-1/2-\delta/2}$$

TO BE COMPARED WITH THE DIFFUSION DOMINATED REGIME: $n(E) \sim E^{-g-d}$.

ELECTRONS FROM SNRs

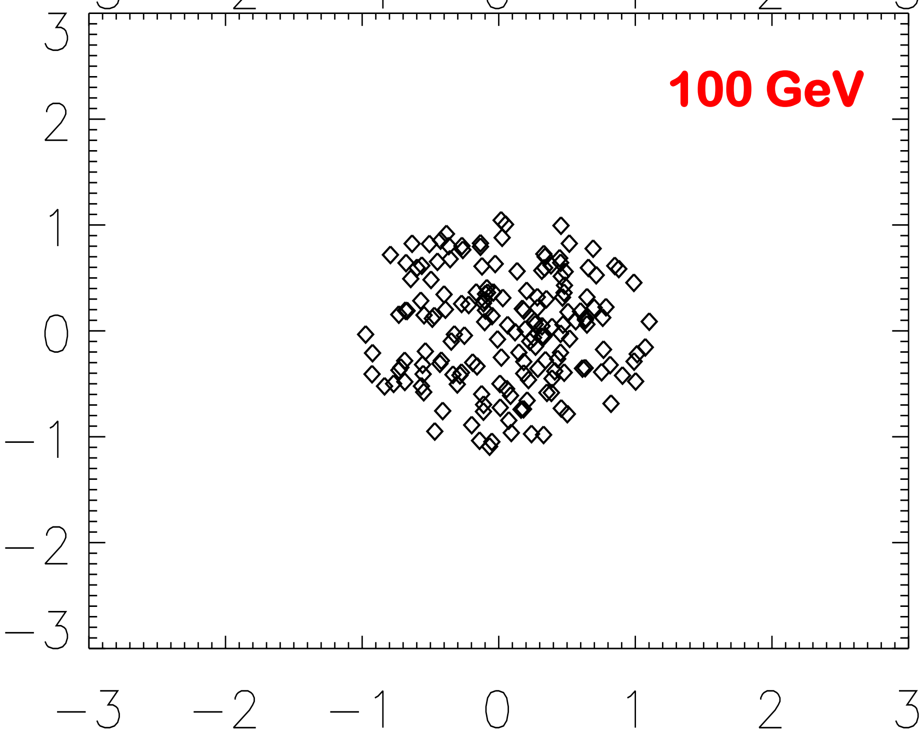


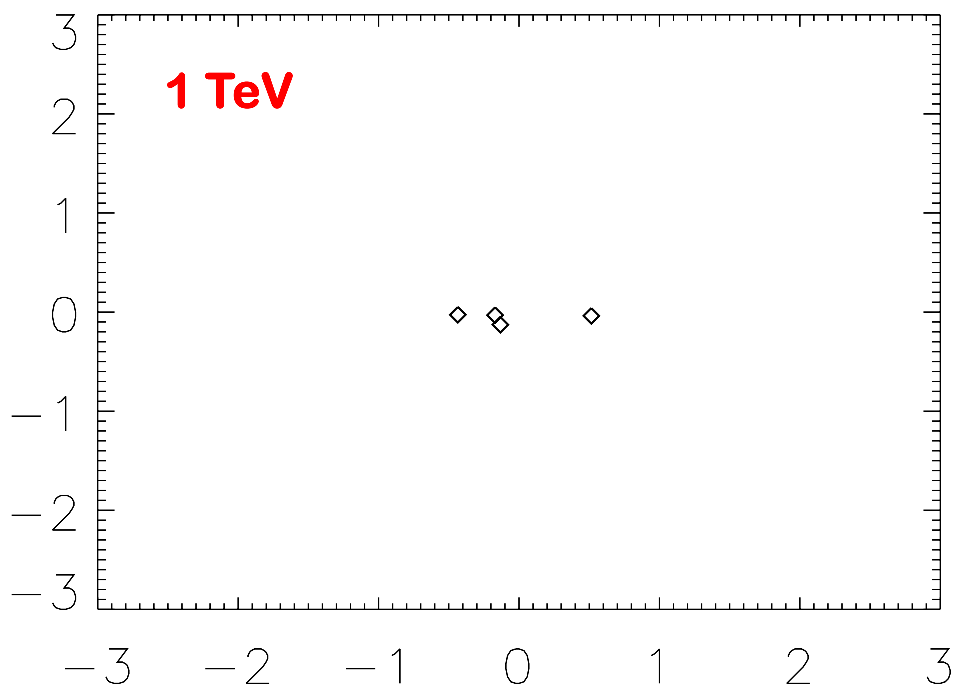
PB & Amato 2012



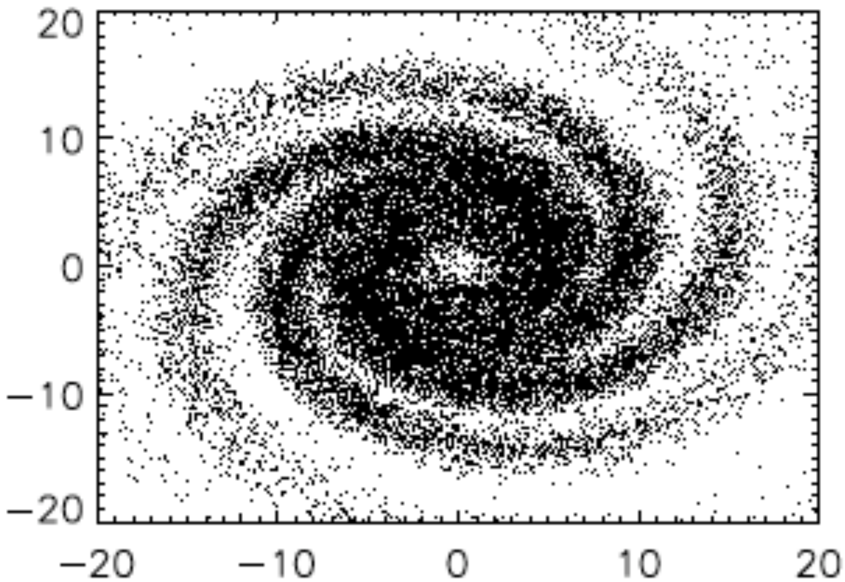
NUMBER OF ELECTRON SOURCES CONTRIBUTING AT GIVEN ENERGIES

PB & Amato 2010

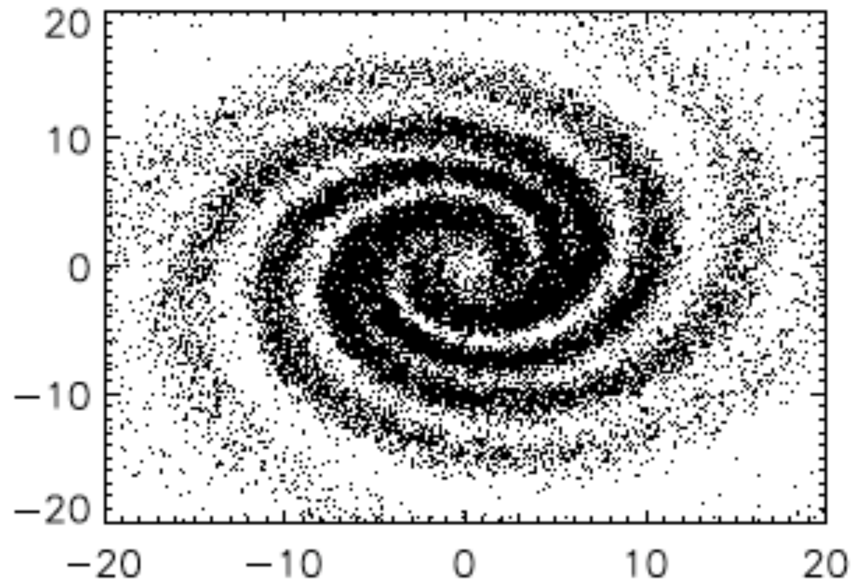




The effect of spiral arms



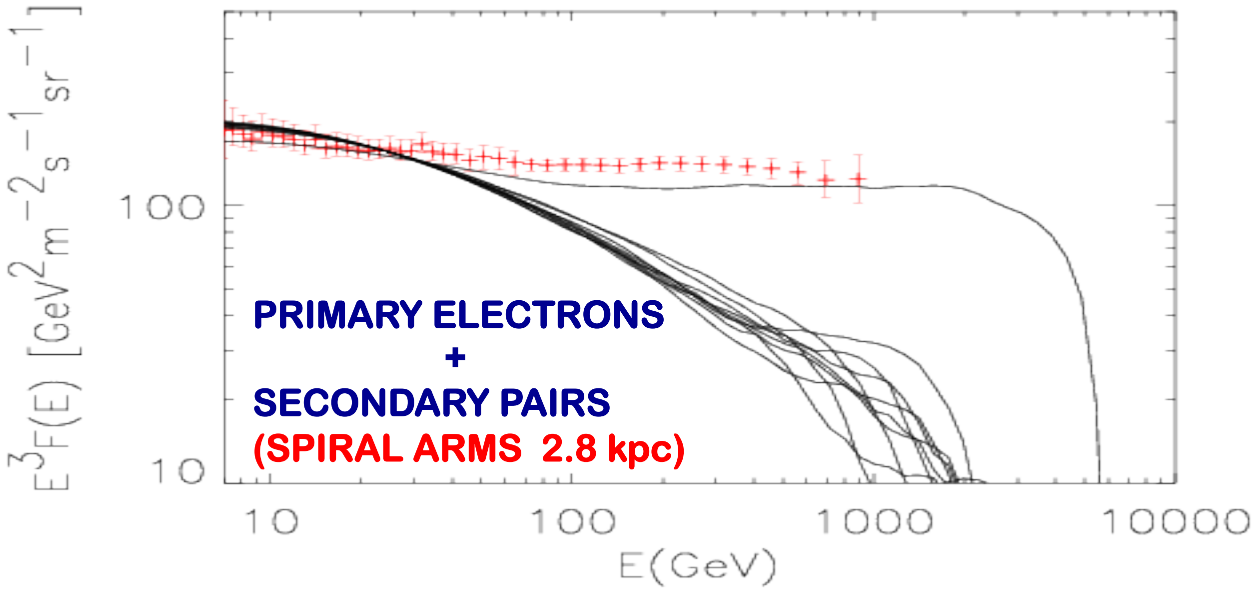
BROAD ARMS



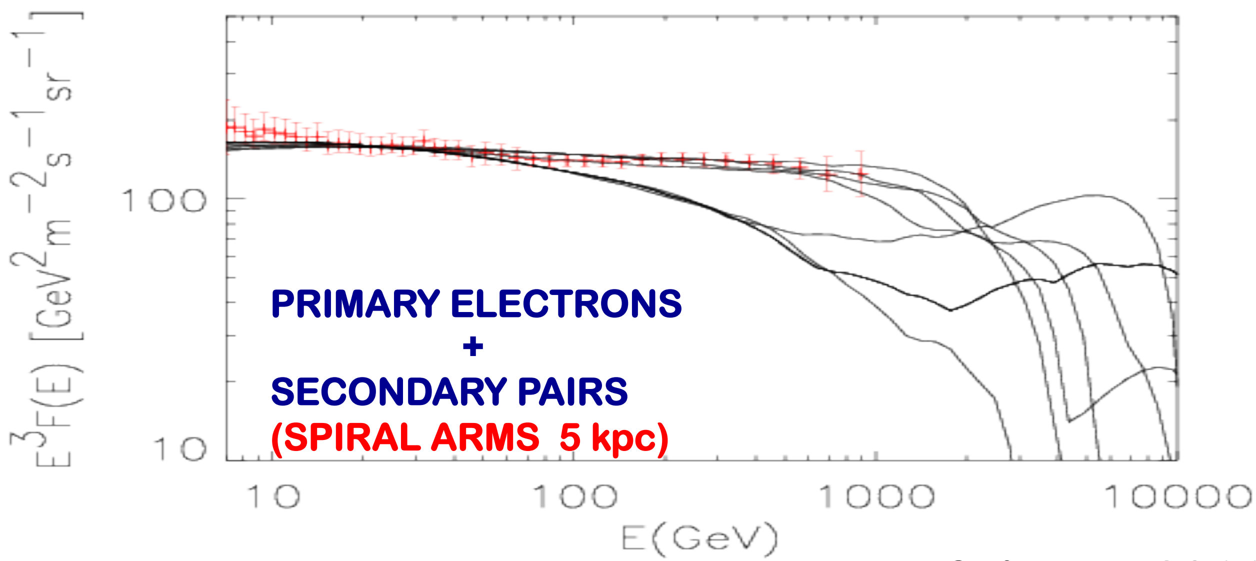
TIGHT ARMS

The effect of spiral arms

TIGHT SPIRAL



BROAD SPIRAL



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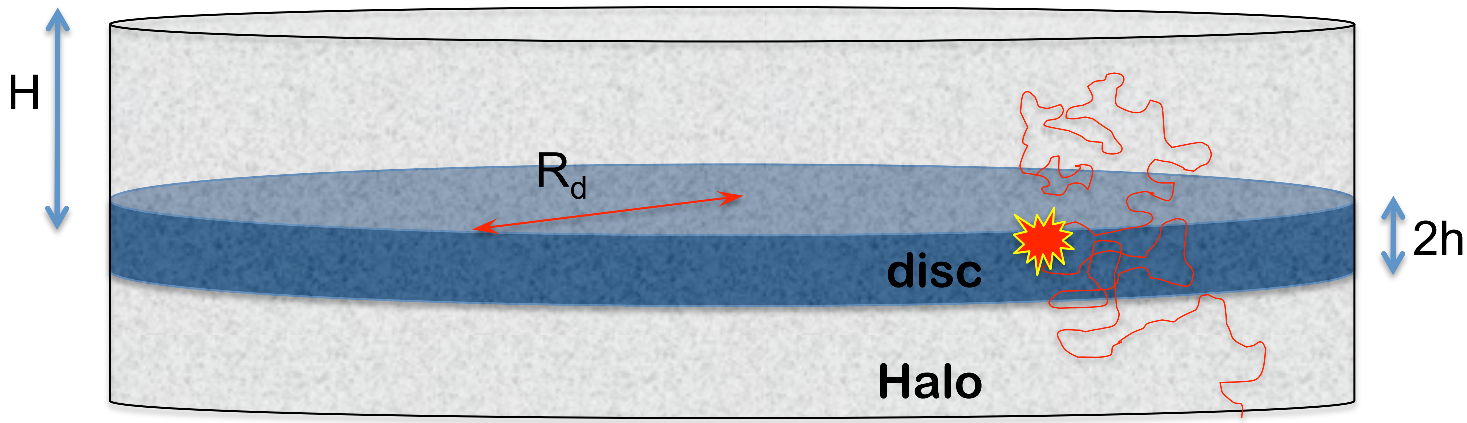
Secondary positrons (1)



PRIMARY COSMIC RAY SPECTRUM AT EARTH

$$n_{CR}(E) = \frac{N(E)\mathcal{R}}{2\pi R_d^2}\frac{H}{D(E)} = \frac{N(E)\mathcal{R}}{2H\pi R_d^2}\frac{H^2}{D(E)} \propto E^{-\gamma-\delta}$$

SPECTRUM OF PRIMARY ELECTRONS AT EARTH

$$n_e(E) \approx \frac{N(E)\Re\tau_{loss}(E)}{\sqrt{D(E)\tau_{loss}(E)}} \propto E^{-\gamma-1/2-\delta/2}$$

IF ENERGY LOSSES ARE DOMINANT UPON DIFFUSION (TYPICALLY E>10 GeV

Secondary positrons (2)

INJECTION RATE OF SECONDARY POSITRONS

$$q_{e^+}(E')dE' = n_{CR}(E)dE\, n_H\, \sigma_{pp}\, c \propto E^{-\gamma-\delta}$$

EQUILIBRIUM SPECTRUM OF SECONDARY POSITRONS (AND ELECTRONS) AT EARTH

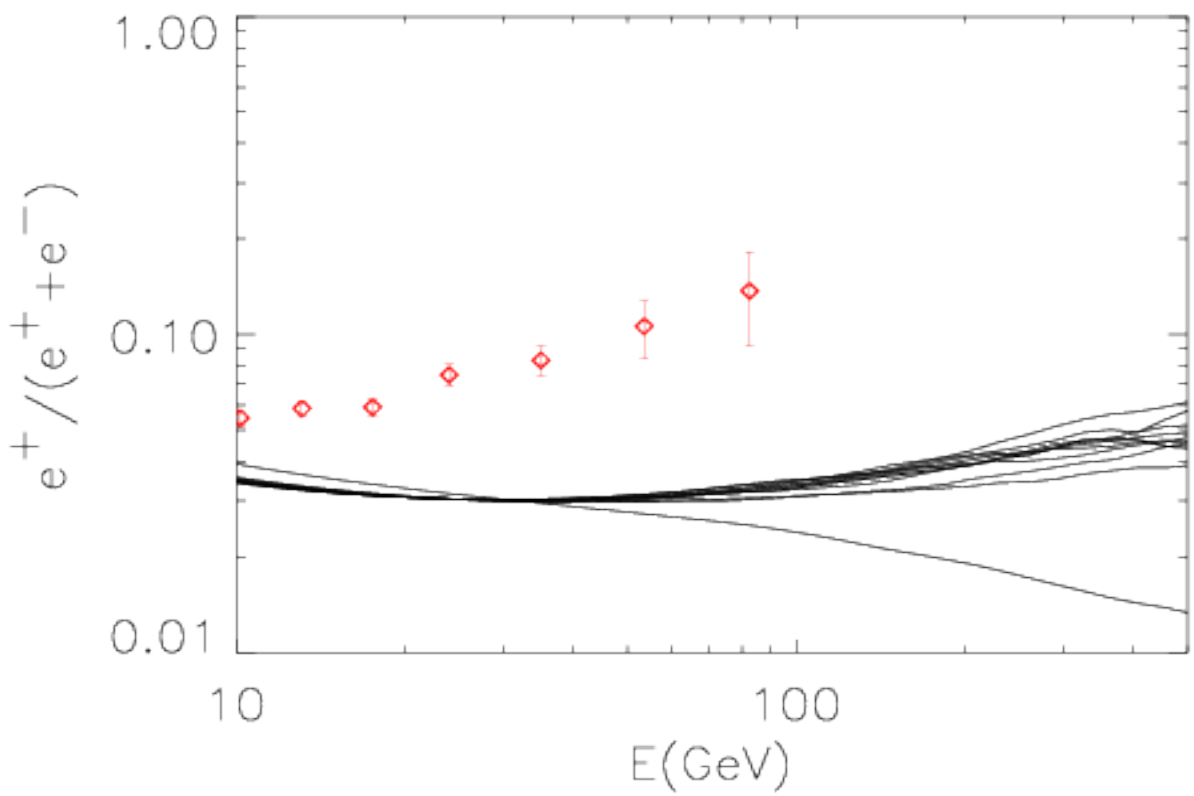
$$n_{e^+}(E) \approx \frac{q_{e^+}(E)\tau_{loss}(E)}{\sqrt{D(E)\tau_{loss}(E)}} \propto E^{-\gamma-1/2-3\delta/2}$$

POSITRON FRACTION

$$\boxed{\frac{\Phi_{e^+}}{\Phi_{e^+}+\Phi_{e^-}} \approx \frac{\Phi_{e^+}}{\Phi_{e^-}} \propto E^{-\delta}}$$

MONOTONICALLY DECREASING FUNCTION OF ENERGY

THE POSITRON FRACTION FOR THE CASE OF TIGHT SPIRAL ARMS



INTERESTING NEW FINDINGS

PAMELA FINDS A GROWING POSITRON RATIO WITH ENERGY (4^{TH} LECTURE)

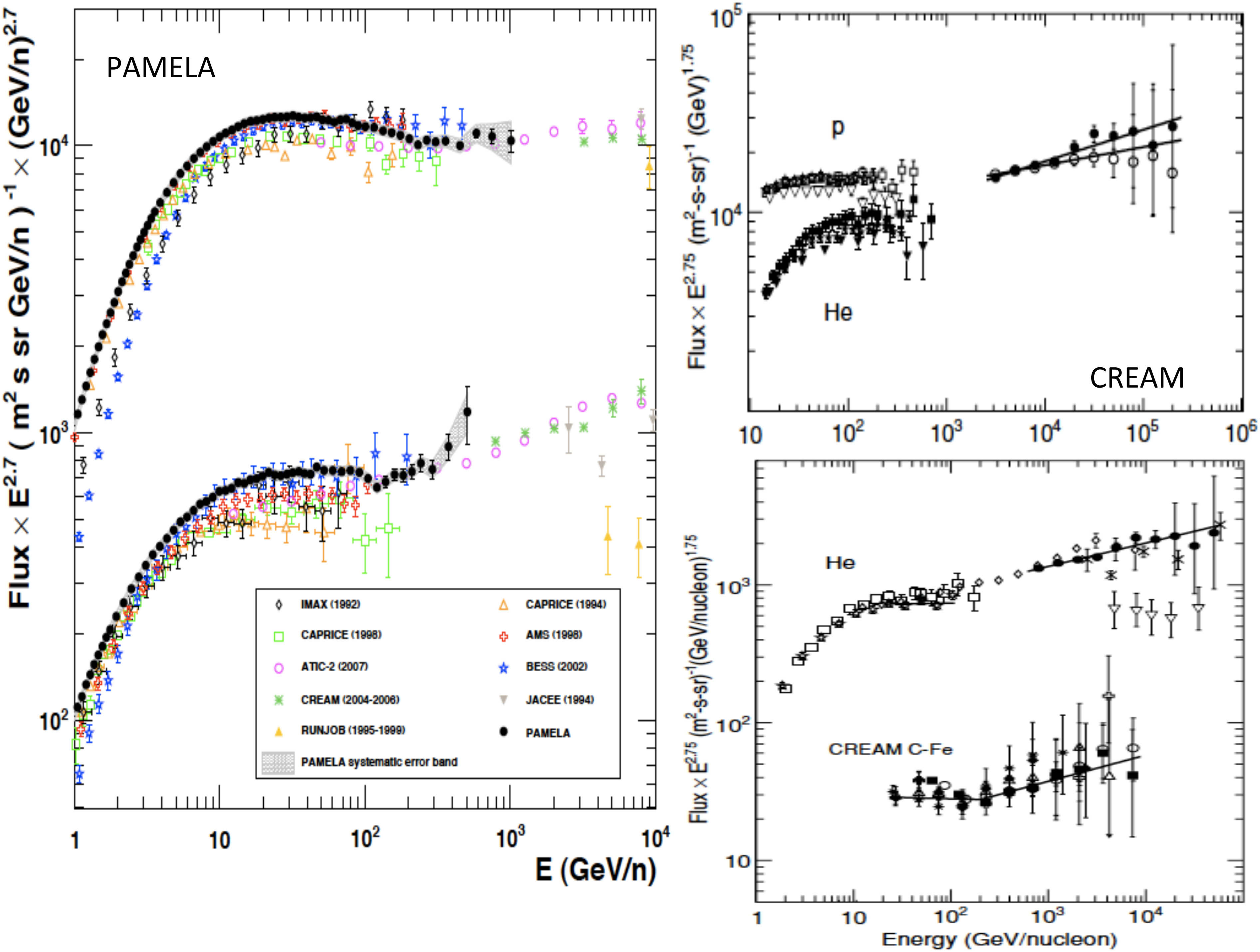
PAMELA AND CREAM FIND THAT THE SPECTRUM OF HYDROGEN AND HELIUM ARE STEEPER BELOW ~200 GeV/n THAN THEY ARE ABOVE THIS RIGIDITY

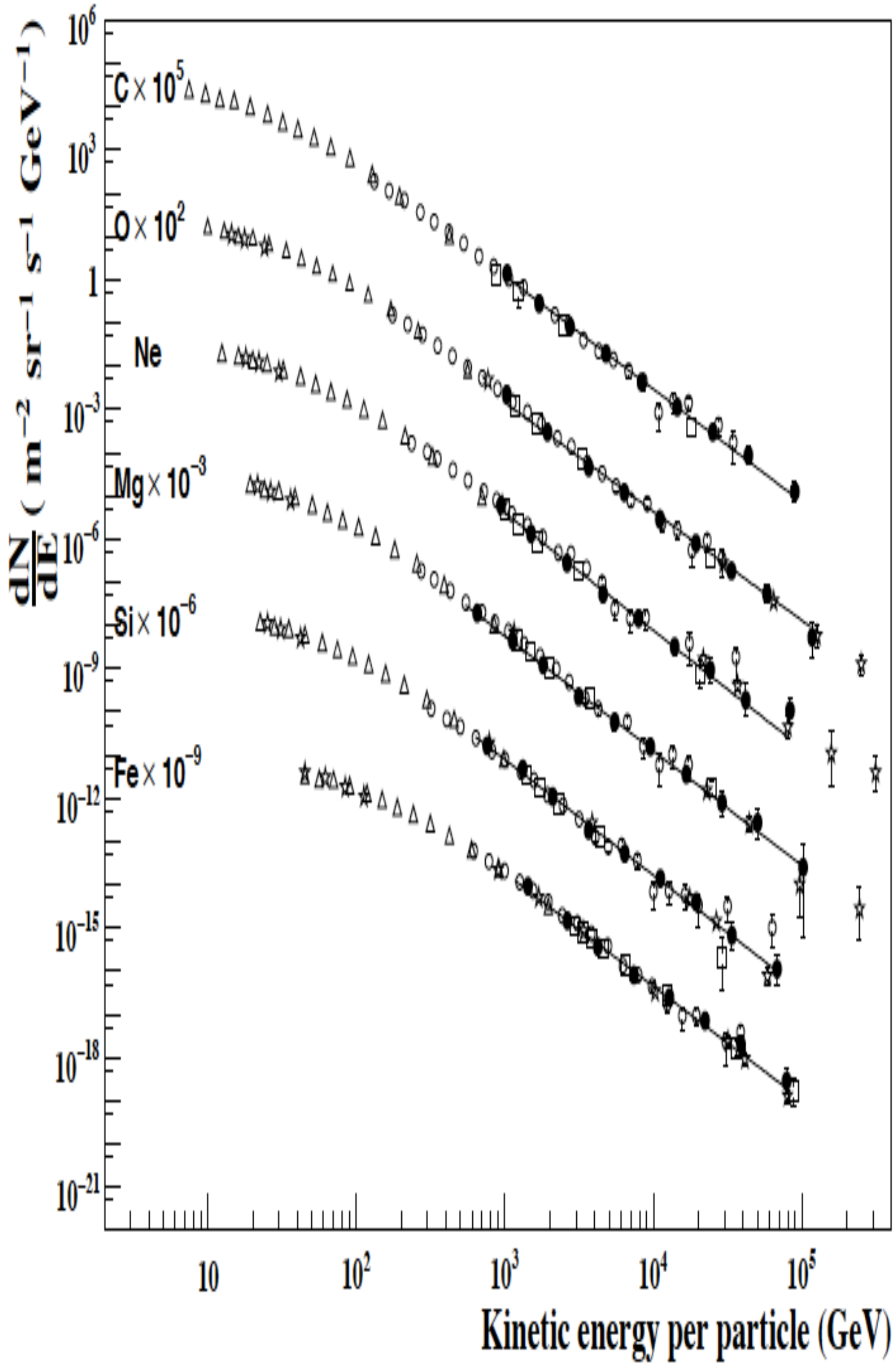
PAMELA AND CREAM BOTH FIND THE HELIUM SPECTRUM HARDER THAN THE PROTON SPECTRUM

AN ANALYSIS OF THE GAMMA RAY EMISSION FROM SOME CLOUDS IN THE GOULD BELT SHOWS A CR SPECTRUM BELOW 200 GeV AS STEEP AS THE ONE MEASURED BY PAMELA

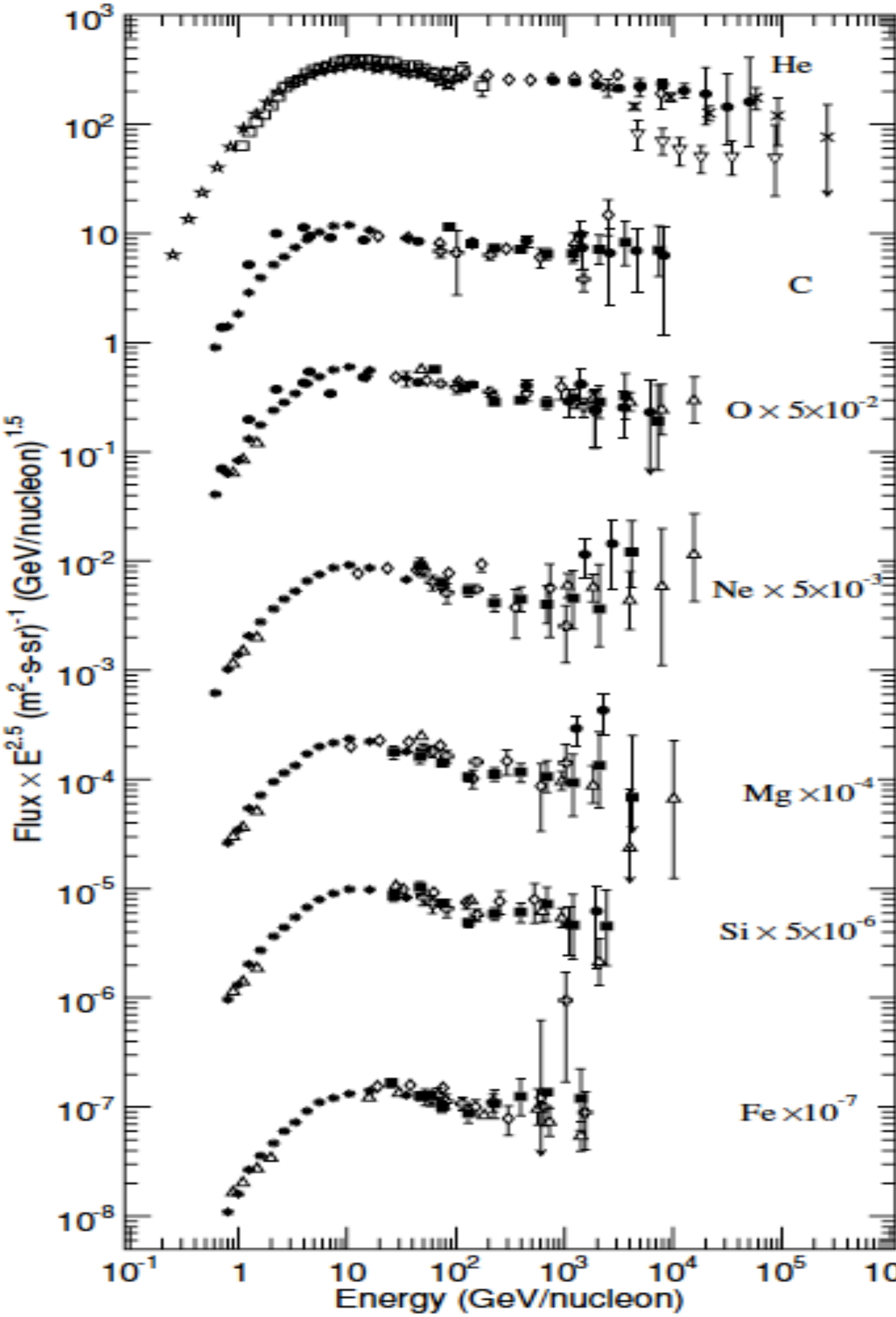
ELECTRON SPECTRUM WIGGLES?

ICECUBE, MILAGRO FIND PUZZLING SMALL SCALE ANISOTROPIES, PROBABLY DUE TO THE BREAKING OF PERFECTLY DIFFUSIVE PROPAGATION IN THE VICINITY OF THE SOLAR SYSTEM.


PAMELA
Flux × E^2.7 (m^2 s sr GeV/n)^-1 × (GeV/n)^2.7
E (GeV/n)
IMAX (1992)
CAPRICE (1994)
CAPRICE (1998)
AMS (1998)
ATIC-2 (2007)
BESS (2002)
CREAM (2004-2006)
JACEE (1994)
RUNJOB (1995-1999)
PAMELA
PAMELA systematic error band
p
He
CREAM
Flux × E^2.75 (m^2-s-sr)^-1 (GeV)^1.75
He
CREAM C-Fe
Flux × E^2.75 (m^2-s-sr)^-1 (GeV/nucleon)^1.75
Energy (GeV/nucleon)

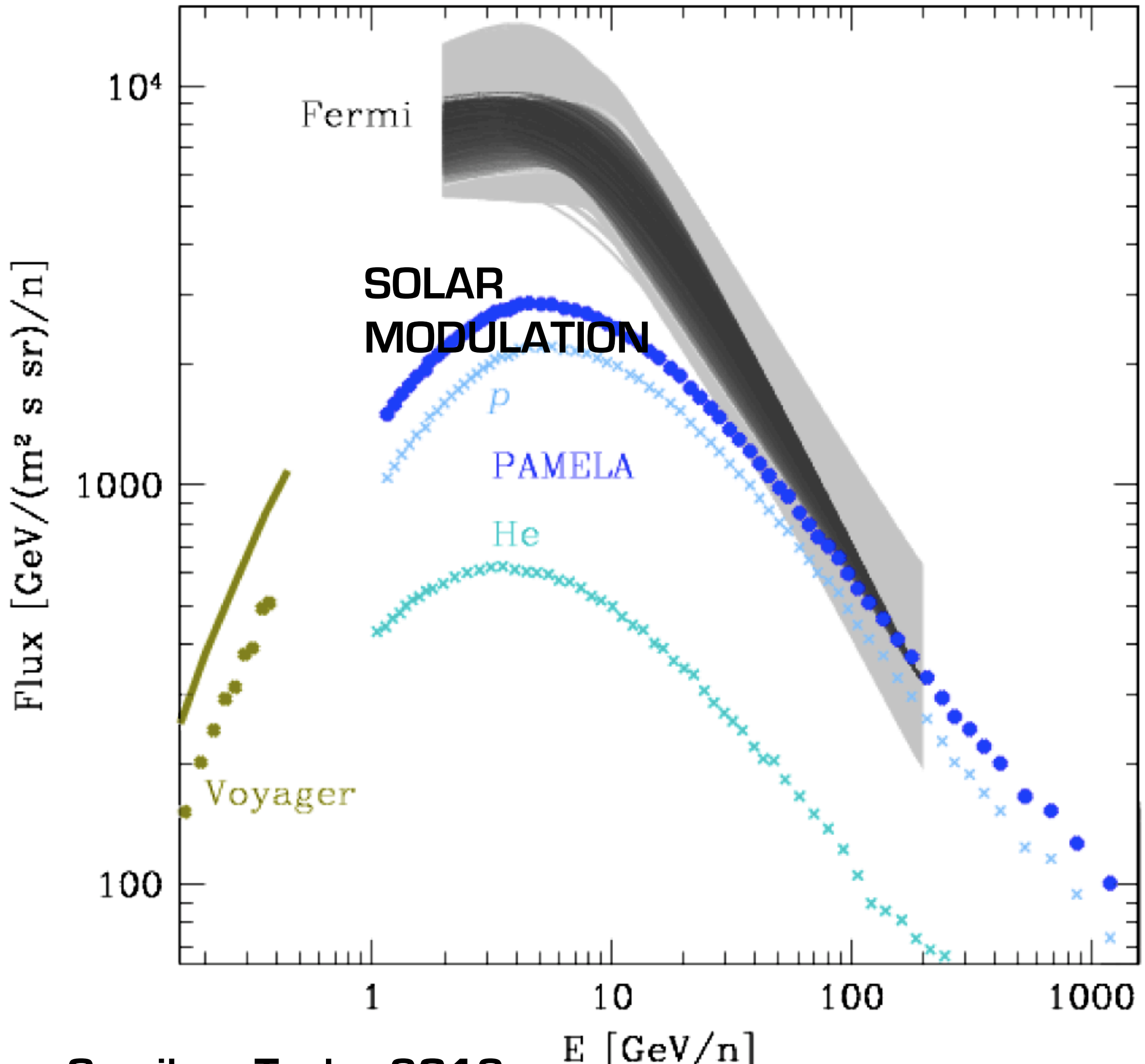


$C \times 10^5$
$O \times 10^2$
Ne
$Mg \times 10^{-3}$
$Si \times 10^{-6}$
$Fe \times 10^{-9}$
$\frac{dN}{dE}$ (m^{-2} sr^{-1} s^{-1} GeV^{-1})
Kinetic energy per particle (GeV)



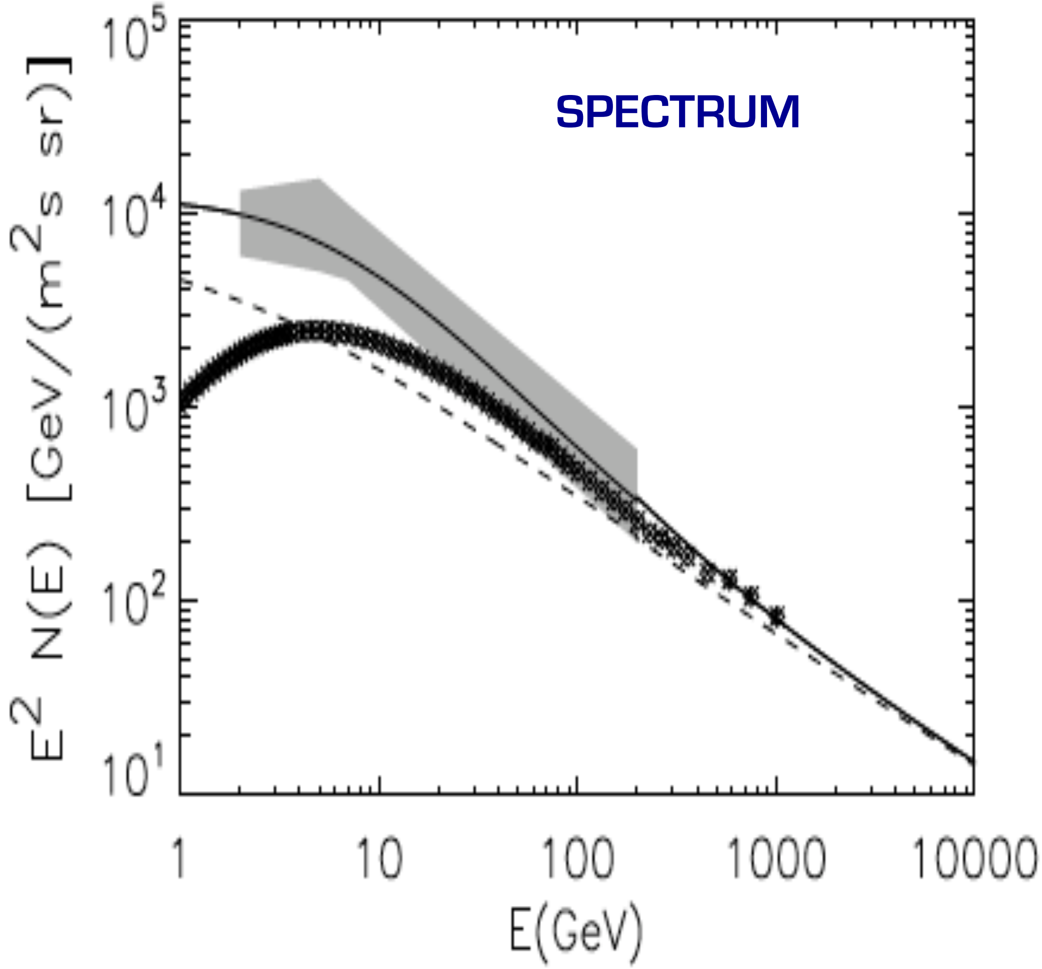
He
C
$O \times 5\times10^{-2}$
$Ne \times 5\times10^{-3}$
$Mg \times 10^{-4}$
$Si \times 5\times10^{-6}$
$Fe \times 10^{-7}$
Flux $\times$ $E^{2.5}$ $(m^2\text{-}s\text{-}sr)^{-1}$ $(GeV/nucleon)^{1.5}$
Energy (GeV/nucleon)

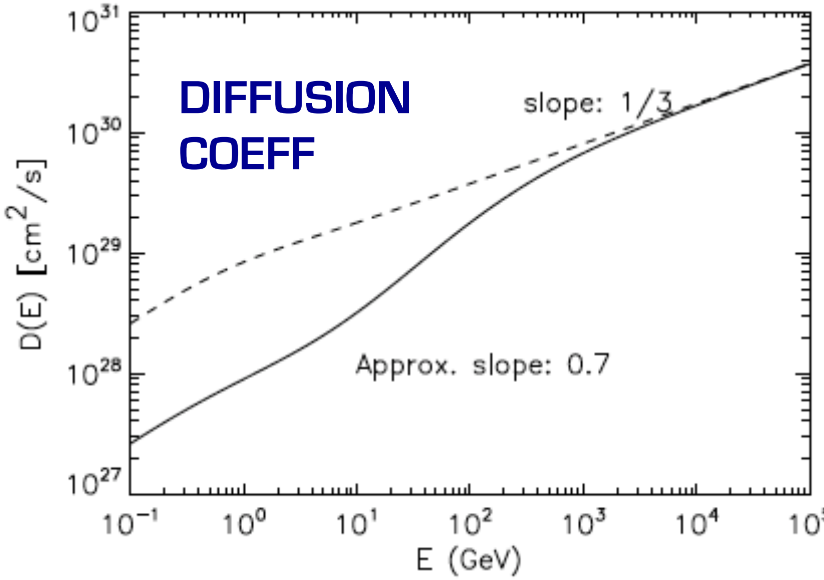
CLOUDS IN THE GOULD BELT

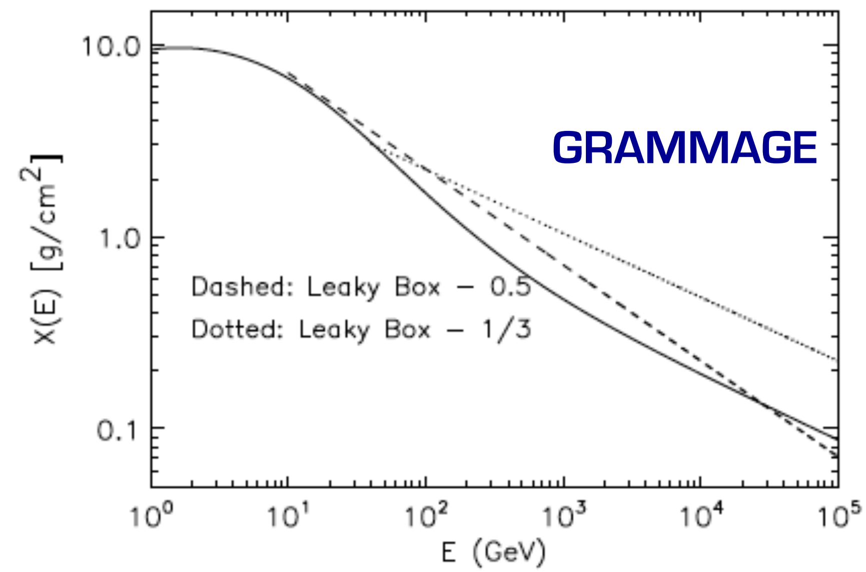


Neronov, Semikoz, Taylor 2012

A POSSIBLE EXPLANATION







PB, Amato & Serpico 2012

ELECTRON SPECTRUM