HIGH-ENERGY LIGHT CURVES IN AN OFFSET POLAR CAP B-FIELD GEOMETRY

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**Aim:** Investigate the effect of different magnetospheric structures on pulsar light curves and additionally what the effect will be when incorporating an $E$-field to modulate the emissivity.

**B-field structures:**
- Static dipole
- Vacuum retarded dipole
- Offset-PC dipole (Harding & Muslimov 2011)

**Geometric models:**
- Two-pole caustic (TPC)
- Outer gap (OG)
- Slot gap (SG)

**Implement an offset-PC solution:**
- Transformation
- PC rim
- SG $E$-field
- Matching parameter
- Transport equation (test curvature radiation reaction – CRR)

Our studies was done on Vela!!!
**OFFSET-PC FIELD**

**B-field expression:**
- Heuristic model of a non-dipolar magnetic structure where the PCs are offset from the magnetic-axis
- Symmetric offset PCs, i.e., offset of both magnetic PCs in the same direction

\[
B'_{\text{offset},s}(r', \theta', \phi') \approx \frac{\mu}{r'^3} \left[ \cos \theta' \hat{r}' + \frac{1}{2} (1 + a) \sin \theta' \hat{\theta}' - \epsilon \sin \theta' \cos \theta' \sin(\phi' - \phi_0) \hat{\phi}' \right]
\]

\[a = \epsilon \cos(\phi' - \phi_0)\]

Distortions due to retardation and asymmetric currents.

**Transformation:**

*B-field specified in the magnetic frame, but transformed the field to the co-rotating frame to determine the PC of the neutron star for this specific B-field solution*
PC rim:

Need to determine rim for new $B$-field
**SG E-field:** Corrected for GR effects (Muslimov & Harding 2003, 2004)

Low-altitude and high-altitude solutions available for offset-PC dipole

$$E_{||,\text{low}} \approx -3E_0 v_{SG} x^a \left\{ \frac{\kappa}{\eta^4} e_1 A \cos \alpha + \frac{1}{4} \frac{\theta_{PC}^{1+a}}{\eta} \left[ e_2 A \cos \phi_{PC} + \frac{1}{4} \epsilon \epsilon_{3A} (2 \cos \phi'_0 - \cos(2 \phi_{PC} - \phi'_0)) \right] \cos \alpha \right\} (1 - \xi^2_*)$$

$$E_{||,\text{high}} \approx -\frac{3}{8} \left( \frac{\Omega R}{c} \right)^3 B_0 v_{SG} x^a \left\{ \left[ 1 + \frac{1}{3} \kappa \left( 5 - \frac{8}{\eta c^3} \right) + 2 \frac{\eta}{\eta_{LC}} \right] \cos \alpha \right\} + \frac{3}{2} \theta_{PC} H(1) \sin \alpha \cos \phi_{PC} (1 - \xi^2_*)$$

Note: magnetic azimuthal angle in $E$-field is $\pi$ out of phase with that of $B$-field

Match these to obtain general SG $E$-field over all altitudes on each B-field line

$$\eta_c(P, \dot{P}, \alpha, \epsilon, \xi, \phi_{PC})$$

$$E_{||,SG} \approx E_{||,\text{low}} \exp \left( \frac{-(\eta - 1)}{(\eta_c - 1)} \right) + E_{||,\text{high}}$$

= 0 “favourably curved” field lines

MATCHING PARAMETER (Barnard et al. 2016, submitted)
**SG E-field:**
Problems encountered when matching

\[ E_{\parallel,SG} \approx E_{\parallel,low} \exp \left( \frac{-(\eta - 1)}{(\eta_c - 1)} \right) + E_{\parallel,high} \]
Transport Equation:
Solve transport equation using general SG $E$-field, on each $B$-field line.

$$\dot{\gamma} = \dot{\gamma}_{\text{gain}} + \dot{\gamma}_{\text{loss}} = \frac{eE_{||,\text{total}}}{mc} - \frac{2e^2\gamma^4}{3\rho_{\text{curv}}mc}$$

$\varepsilon = 0$ (thick lines), $\varepsilon = 0.18$ (thin lines)
RESULTS: Uniform emissivity (TPC model)

\( \varepsilon = 0 \)

\( \zeta = 60^\circ \)

\( \alpha = 0^\circ \)  
\( \alpha = 15^\circ \)  
\( \alpha = 30^\circ \)  
\( \alpha = 45^\circ \)  
\( \alpha = 60^\circ \)  
\( \alpha = 75^\circ \)  
\( \alpha = 90^\circ \)

\( \varepsilon = 0.18 \)

\( \zeta = 60^\circ \)

\( \alpha = 0^\circ \)  
\( \alpha = 15^\circ \)  
\( \alpha = 30^\circ \)  
\( \alpha = 45^\circ \)  
\( \alpha = 60^\circ \)  
\( \alpha = 75^\circ \)  
\( \alpha = 90^\circ \)
RESULTS: Variable emissivity (SG model)

\( \varepsilon = 0 \)

\( \varepsilon = 0.18 \)
RESULTS: Variable emissivity (vary the magnitude of the SG E-field)

Case 1: $\varepsilon = 0.18$
- Lowered minimum energy to 1 MeV
- Hard X-ray band

Case 2: $\varepsilon = 0.18$
- Increased $E$-field by a factor 100
- Gamma-ray band
**Transport Equation:**
Solve transport equation using general SG $E$-field increased by a factor 100.

\[
\dot{\gamma} = \dot{\gamma}_{\text{gain}} + \dot{\gamma}_{\text{loss}} = \frac{eE_{||,\text{total}}}{mc} - \frac{2e^2\gamma_e A}{3\rho_{\text{curv}}^2mc}
\]

$\varepsilon = 0$ (thick lines), $\varepsilon = 0.18$ (thin lines)
RESULTS:

Fitting model light curves to Vela data

- Used a usual chi-squared method (Breed et al. 2014, 2015)
- search the multivariate solution space for optimal model parameters

Chi-squared contour

Best fit to the data for offset-PC field: SG model for $\varepsilon = 0.15$
**RESULTS:**

Fitting model light curves to Vela data
Increased $E$-field by a factor 100

**Best-fit model light curve:**

*Chi-squared contour*

Best fit to the data for offset-PC field:
TPC model for $\varepsilon = 0$
\[ \Delta \xi^2 = \xi^2 - \xi^2_{\text{opt}} = N_{\text{dof}} \left( \frac{\chi^2}{\chi^2_{\text{opt}}} - 1 \right) \]

**Results:**

First:

Compared the optimal and alternative models for each B-field

Second:

Compared all B-field and model combinations to the **OVERALL** optimal B-field and model combination

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Model</th>
<th>( \epsilon )</th>
<th>( \chi^2 ) (( \times 10^5 ))</th>
<th>( \alpha ) (°)</th>
<th>( \zeta ) (°)</th>
<th>( A )</th>
<th>( \Delta \phi_L )</th>
<th>( \Delta \xi^2_B )</th>
<th>( \Delta \xi^2_{\text{all}} )</th>
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<tbody>
<tr>
<td><strong>Static dipole B-field:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>TPC</td>
<td></td>
<td>0.819</td>
<td>0.73^±^3</td>
<td>45^±^4</td>
<td>1.3</td>
<td>0.55</td>
<td>0.00</td>
<td>0.00</td>
<td>108.75</td>
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<tr>
<td>OG</td>
<td></td>
<td>0.891</td>
<td>64^±^5</td>
<td>86^±^4</td>
<td>1.3</td>
<td>0.05</td>
<td>8.44</td>
<td>126.75</td>
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<td><strong>RVD B-field:</strong></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>TPC</td>
<td></td>
<td>3.278</td>
<td>54^±^5</td>
<td>67^±^5</td>
<td>0.5</td>
<td>0.05</td>
<td>723.50</td>
<td>723.50</td>
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<tr>
<td>OG</td>
<td></td>
<td>0.384</td>
<td>78^±^4</td>
<td>69^±^4</td>
<td>1.3</td>
<td>0.00</td>
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<table>
<thead>
<tr>
<th>Other Multi-wavelength Fits</th>
</tr>
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<tbody>
<tr>
<td>( \alpha ) (°)</td>
</tr>
<tr>
<td>53</td>
</tr>
<tr>
<td>63.6^±^0.07</td>
</tr>
<tr>
<td>62–68</td>
</tr>
<tr>
<td>75</td>
</tr>
<tr>
<td>44^±^4</td>
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<td>65^±^2</td>
</tr>
<tr>
<td>88^±^2</td>
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<td>15^±^1</td>
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<tr>
<td>55^±^10</td>
</tr>
<tr>
<td>80^±^1</td>
</tr>
<tr>
<td>3^±^2</td>
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<tr>
<td>45^±^2</td>
</tr>
<tr>
<td>71^±^2</td>
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<tr>
<td>56^±^2</td>
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<thead>
<tr>
<th>X-ray torus</th>
<th>RVD &amp; TPC</th>
<th>RVD &amp; OG</th>
<th>RVD &amp; Symmetric SG</th>
<th>RVD &amp; Asymmetric SG</th>
<th>RVD &amp; OG</th>
<th>FF &amp; Symmetric SG</th>
<th>FF &amp; Asymmetric SG</th>
<th>FF &amp; OG</th>
<th>RVD &amp; PC</th>
<th>RVD &amp; SG</th>
<th>RVD &amp; OG</th>
<th>RVD &amp; OPC</th>
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<tbody>
<tr>
<td>53</td>
<td>63.6^±^0.07</td>
<td>64</td>
<td>3</td>
<td>4</td>
<td>4</td>
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<td>5</td>
<td>5</td>
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RESULTS: Comparison of the best-fit solutions (Pierbattista et al. 2015)
CONCLUSIONS

For an offset-PC magnetosphere:

- Therefore both the $B$-field and $E$-field have an impact on the predicted light curves.

- Solving the particle transport equation shows that the particle energy only becomes large enough to yield significant curvature radiation at large altitudes above the stellar surface, given this relatively low $E$-field. Therefore, particles do not always attain the radiation-reaction limit.

- Our overall optimal light curve is for the retarded vacuum dipole field and outer gap model. But the offset-PC dipole delivers an second overall optimal fit.