

# Photosphere – Internal Shock Model of Gamma-Ray Bursts: Implications for the Fermi/LAT Results

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# Outline

1. Prompt Emission: Observations and Models
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3. Ph-IS Model: Temporal Properties
4. Ph-IS Model: Spectral Properties
5. Case Studies: GRB 080916C, 090902B, 090510
6. Summary & Discussion

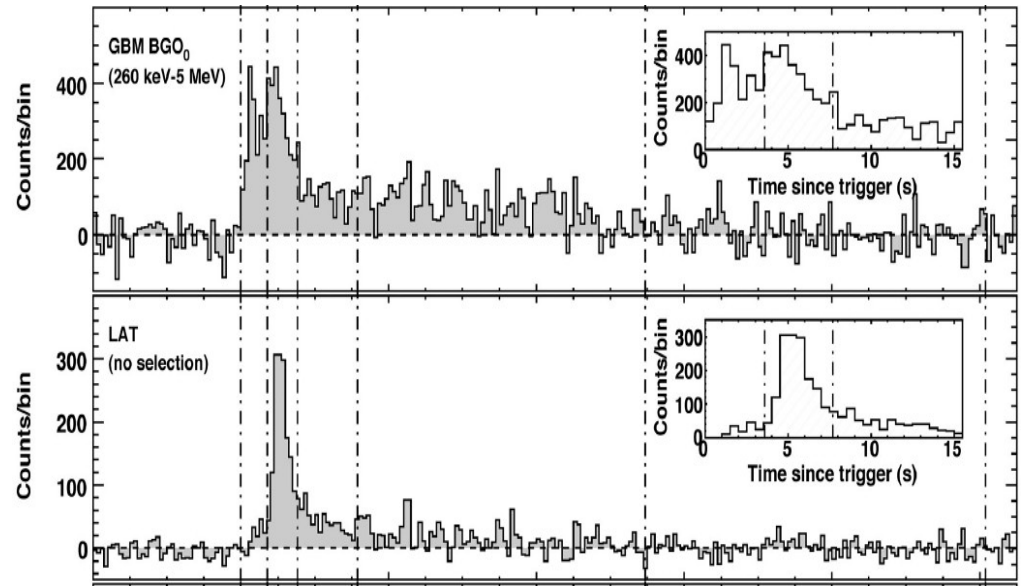
# Prompt Emission: Observations

## Before *Fermi*

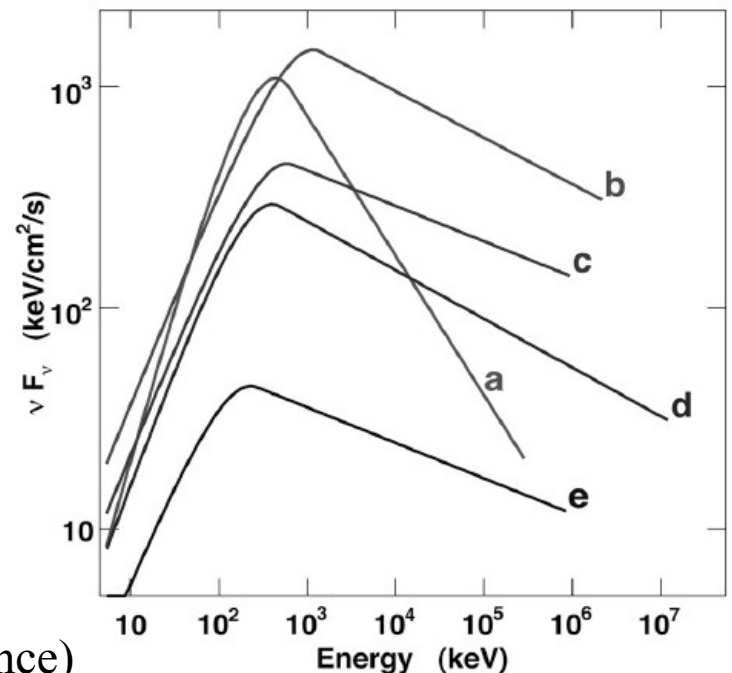
- mainly observed in 10 keV – 1 MeV
- Band or CPL spectrum (Preece+00; Ghirlanda+02; Kaneko+06)
- variable light curve

## *Fermi* observations (Fermi team papers 09;10)

- observed in 8 keV – 300 GeV
- **Band spectrum in 10 keV – 10 GeV** (GRB 080825C, 080916C, ...)
- **Band + distinct PL** (GRB 090510, 090902B, 090926A)
- **delayed onset in  $> 100$  MeV** (due to the flux increase and/or spectral change)
- **variable light curve in  $> 100$  MeV** (GRB 090510, 090926A)



GRB 080916C  
(Abdo+09, Science)



# Prompt Emission: Models

Three types of models actively discussed for the Band component (<10MeV)

- Photospheric emission models (e.g. Paczynski 86; Thompson 94; Meszaros & Rees 00)
- Internal shock models (e.g. Rees & Meszaros 94; Kobayashi+ 97; Daigne & Mochkovitch 98)
- Magnetic dissipation models (e.g. Drenkhahn & Spruit 02; Lyutikov 06)

Clarifying the origin of the prompt emission would help understand the nature of the relativistic jets and the central compact objects.

Origin of the distinct components and the onset delays in the high-energy range (> 100 MeV) ?

- External shock models (Kumar & Barniol Duran 09; Ghisellini+10) → variability ?

- Internal shock models (Abdo+09; Corsi+09; Daigne+10) → SSC onset delay not larger than variability timescale

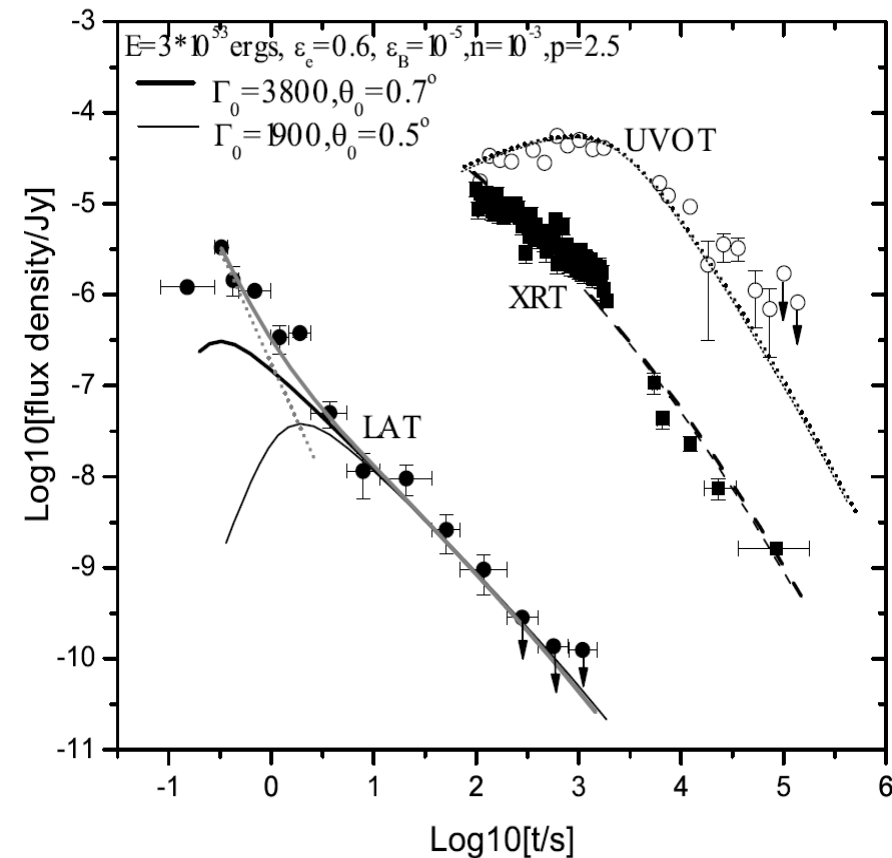
- Cocoon EIC model (Toma+09) needs  $\epsilon_B \sim 10^{-5}$

- Hadronic emission models (Asano+09; Razzaque+10) need large energy budgets

- Magnetic dissipation models → ? (see Zhang10, Fan 10)

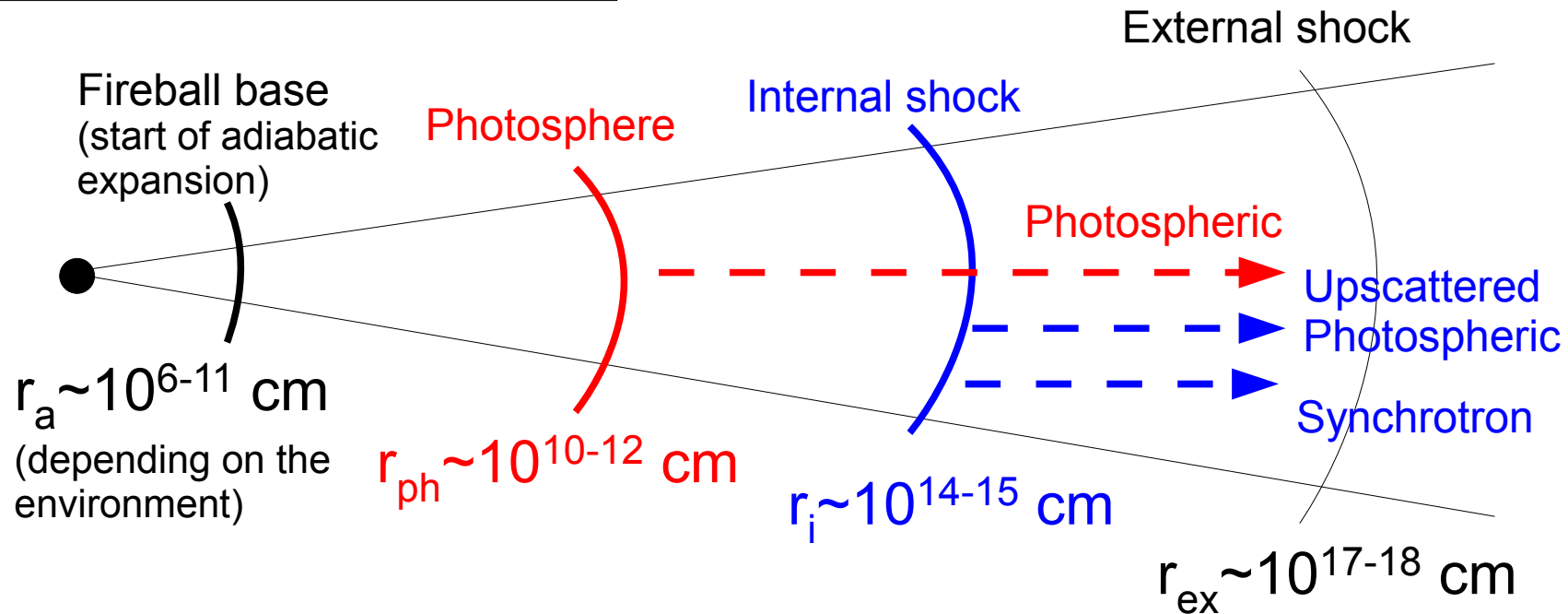
We focus on the photospheric emission models.

External shock emission is suggested to be insufficient (He+ 10; Liu & Wang 10)



# 2. Photospheric Emission Models

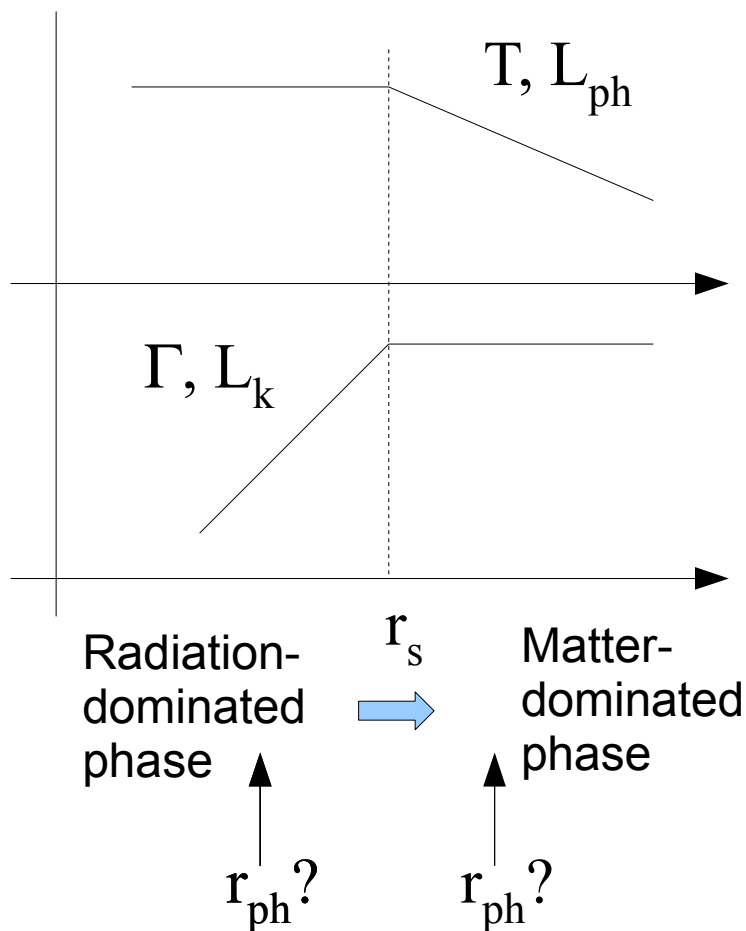
## The standard fireball model



- The photospheric emission is naturally bright around  $\sim 1$  MeV (Paczynski 86; Goodman 86)
- There could be some effects which cause a non-thermal dissipation of the jet at  $r_a < r < r_{ph}$  and make a non-thermal tail of the photospheric emission. These may be associated with copious pair creation (Rees & Meszaros 05; Pe'er+05; Ioka+07; Beloborodov 10; Lazzati & Begelman 10; Thompson 94)
- The high-energy emission may not be from the photosphere, but may be instead from a dissipation region out of the photosphere (Gao+09; Ryde+10)
- The UP emission is a good candidate for the high-energy emission (This work; Pe'er+10)

# Photospheric Emission

Fireball dynamics:



Key parameter:  $\eta \equiv L/\dot{M}c^2$ ,

Critical value:

$$\eta_* = \left( \frac{\sigma_T \mathcal{R} L \Gamma_a}{8\pi r_a m_p c^3} \right)^{1/4} \simeq 2.8 \times 10^3 L_{53}^{1/4} \left( \frac{r_{a,7}}{\Gamma_a} \right)^{-1/4} \mathcal{R}_1^{1/4}.$$

$$n'_l = \mathcal{R} n',$$

Low baryon load case:  $\eta > \eta_*$   $r_{\text{ph}} < r_s$

$$L_{\text{ph}} \simeq L, \quad \varepsilon_{\text{ph}} \simeq 4 kT_a, \quad \Gamma_f \simeq \eta_*, \quad L_k \simeq L \left( \frac{\eta}{\eta_*} \right)^{-1}.$$

High baryon load case:  $\eta < \eta_*$   $r_{\text{ph}} > r_s$

$$L_{\text{ph}} \simeq L \left( \frac{\eta}{\eta_*} \right)^{8/3}, \quad \varepsilon_{\text{ph}} \simeq 4 kT_a \left( \frac{\eta}{\eta_*} \right)^{8/3}$$

$$\Gamma_f = \eta, \quad L_k \simeq L.$$

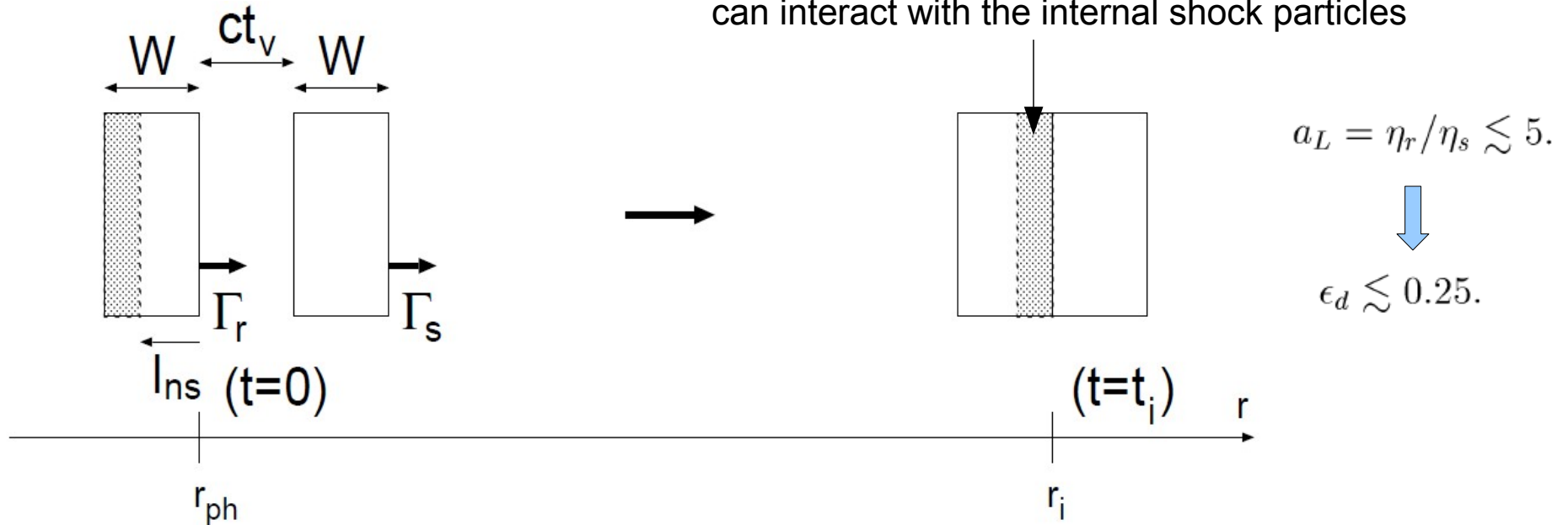
The observer-frame temperature at the base:

$$T_a = \left( \frac{L \Gamma_a^2}{4\pi r_a^2 c a} \right)^{1/4} \simeq 2 L_{53}^{1/4} \left( \frac{r_{a,7}}{\Gamma_a} \right)^{-1/2} \text{ MeV}/k, \quad \rightarrow$$

Photospheric emission can be dominant in the MeV energy range.

# 3. Ph-IS Model: Temporal Properties

Simple kinematic consideration:



$$l_{\text{ns}} = c(1 - \beta_r)t_i \approx a_L^{-2}ct_v. \quad < W/2: \text{ efficient scattering regime}$$

(Typical case  $W \sim ct_v$  is included)

Up-scattered photospheric (UP) emission pulses are correlated with the photospheric pulses from rapid shells, and lagged from the photospheric pulses from slow shells.  $t_{\text{lag}} = (W + ct_v + l_{\text{ns}})/c \sim (W/c) + t_v.$

This lag is comparable to  $t_v$  or  $\delta t_{\text{ph}} \sim \frac{W}{c}$ . The observed LAT onset delays are very large compared with the variability timescale. We will propose another explanation for the observed delays in the end of this talk.

# 4. Ph-IS Model: Spectral Properties (3 slides)

Internal shock radius:

$$r_i \simeq 2ct_v \eta_s^2 = 2ct_v \eta^2 a_L^{-2} \simeq 2 \times 10^{13} \eta_3^2 t_{v,-2} (a_L/5)^{-2} \text{ cm.}$$

Minimum injection energy of accelerated leptons:

$$\gamma_m = \frac{m_p p - 2}{m_e p - 1} \mathcal{R}^{-1} \epsilon_d \epsilon_e \simeq 4 \mathcal{R}_1^{-1} \left( \frac{\epsilon_d \epsilon_e}{0.1} \right) f(p),$$

Magnetic field amplification:

$$U'_B = B'^2 / (8\pi) = L \epsilon_d \epsilon_B / (4\pi r_i^2 c \eta^2).$$

Synchrotron, SSC, UP emission energy densities:

$$U'_{\text{syn}} = t'_{\text{dyn}} \frac{4}{3} \sigma_T c U'_B \int \gamma^2 \frac{dn'_l}{d\gamma} d\gamma = x U'_B, \quad U'_{\text{SSC}} = x U'_{\text{syn}}, \quad U'_{\text{up}} = x U'_{\text{ph}},$$

$$L_{\text{up}} \simeq x L_{\text{ph}},$$

$$L_{\text{syn}} \simeq \epsilon_d \epsilon_B x L \simeq k x L_{\text{ph}},$$

$$L_{\text{SSC}} \simeq \epsilon_d \epsilon_B x^2 L \simeq k x^2 L_{\text{ph}},$$

$$x = \frac{4}{3} \sigma_T \frac{r_i}{2\eta} \int \gamma^2 \frac{dn'_l}{d\gamma} d\gamma \simeq \frac{4(p-1)}{3(p-2)} \tau_{l,i} \gamma_m \gamma_c h(\gamma_m, \gamma_c),$$

$$k \equiv \frac{\epsilon_d \epsilon_B}{(\eta/\eta_*)^{8/3}}.$$



The cooling energy of leptons can be estimated by:

$$\gamma_e m_e c^2 = P(\gamma_e) r_i / (2c\eta) \quad P(\gamma) = (4/3)\sigma_T c \gamma^2 (U'_B + U'_{\text{syn}} + U'_{\text{ph}})$$

**Table 1.** Ordering of emission luminosities for various cases of  $\mathcal{G} = (\eta/\eta_*)^{8/3}$ ,  $\mathcal{E} = \epsilon_d \epsilon_e h$ , and  $\mathcal{B} = \epsilon_d \epsilon_B$ .

Case	$\mathcal{G}$	$k(\text{or } k'), x$	Luminosities	$\mathcal{E}$ and $\mathcal{B}$	
1	$\eta < \eta_*$ ( $\mathcal{G} < 1$ )	$k \ll 1, kx \ll 1$	$x \ll 1$	$L_{\text{ph}} \gg L_{\text{up}} \gg L_{\text{syn}} \gg L_{\text{ssc}}$	$\mathcal{G} \gg \max(\mathcal{E}, \mathcal{B})$
2			$x \gg 1, kx^2 \ll 1$	$L_{\text{up}} \gg L_{\text{ph}} \gg L_{\text{ssc}} \gg L_{\text{syn}}$	$\mathcal{E} \gg \mathcal{G} \gg (\mathcal{E}^2 \mathcal{B})^{1/3} \gg \mathcal{B}$
3			$x \gg 1, kx^2 \gg 1$	$L_{\text{up}} \gg L_{\text{ssc}} \gg L_{\text{ph}} \gg L_{\text{syn}}$	$\mathcal{E} \gg (\mathcal{E}^2 \mathcal{B})^{1/3} \gg \mathcal{G} \gg \mathcal{B}$
4		$k \ll 1, kx \gg 1$		$L_{\text{ssc}} \gg L_{\text{up}} \gg L_{\text{syn}} \gg L_{\text{ph}}$	$\mathcal{E} \gg (\mathcal{E} \mathcal{B})^{1/2} \gg \mathcal{G} \gg \mathcal{B}$
5		$k \gg 1$	$x \gg 1$	$L_{\text{ssc}} \gg L_{\text{syn}} \gg L_{\text{up}} \gg L_{\text{ph}}$	$\mathcal{E} \gg \mathcal{B} \gg \mathcal{G}$
6			$x \ll 1, kx \gg 1, kx^2 \gg 1$	$L_{\text{syn}} \gg L_{\text{ssc}} \gg L_{\text{ph}} \gg L_{\text{up}}$	$\mathcal{B} \gg \mathcal{E} \gg \mathcal{E}^2/\mathcal{B} \gg \mathcal{G}$
7			$x \ll 1, kx \gg 1, kx^2 \ll 1$	$L_{\text{syn}} \gg L_{\text{ph}} \gg L_{\text{ssc}} \gg L_{\text{up}}$	$\mathcal{B} \gg \mathcal{E} \gg \mathcal{G} \gg \mathcal{E}^2/\mathcal{B}$
8			$x \ll 1, kx \ll 1$	$L_{\text{ph}} \gg L_{\text{syn}} \gg L_{\text{up}} \gg L_{\text{ssc}}$	$\mathcal{B} \gg \mathcal{G} \gg \mathcal{E}$
9	$\eta > \eta_*$ ( $\mathcal{G} > 1$ )	$k' \ll 1, k'x \ll 1$	$x \ll 1$	$L_{\text{ph}} \gg L_{\text{up}} \gg L_{\text{syn}} \gg L_{\text{ssc}}$	$\mathcal{G} \gg \max(\mathcal{E}, \mathcal{B})$

Cases 1, 2, 3, 9:

$$x \approx \frac{\epsilon_d \epsilon_e h}{(\eta/\eta_*)^{8/3}} \quad (k \ll 1, kx \ll 1).$$

Cases 4:

$$x \approx \sqrt{\epsilon_e h / \epsilon_B} \quad (k \ll 1, kx \gg 1).$$

Cases 5, 6, 7, 8:

$$x \approx \begin{cases} \sqrt{\epsilon_e h / \epsilon_B}, & (x \gg 1), \\ \epsilon_e h / \epsilon_B, & (x \ll 1). \end{cases} \quad (k \gg 1)$$

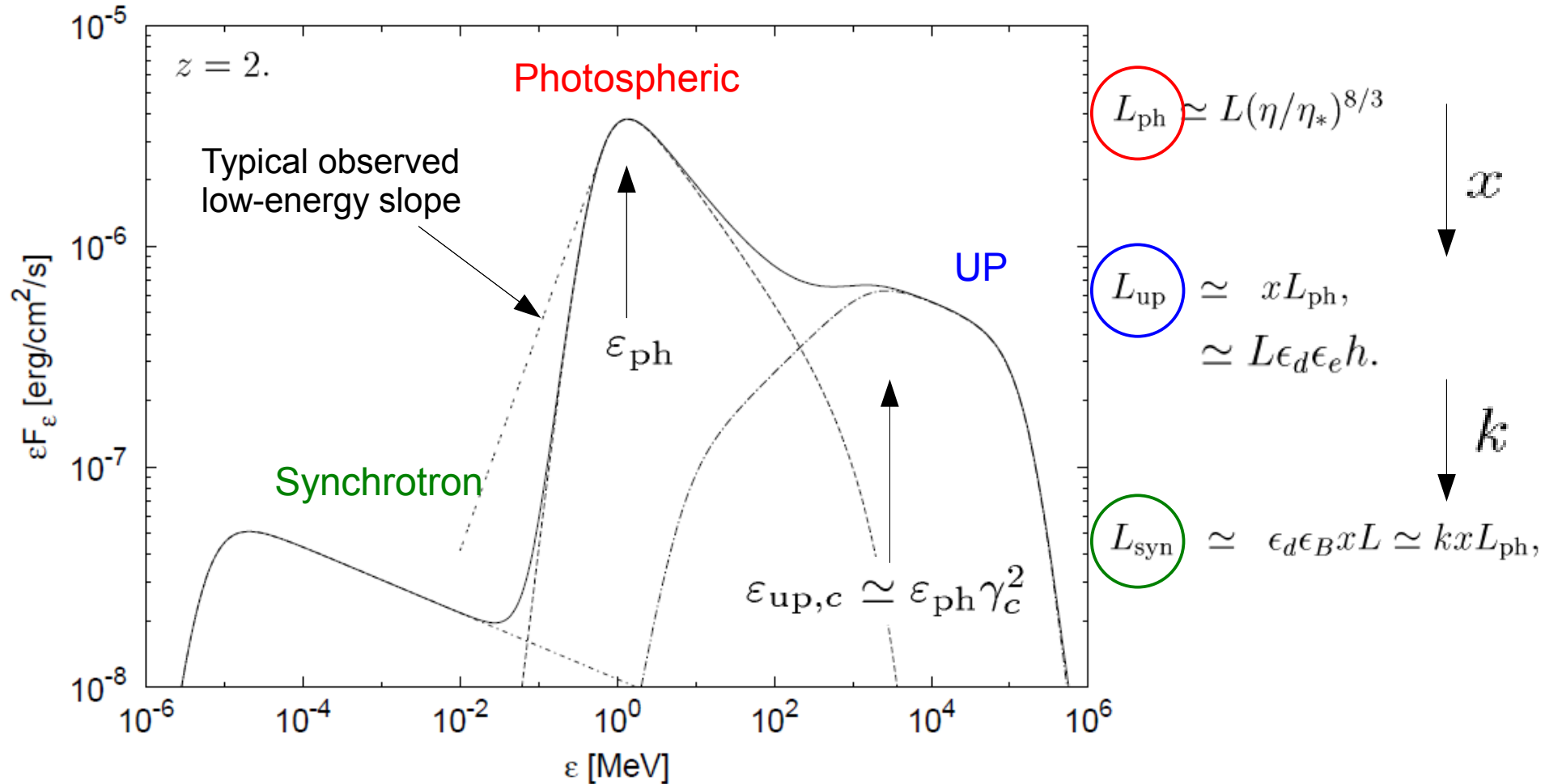
$\mathcal{G} \equiv (\eta/\eta_*)^{8/3}$ ,  $\mathcal{E} \equiv \epsilon_d \epsilon_e h$ , and  $\mathcal{B} \equiv \epsilon_d \epsilon_B$   
 divide spectral models into two:  
**photospheric emission models** and  
 synchrotron/SSC emission models.

Consistent with model of  
 synchrotron/SSC only (Sari & Esin 01)

An example of case 1:  $L_{53} = 3, \eta_3 = 3, r_{a,7}/\Gamma_a = 1, \mathcal{R}_1 = 2, \beta_{\text{ph}} = -2.5,$

$t_{v,-2} = 1, p = 2.3, \epsilon_d \epsilon_e = 0.1, \epsilon_d \epsilon_B = 0.03, \text{ and } a_L = 5.$

→  $(\eta/\eta_*)^{8/3} \sim 0.4 > \max(\epsilon_d \epsilon_d h, \epsilon_d \epsilon_B), \quad r_{\text{ph}} \sim 1 \times 10^{11} \text{ cm} \quad r_i \sim 2 \times 10^{14} \text{ cm}$



An approximate analytical form with smoothed breaks of each spectral component is used. **Photospheric and UP components are dominant in the MeV and high-energy ranges, respectively.**

# Model Fit: GRB 080916C

## Second time-bin:

$$L_{\text{up}}/L_{\text{ph}} = x \sim 0.2. \quad \epsilon_d \epsilon_e \lesssim 0.1$$

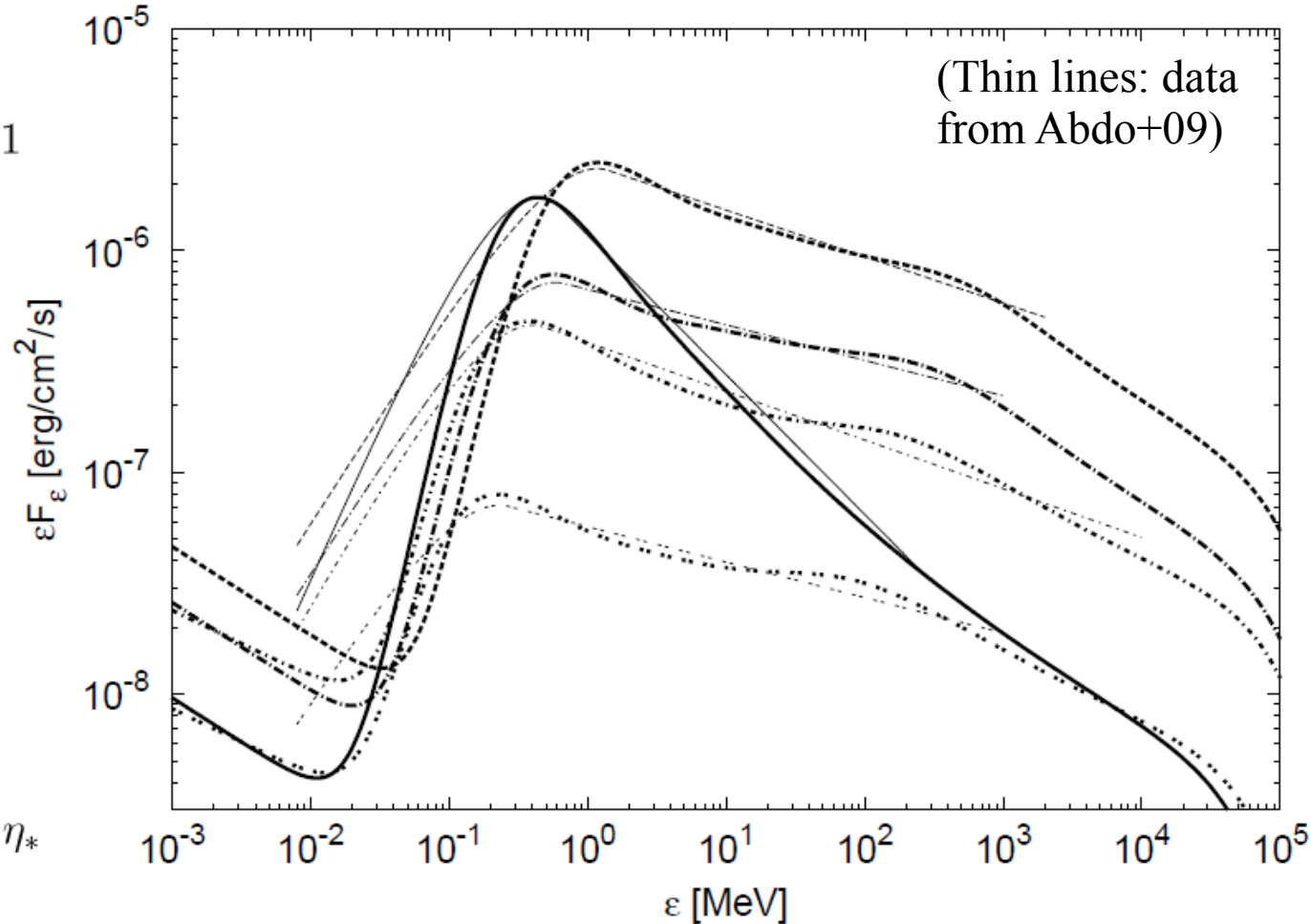
$$x \approx \frac{\epsilon_d \epsilon_e h}{(\eta/\eta_*)^{8/3}} \quad \rightarrow \quad \eta < \eta_*$$

## Seven model parameters

$L$ ,  $\eta$ ,  $r_a/\Gamma_a$ ,  $\mathcal{R}$ ,  $\epsilon_d \epsilon_e$ ,  $t_v$ , and  $a_L$ .  
 can be determined uniquely by  
 $L_{\text{ph}}$ ,  $\epsilon_{\text{ph}}$ ,  $L_{\text{up}}$ ,  $\gamma_c$ , and  $\gamma_m$ ,  
 for reasonable values of  
 $\epsilon_d \epsilon_e \sim 0.1$  and  $a_L \sim 5$ ,

## First time-bin:

$$L_{\text{up}}/L_{\text{ph}} = x \lesssim 0.06. \quad \rightarrow \quad \eta \gtrsim \eta_*$$

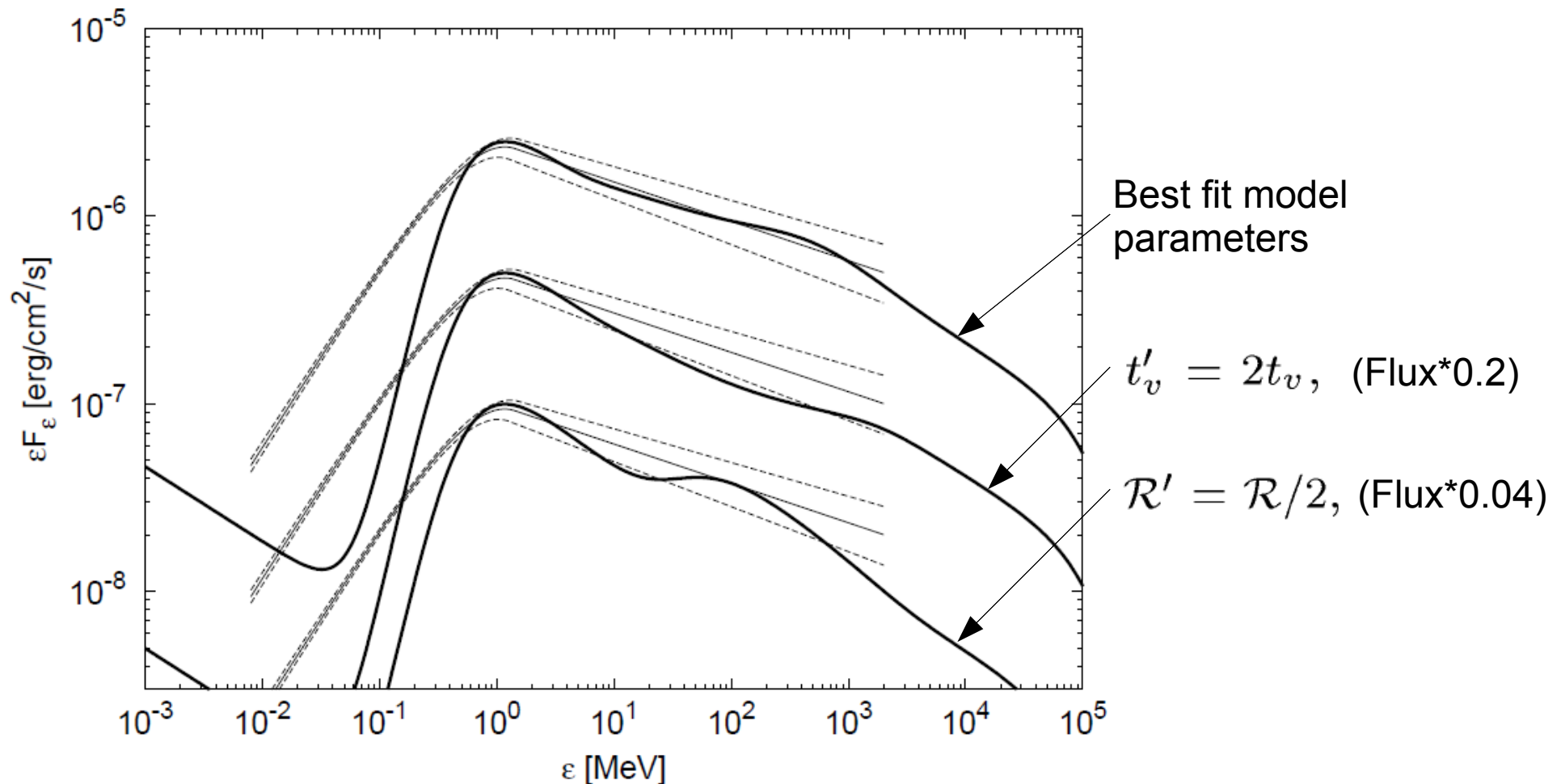


UP luminosity increase is due to the shift of the parameter regime; from  $\eta > \eta_*$  into  $\eta < \eta_*$

GRB 080916C <sup>a</sup>	Time-bin	$L_{53}$	$r_{a,7}/\Gamma_a$	$\eta_3$	$\mathcal{R}_1$	$\beta_{\text{ph}}$	$t_{v,-2}$	$p$	$\epsilon_d \epsilon_e$	$\epsilon_d \epsilon_B$	$\eta/\eta_*$
	0.0 – 3.6 s	3	20	3.2	2	-3.0	0.3	2.8	0.1	< 1	2
		3	20	3.1	2	-2.7	0.1	2.8	0.01	< 1	2
	3.6 – 7.7 s	10	0.9	4.9	3	-2.5	0.2	2.8	0.1	$\lesssim 0.1$	0.7
	7.7 – 15.8 s	5	1	3.2	3	-2.5	0.6	2.8	0.1	$\lesssim 0.1$	0.6
	15.8 – 54.7 s	3	1	2.5	3	-2.5	0.7	2.6	0.1	$\lesssim 0.1$	0.5
	54.7 – 100.8 s	0.7	2	1.6	3	-2.5	1.5	2.6	0.1	$\lesssim 0.2$	0.5

# Fine tuning not needed for Band-like shapes

The second time-bin spectrum of GRB 080916C



The model spectra with  $t'_v$  two times larger or  $\mathcal{R}$  two times smaller than the best fit parameters seem to be still well fitted by a Band-like function, and consistent with the observed flux within a 1 sigma error.

# Model Fit: GRB 090902B

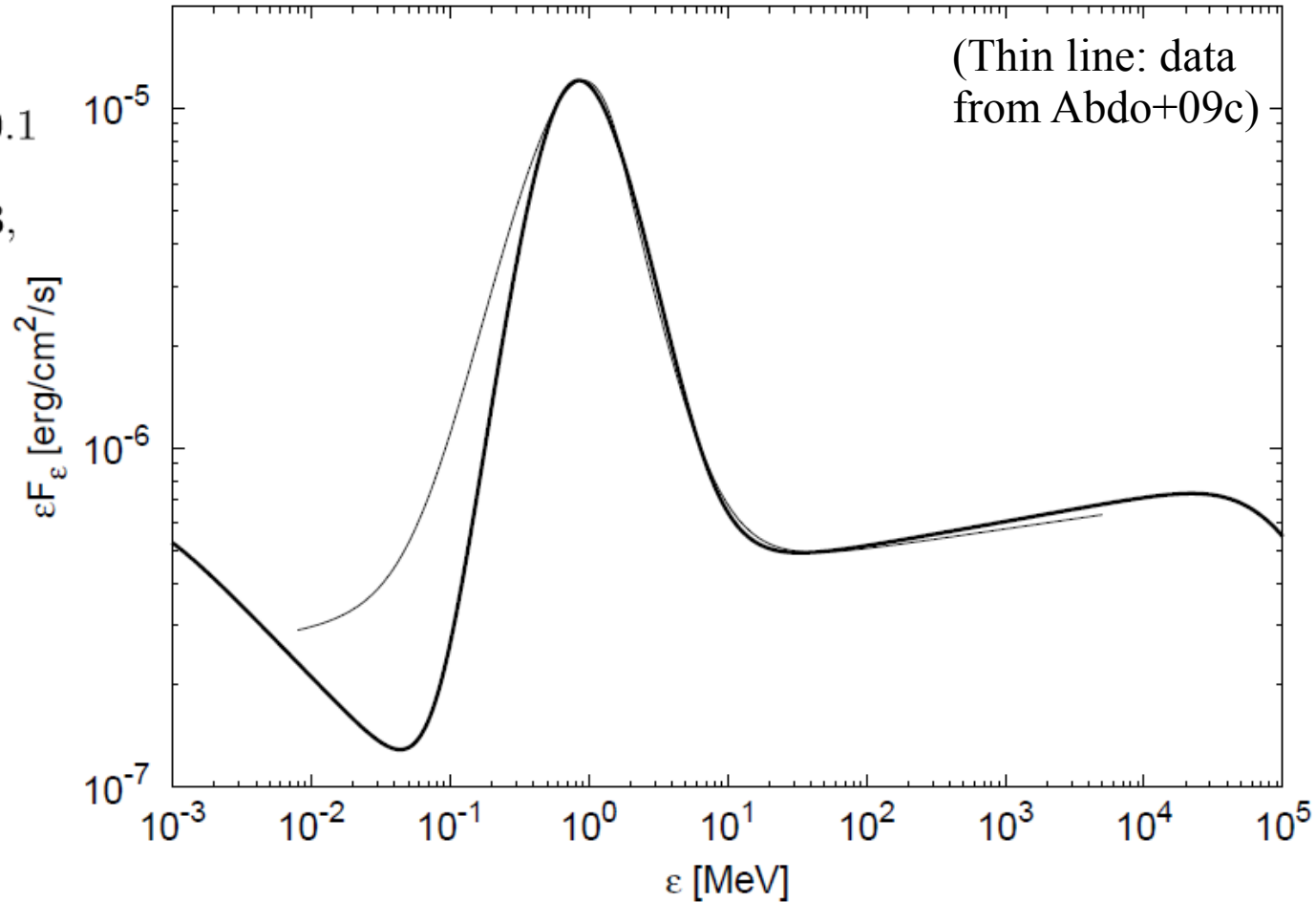
## Second time-bin:

$$L_{\text{up}}/L_{\text{ph}} = x \sim 0.05. \quad \epsilon_d \epsilon_e \lesssim 0.1$$

$$h = (\gamma_c/\gamma_m)^{2-p}/(3-p) \lesssim 0.3,$$

$$x \approx \frac{\epsilon_d \epsilon_e h}{(\eta/\eta_*)^{8/3}} \quad \rightarrow \quad \eta < \eta_*$$

Best fit model parameters are similar to the second time-bin of GRB 080916C except for  $t_v$  and  $\beta_{\text{ph}}$



GRB 090902B	Time-bin	$L_{53}$	$r_{a,7}/\Gamma_a$	$\eta_3$	$\mathcal{R}_1$	$\beta_{\text{ph}}$	$t_{v,-2}$	$p$	$\epsilon_d \epsilon_e$	$\epsilon_d \epsilon_B$	$\eta/\eta_*$
	4.6 – 9.6 s	12	2	3.3	3	-4.0	10	2.85	0.1	$\lesssim 0.2$	0.6

GRB 080916C <sup>a</sup>	Time-bin	$L_{53}$	$r_{a,7}/\Gamma_a$	$\eta_3$	$\mathcal{R}_1$	$\beta_{\text{ph}}$	$t_{v,-2}$	$p$	$\epsilon_d \epsilon_e$	$\epsilon_d \epsilon_B$	$\eta/\eta_*$
	0.0 – 3.6 s	3	20	3.2	2	-3.0	0.3	2.8	0.1	< 1	2
		3	20	3.1	2	-2.7	0.1	2.8	0.01	< 1	2
	3.6 – 7.7 s	10	0.9	4.9	3	-2.5	0.2	2.8	0.1	$\lesssim 0.1$	0.7

# Model Fit: GRB 090510

## Second time-bin:

$$L_{\text{up}}/L_{\text{ph}} = x \sim 0.1, \quad \epsilon_d \epsilon_e \lesssim 0.1$$

$$h \lesssim 0.7.$$

$$x \approx \frac{\epsilon_d \epsilon_e h}{(\eta/\eta_*)^{8/3}} \quad \rightarrow \quad \eta < \eta_*$$

## First time-bin:

$$L_{\text{up}}/L_{\text{ph}} = x \lesssim 3 \times 10^{-3}.$$

$$\rightarrow \quad \eta \gtrsim \eta_*$$

## Third time-bin:

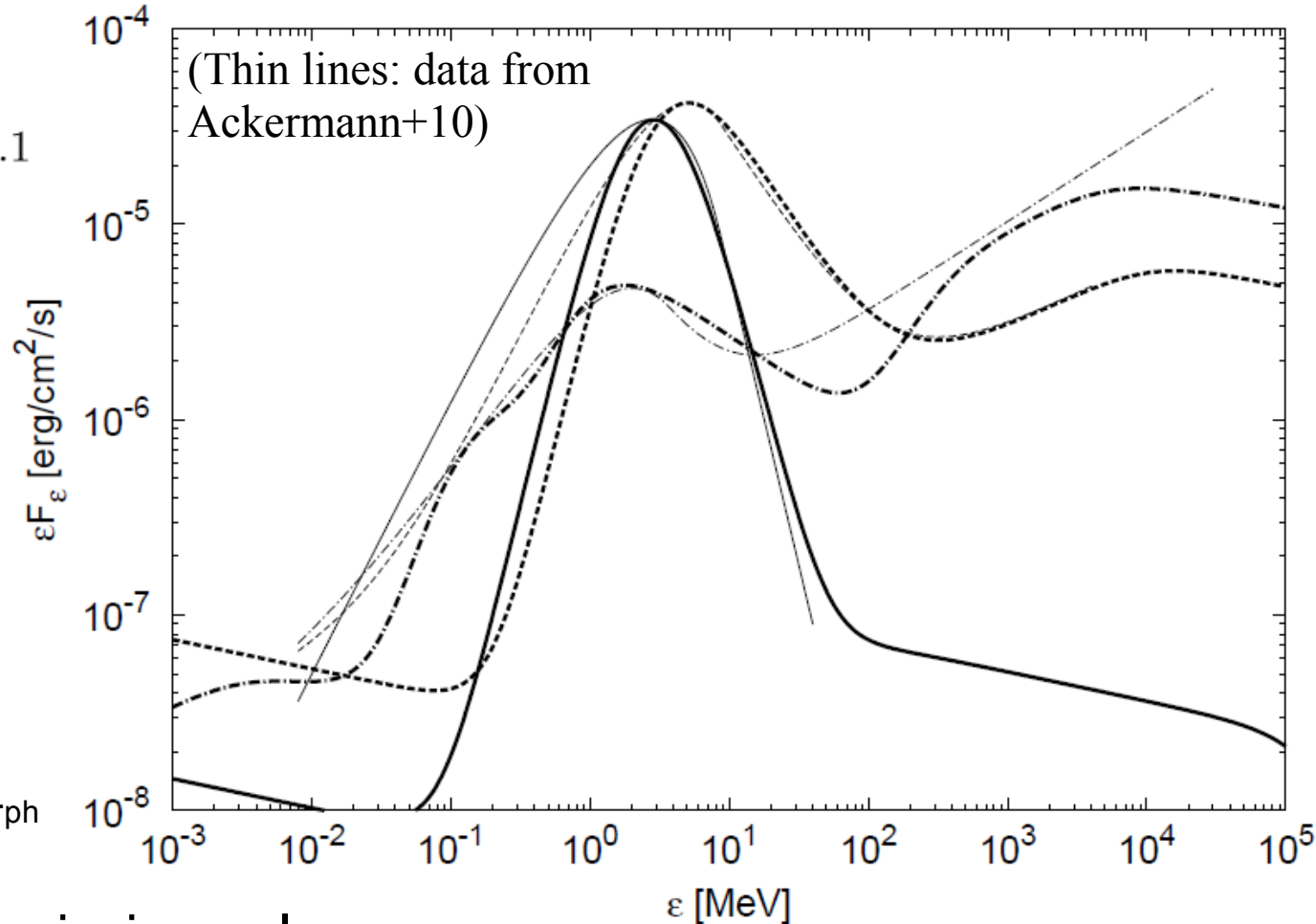
This spectrum is unique:  $L_{\text{up}} > L_{\text{ph}}$

$\rightarrow$  Case 2

## Fourth time-bin: the LAT emission only

Could be the high-latitude emission with a very large  $t_v$

$$\delta t_{\text{up}} \sim \frac{W}{c} + \frac{3r_i}{2c\Gamma_m^2} \simeq \frac{W}{c} + 3a_L^{-1} t_v.$$



GRB 090510	Time-bin	$L_{53}$	$r_{a,7}/\Gamma_a$	$\eta_3$	$\mathcal{R}_1$	$\beta_{\text{ph}}$	$t_{v,-2}$	$p$	$\epsilon_d \epsilon_e$	$\epsilon_d \epsilon_B$	$\eta/\eta_*$
	0.5 – 0.6 s	1	3	73	2	-4.8	0.3	2.3	0.1	< 1	30
		1	3	5.4	2	-4.8	0.1	2.3	$5 \times 10^{-3}$	< 1	2
	0.6 – 0.8 s	4	0.3	4.7	2	-3.0	0.2	2.3	0.1	$\lesssim 7 \times 10^{-3}$	0.7
	0.8 – 0.9 s	4	0.02	2.1	0.5	-2.5	0.7	2.25	0.15	$\lesssim 2 \times 10^{-4}$	0.3
		4	0.04	1.2	0.5	-2.5	20	2.25	0.15	$\lesssim 5 \times 10^{-4}$	0.2

# Summary & Discussion (3 slides)

1. Radially inhomogeneous jets naturally produce **variable photospheric emission around the MeV energy range**, and can lead to internal shocks out of the photosphere. We have shown that the photospheric emission is efficiently up-scattered by the internal shocked electrons (and positrons).
2. We have derived various spectral types which depends on the values of  $(\eta/\eta_*)^{8/3}$ ,  $\epsilon_d\epsilon_e h$ , and  $\epsilon_d\epsilon_B$ , and obtained **necessary conditions for photospheric emission models**.

$$(\eta/\eta_*)^{8/3} \gg \max(\epsilon_d\epsilon_e h, \epsilon_d\epsilon_B) \quad \longrightarrow \quad \text{Photospheric around MeV, UP in high-energy range}$$

3. This case can be consistent with the time-binned spectra of LAT GRBs (as well as other GRBs observed at  $< 10$  MeV). The low-energy spectrum of the photospheric emission is  $\alpha_{\text{ph}} = 1$ , which is much harder than observed,  $\alpha_{\text{ph}} \sim -1$ . **The superposition of the emission from the multiple shells have the potential of reproducing the observed spectral slope.**

4. **Observed LAT onset delays larger than the variability timescales** may not be explained as the simple kinematic effect but instead due to the shift of the parameter regime:

$$\eta > \eta_* \quad \text{small } L_{\text{up}}/L_{\text{ph}} = x \quad \Rightarrow \quad \eta < \eta_* \quad \text{large } L_{\text{up}}/L_{\text{ph}} = x$$

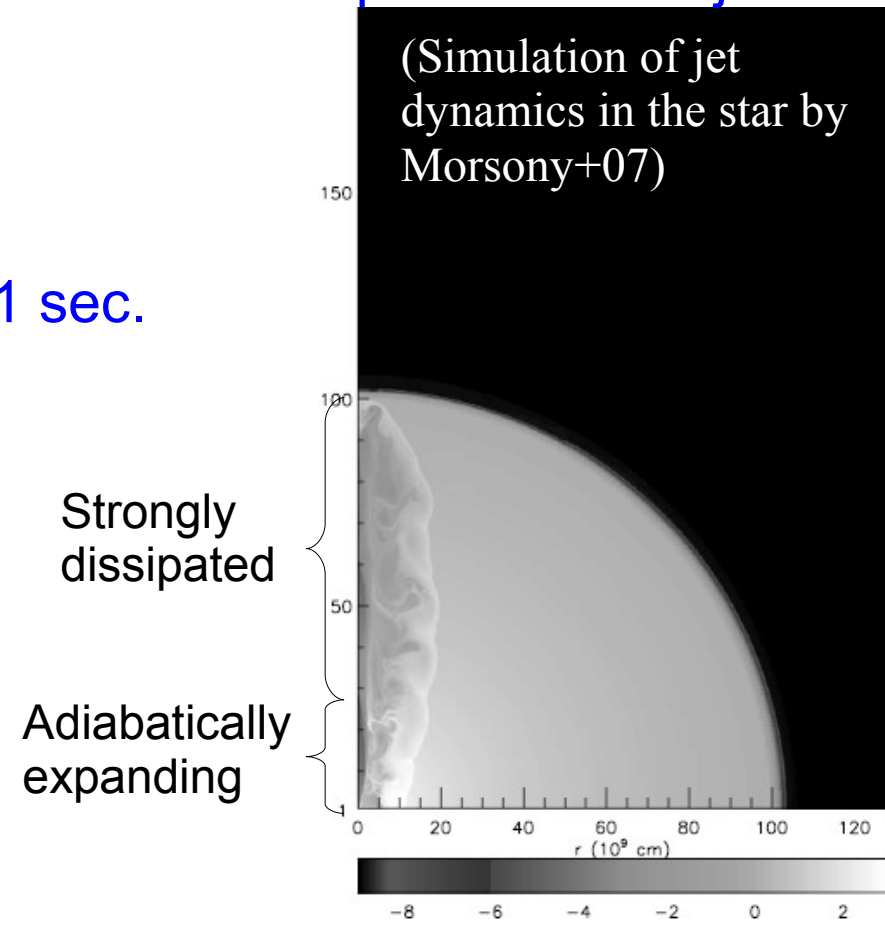
The model fits of the data indicates that this shift is related to the **decrease of  $r_a/\Gamma_a$** .

For GRB 080916C,  $r_a/\Gamma_a$  decreases from  $\sim 10^8 \text{cm}$  to  $\sim 10^7 \text{cm}$ . This may be consistent with a collapsar model, in which the front portion of the jet is expected to be strongly dissipated.

$$\begin{array}{ccc} r_a \sim 5 \times 10^{10} \text{ cm.} & \Rightarrow & r_a \sim 10^7 \text{ cm} \\ \Gamma_a \sim 500. & & \Gamma_a \sim 1. \end{array}$$

This may lead to a delay timescale  $r_a/c \sim 1 \text{ sec}$ .

For GRB 090510,  $r_a/\Gamma_a$  decreases from  $\sim 3 \times 10^7 \text{cm}$  to  $\sim 3 \times 10^5 \text{cm}$ , suggesting a smaller size of the progenitor and the central object. This can be consistent with a smaller delay  $\sim 0.1 \text{sec}$ .





5. Rough calculations of the prompt emission efficiency,  $\Sigma(L_{\text{ph}}+L_{\text{up}})t_{\text{bin}}/\Sigma L t_{\text{bin}}$ , are  $\sim 30\%$  for GRB 080916C,  $\sim 20\%$  for GRB 090902B, and  $\sim 40\%$  for GRB 090510. The standard external shock model of the late radio/opt/X afterglow of GRB 090902B indicates the efficiency  $\sim 80\%$  (Cenko+10; see also Pe'er+10). This value can be reduced if only a fraction of the electrons are accelerated in the external shock (Eichler & Waxman 05; Toma+08).

6. The spectrum in the high-energy range may have spectral breaks at  $\varepsilon_{\text{up},h}$  and at  $\varepsilon_{\text{KN}}$ , while a break due to the electron-positron pair creation in the emission site is estimated to be typically much above those breaks in our model. Detecting spectral breaks in the high-energy range by Fermi/LAT ( $<100\text{GeV}$ ) and/or by the future CTA ( $>30\text{GeV}$ ) would be very helpful to constrain the models.



# Low-energy spectral slope

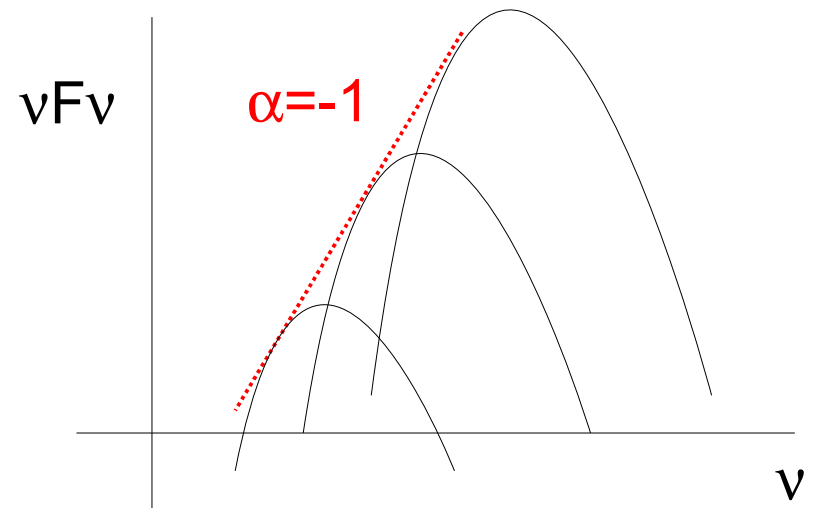
An outstanding problem in the photospheric emission models, as well as in the synchrotron/SSC models is that **the low-energy spectrum of the Band component. The Rayleigh-Jeans slope  $\alpha_{\text{ph}}=1$  is much harder than observed slope  $\alpha \sim -1$ .** The superposition of the photospheric emissions from multiple shells with different peak energies has the potential of reproducing the low-energy spectrum.

$$L_{\text{ph}} \simeq L \left( \frac{\eta}{\eta_*} \right)^{8/3}, \quad \varepsilon_{\text{ph}} \simeq 4 kT_a \left( \frac{\eta}{\eta_*} \right)^{8/3}$$

→  $L_{\text{ph}} \propto \varepsilon_{\text{ph}}$

with different  $\eta$  and similar  $L$ ,  $r_a/\Gamma_a$ , and  $\mathcal{R}$

This is just a speculation, and we need more detailed consistency checks.



Our model fits indicate that  $t_{\nu} \sim 0.1\text{s}$  for GRB 090902B and  $t_{\nu} \sim 0.001\text{s}$  for GRB 080916C. This might be consistent with the report by Zhang B.B.+10 that the spectrum of GRB 090902B is more like blackbody for smaller time-bin, while the spectrum of GRB 080916C keeps highly non-thermal for smaller time-bin ( $> \sim 1\text{s}$ ).

# Yonetoku relation in a burst

The spectral peak of the Band components of the time-binned spectra obeys the so-called Yonetoku relation.

$$\varepsilon_{\text{ph}} \propto L_{\text{ph}}^{1/2}$$

This may be a problem for all the emission models.

In our type of the photospheric emission model, this data require a relation between the physical parameters

$$L_{\text{ph}} \simeq L \left( \frac{\eta}{\eta_*} \right)^{8/3}, \quad \varepsilon_{\text{ph}} \simeq 4 kT_a \left( \frac{\eta}{\eta_*} \right)^{8/3} \quad \longrightarrow \quad \eta \propto L^{7/16} (r_a/\Gamma_a)^{1/8} \mathcal{R}^{1/4}$$

