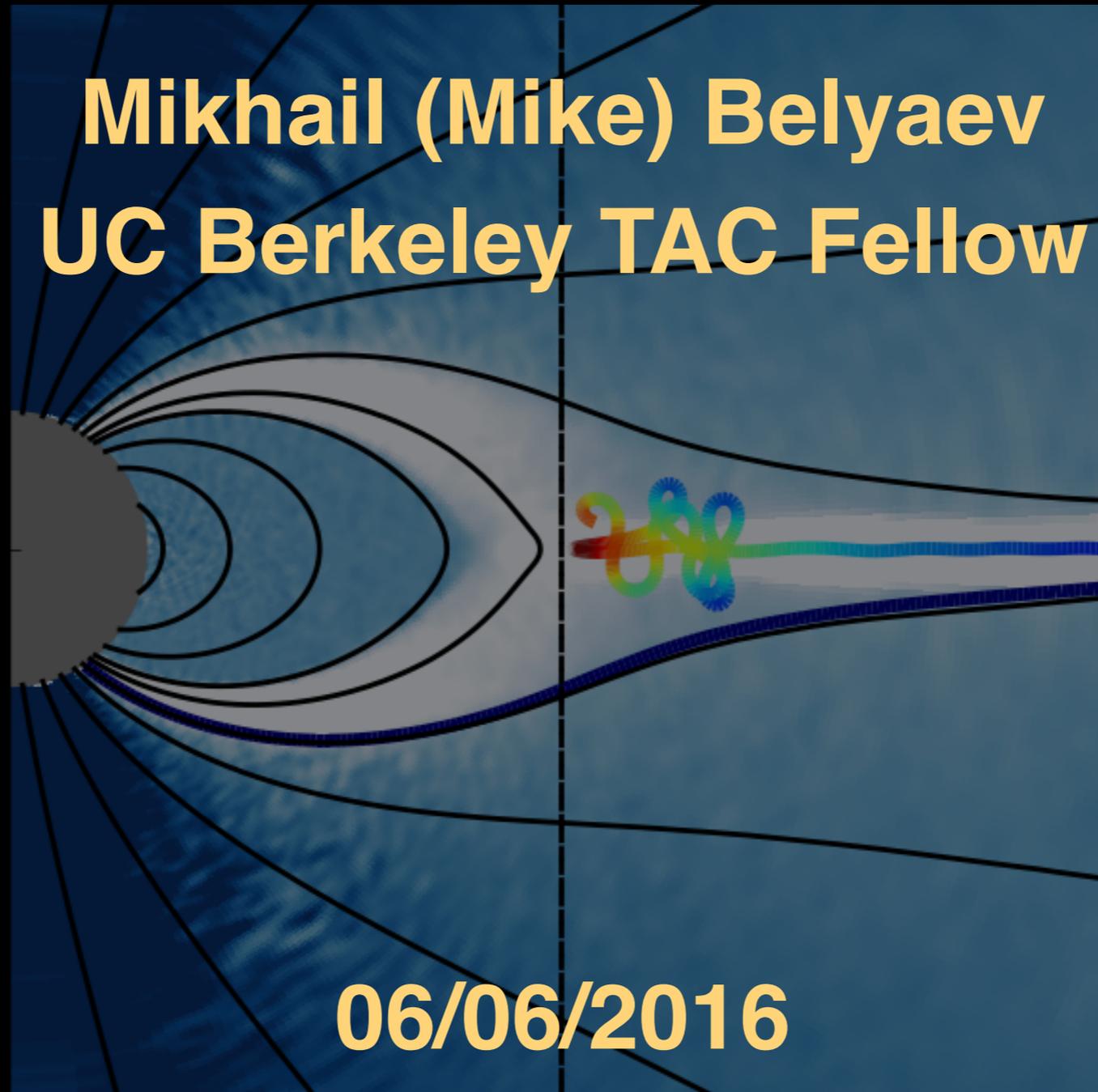


# *Polar Cap & Y-Point* *Theory & PIC Simulation*

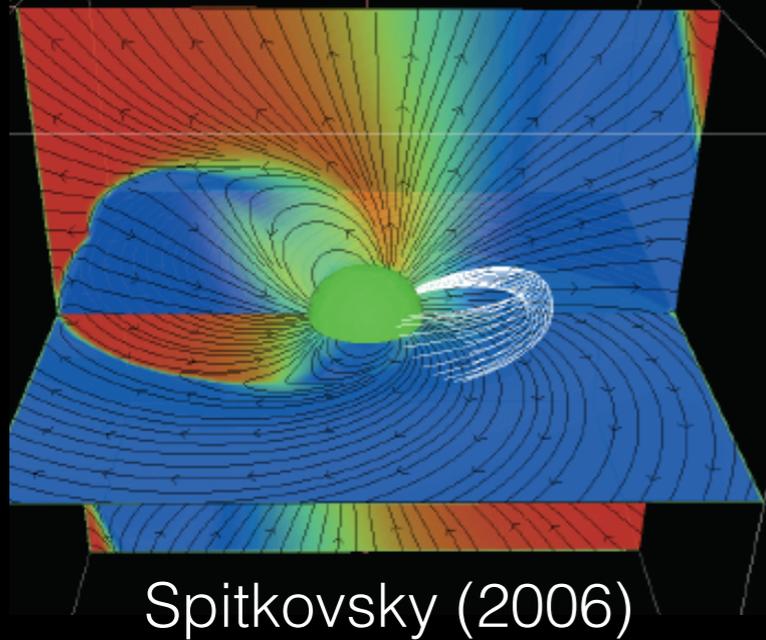
**Mikhail (Mike) Belyaev**  
**UC Berkeley TAC Fellow**



**06/06/2016**

# Theoretical Backgr

We need to go beyond force-free to understand the emission and connect to observations!



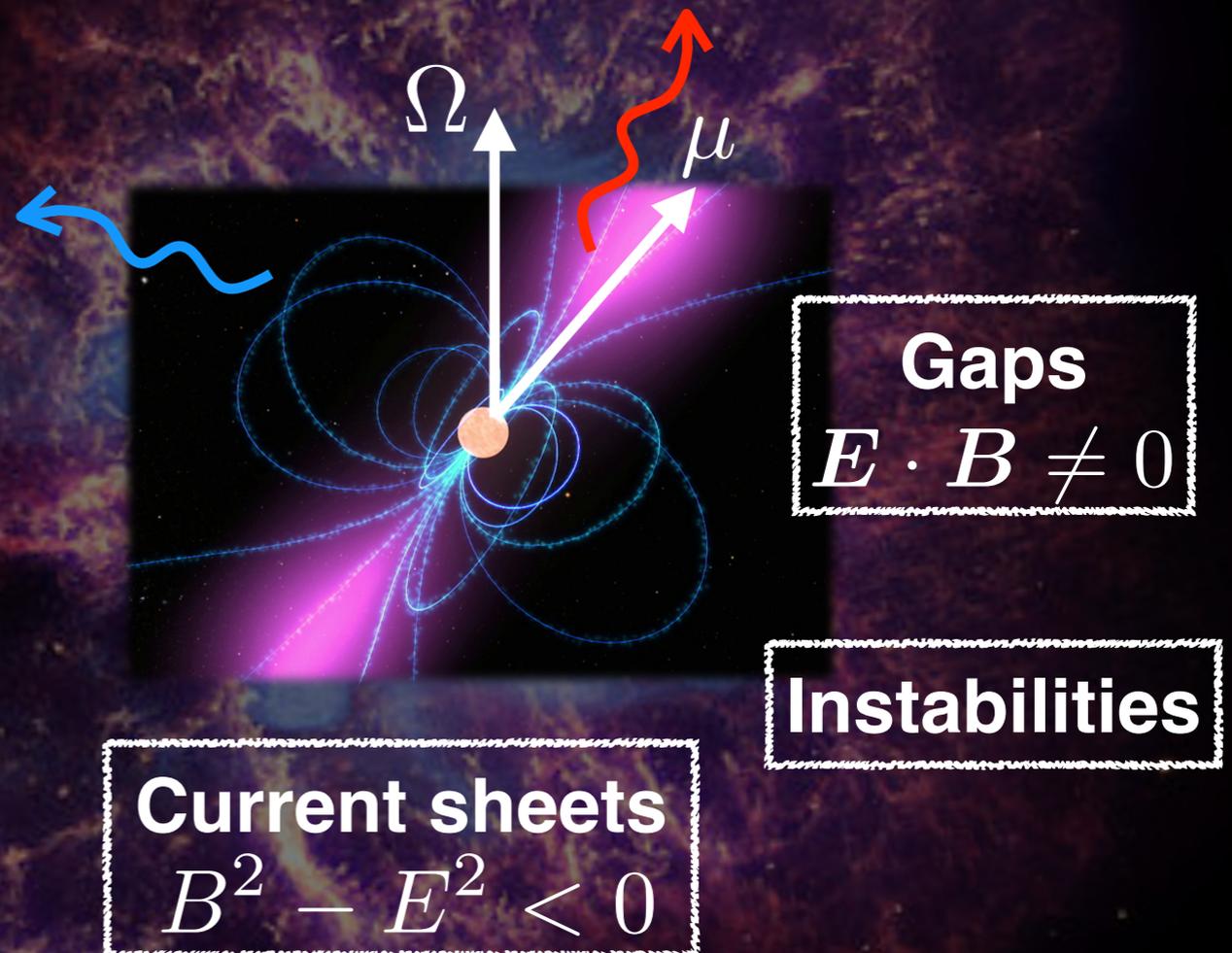
**Magnetosphere approximated as force-free due to a high plasma density.**

$$\rho \mathbf{E} + c^{-1} \mathbf{J} \times \mathbf{B} = 0 \implies \mathbf{E} \cdot \mathbf{B} = 0$$

$$B^2 - E^2 > 0 \implies \mathbf{V}_d = c \mathbf{E} \times \mathbf{B} / B^2$$

**Emission in the magnetosphere requires non-force-free effects.**

$$\frac{\partial u}{\partial t} + \frac{c}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{B}) = -\mathbf{E} \cdot \mathbf{J}$$



**Gaps**  
 $E \cdot B \neq 0$

**Current sheets**  
 $B^2 - E^2 < 0$

**Instabilities**

# *Part I: Polar Cap*

## **Goals:**

- Analytically determine distribution of current over the polar cap for a force-free magnetosphere with aligned spin and magnetic axes.
- Relate spatial distribution of current to spatial distribution of polar cap pair production.

## **Results:**

- Pair production occurs at the inner and outer edges of the polar cap when general-relativity taken into account.
- No pair production at mid-latitudes on polar cap for simple surface field structure (e.g. dipole).

# Polar Cap Pair Production

$$J^\mu J_\mu \equiv -(\rho c)^2 + J^2$$

$\gamma - B$  pair production

Pairs generated when there is backflow of particles onto the polar cap.

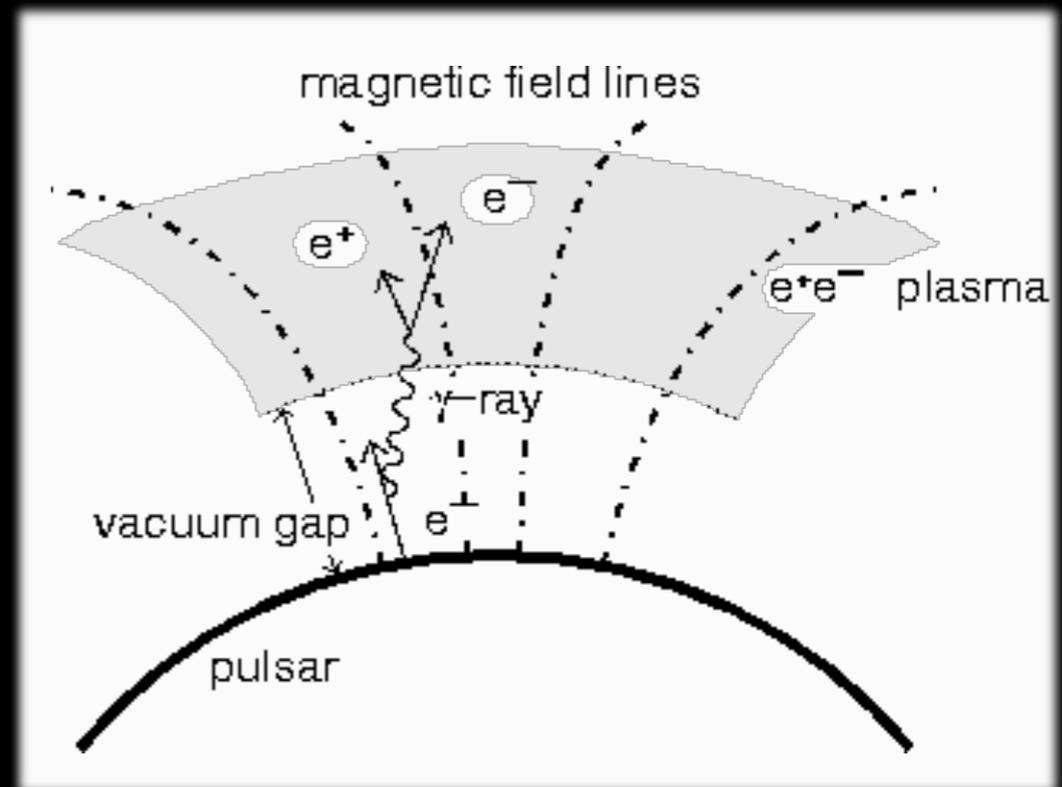
Pairs produced locally:  
Field line by field line basis

Beloborodov (2008)

Timokhin & Arons (2013)

$$J^\mu J_\mu < 0 \begin{cases} \mathbf{J} \cdot \hat{\mathbf{r}} / \rho_{GJ} > 0 : \text{no pairs} \\ \mathbf{J} \cdot \hat{\mathbf{r}} / \rho_{GJ} < 0 : \text{pairs} \end{cases}$$

$$J^\mu J_\mu > 0 : \text{pairs}$$



# General Solution Method

Density is determined *locally* as GJ density

$$V_0 \equiv \begin{cases} \Omega \times r, & \text{flat} \\ \alpha^{-1} (\Omega - \omega_{LT}) \times r, & \text{Kerr} \end{cases}$$

$$\rho_G = -\frac{(\Omega - \omega_{LT}) \cdot B}{2\pi c \alpha} + \frac{V_0 \cdot (\nabla \times B)}{4\pi c}$$

Current is set by *global* magnetospheric structure

Poloidal current flows along magnetic flux surfaces

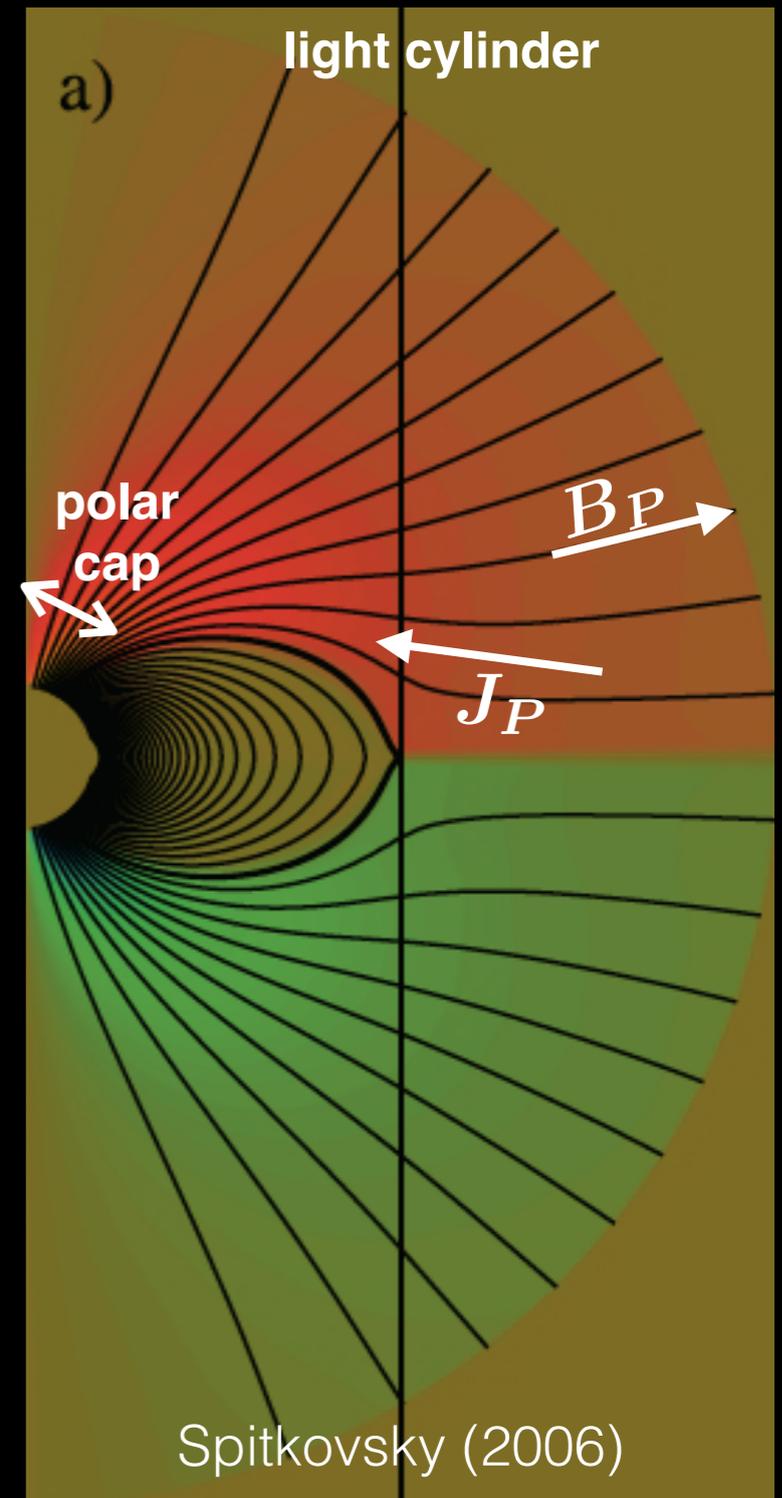
$$\mathbf{J}_P \times \mathbf{B}_P = 0$$

$$\nabla \cdot (\alpha \mathbf{J}) = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

flux surface invariant

$$\alpha \mathbf{J}_P \propto \mathbf{B}_P$$



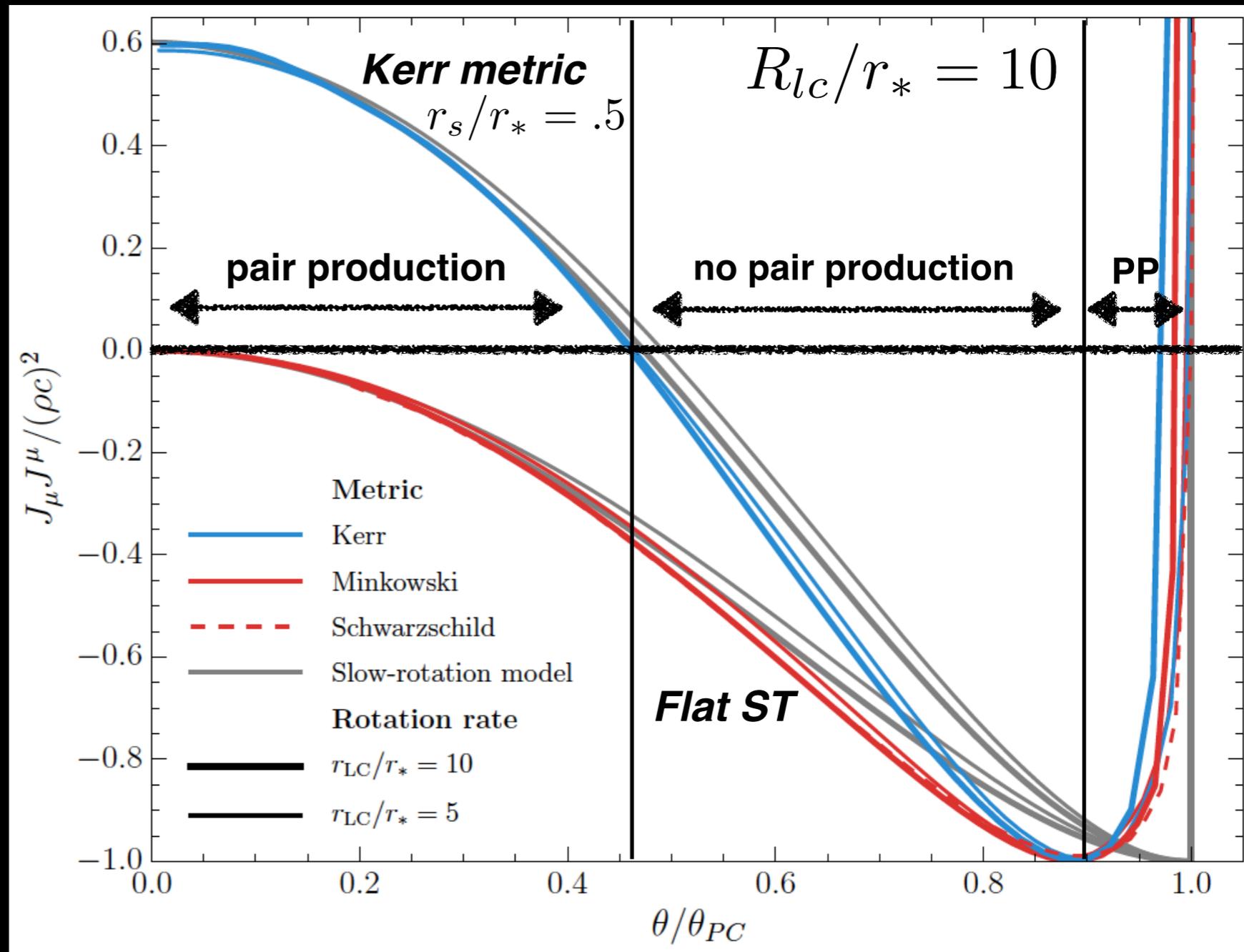
Trace back current on open B field lines from beyond light cylinder  
(simple current distribution) to the polar cap (complicated distribution)

# Dipole: Computational Results

Difference between GR and flat ST due exclusively to frame dragging.

With GR, **two PP regions**.  
Second region due to distributed return current.

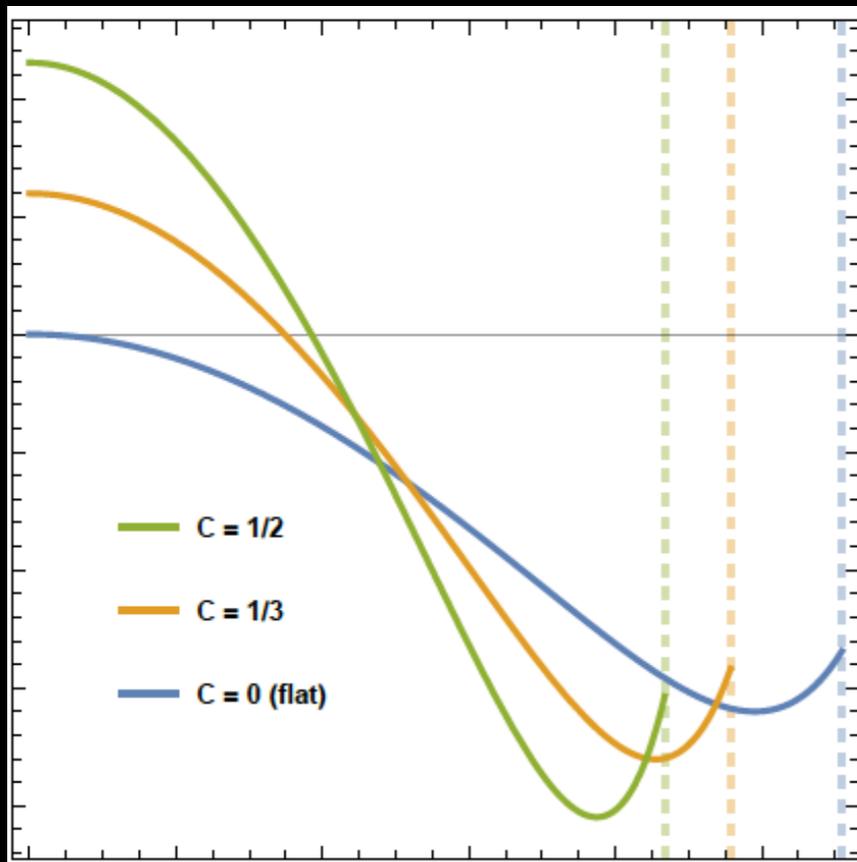
No PP region **always** exists, because poloidal current changes sign.



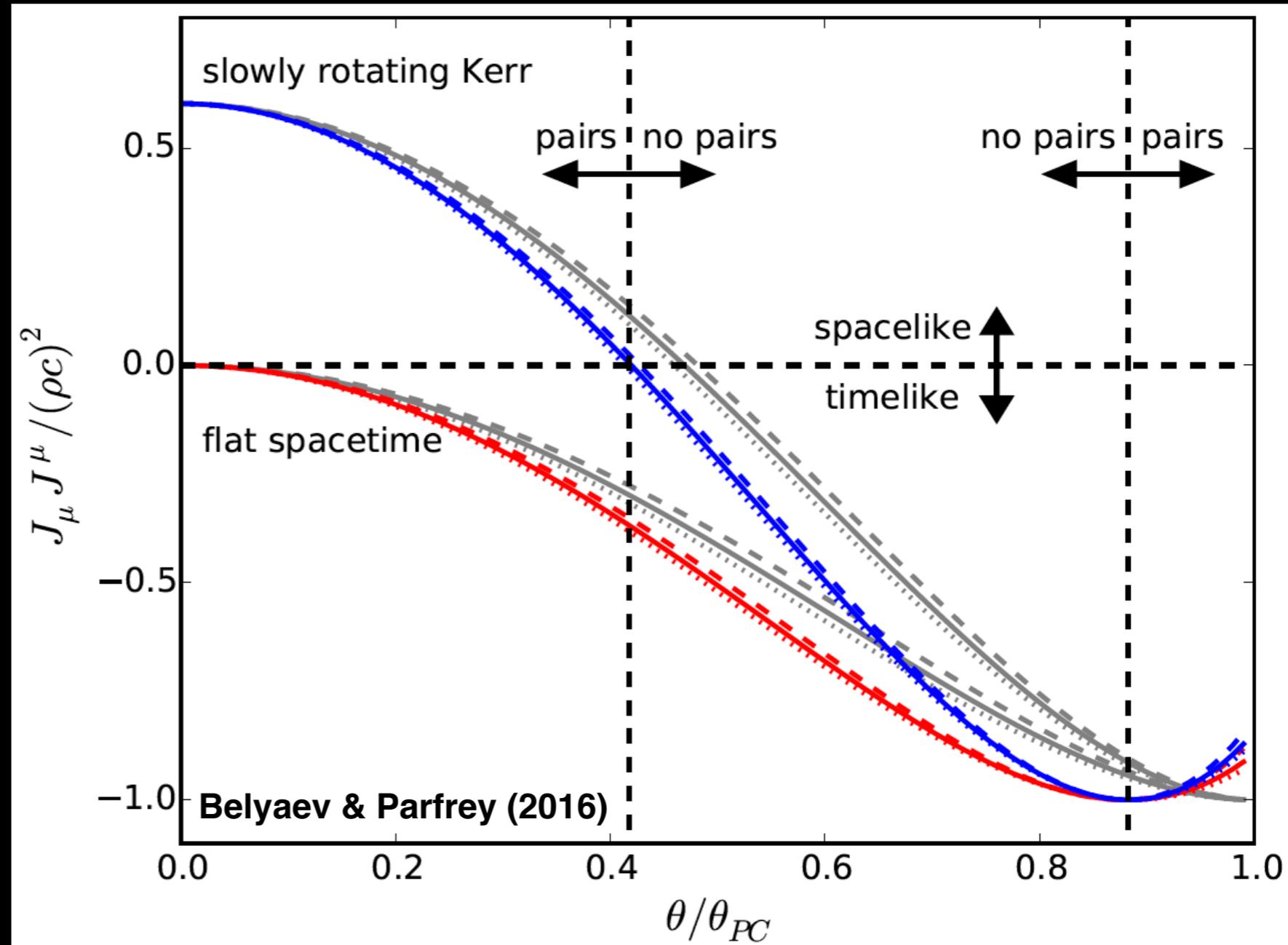
Belyaev & Parfrey (2016)

# Dipole: Analytical Results

- Gray curves — split monopole
- Color curves — SM + 1st correction
- Different linetypes — different amounts of open flux and different ratios of  $R_{lc}/r_*$ .



Gralla, Lupsascu, Philippov (2016)



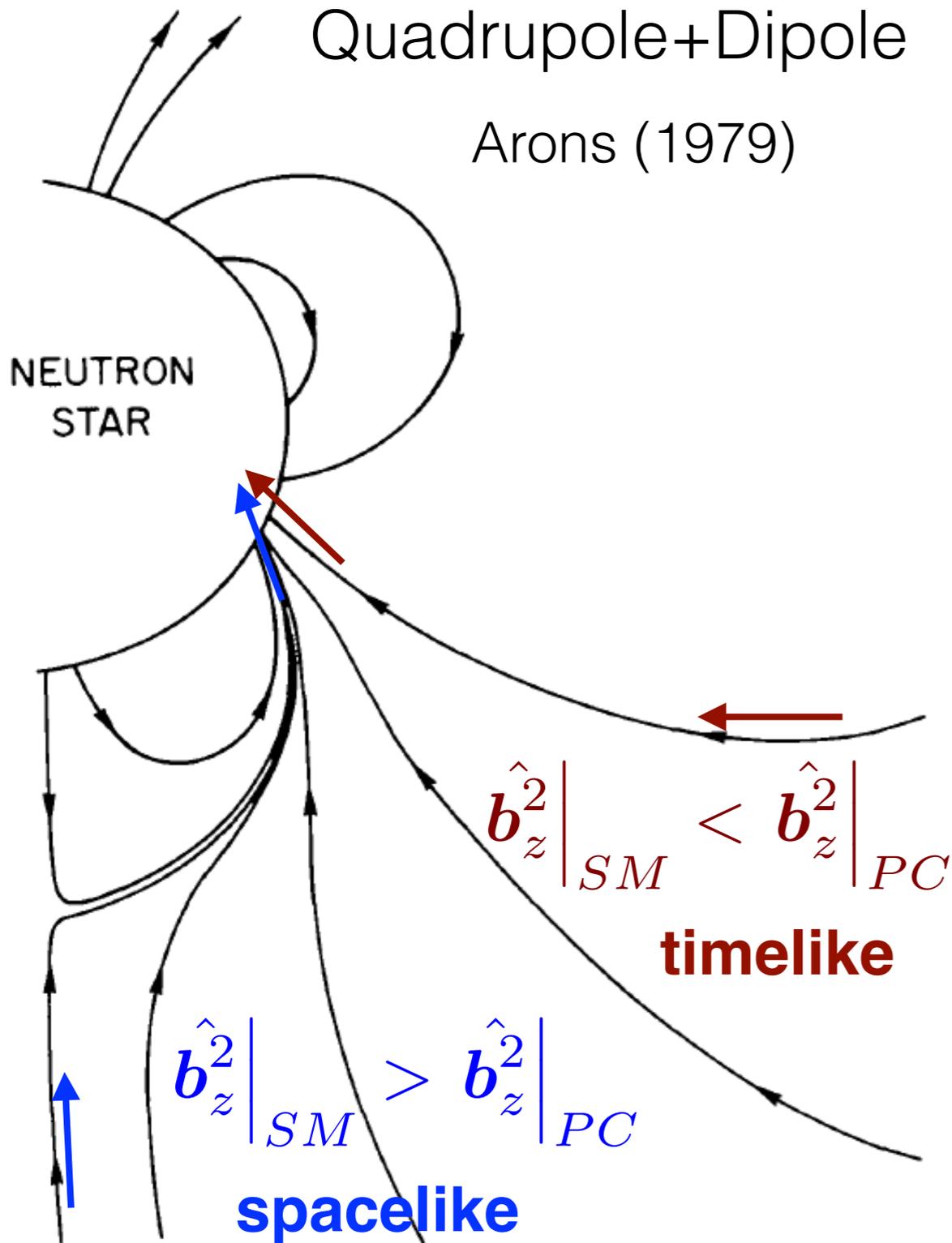
$$B_r^{(1)}(\theta) = B_0(r/r_0)^{-2} [1 + 0.02 \sin \theta + 0.22(\cos \theta - 1) - 0.07(\cos \theta - 1)^4] \times \text{sign} \cos \theta.$$

Tchekhovskoy, Philippov & Spitkovsky (2016)

# More General Surface Field

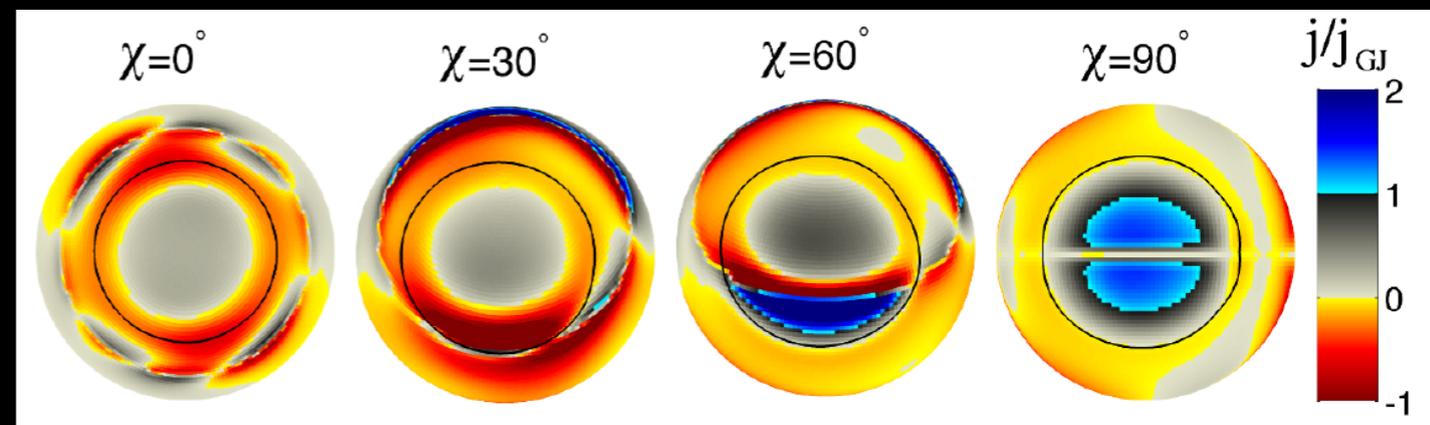
Quadrupole+Dipole

Arons (1979)



## Extension to 3D

Bai & Spitkovsky (2010)



Gruzinov (2005)

$$\mathbf{B} \times \left[ \nabla \times \left( \mathbf{B} + \frac{\mathbf{V}_0}{c} \times \left( \frac{\mathbf{V}_0}{c} \times \mathbf{B} \right) \right) \right] = 0$$

**Current-like 3D vector invariant  
along magnetic field lines.**

# *Part II: Pulsar Y-Point*

## **Goals:**

- Understand particle trajectories at the Y-point and in the current sheet beyond it.
- Understand dissipation at the pulsar Y-point, i.e. the role of pair multiplicity on dissipation and kink instability.

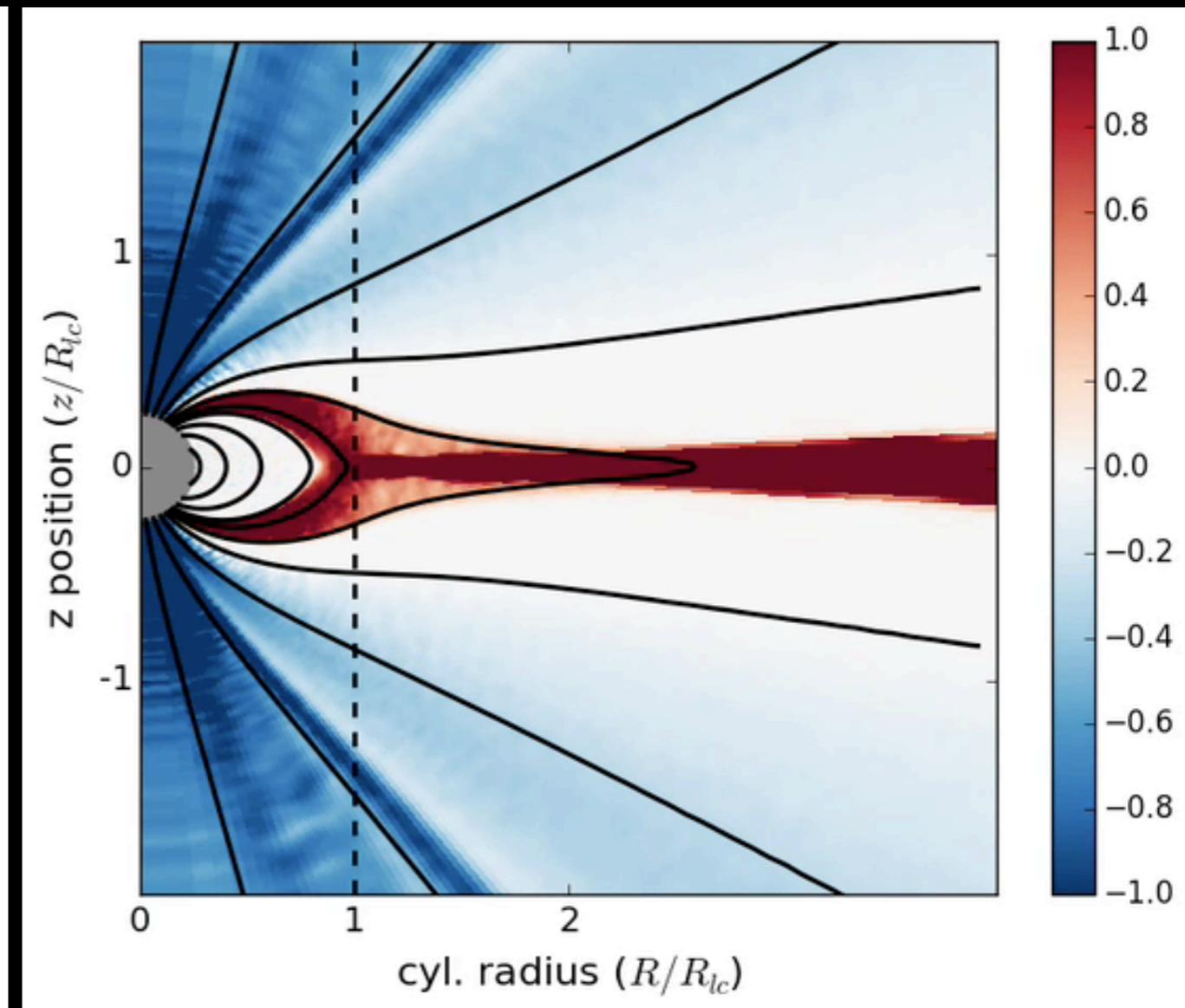
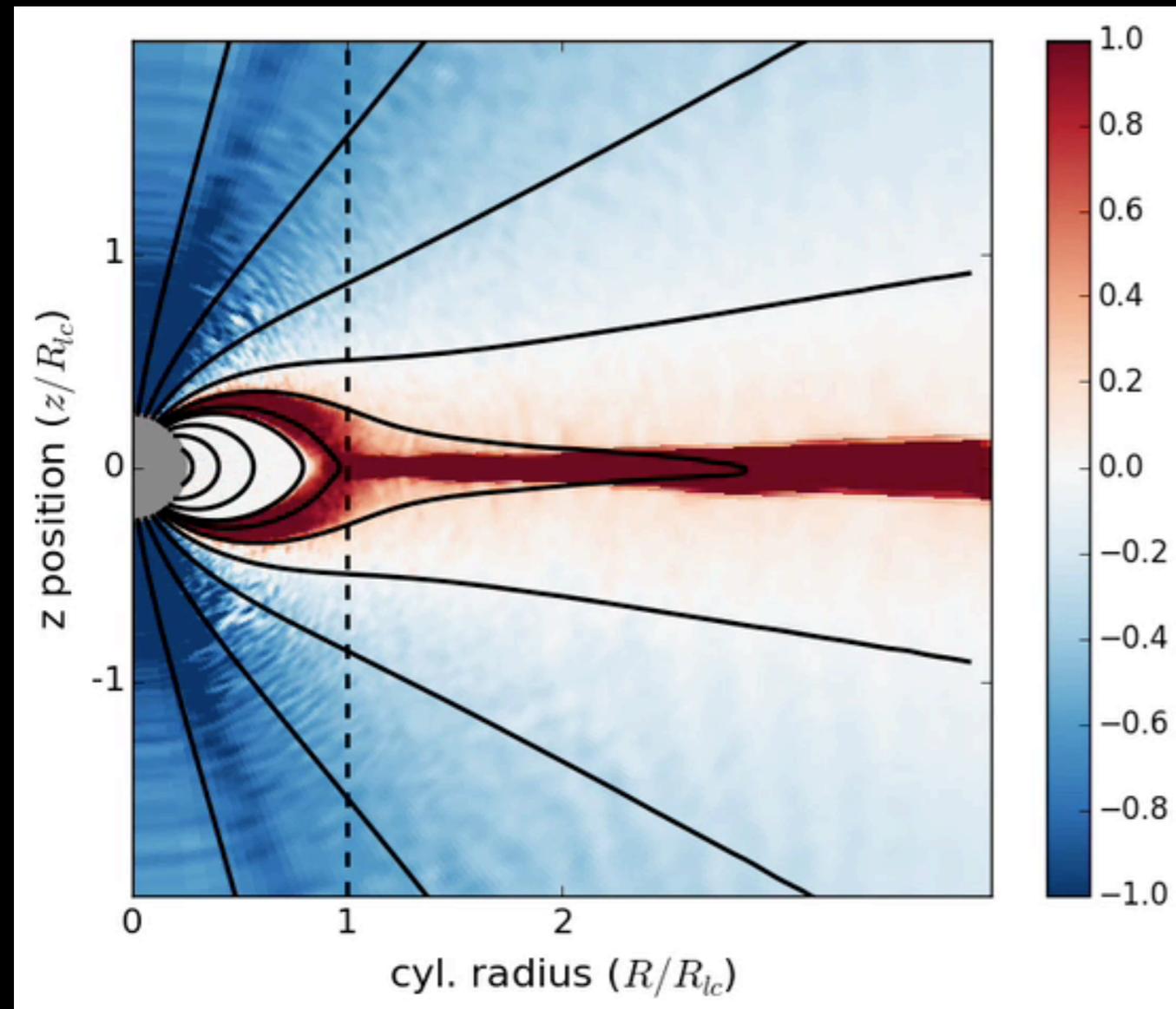
## **Results:**

- Axisymmetric magnetosphere is inherently dissipative.
- Y-point can extend inside light cylinder with PIC due to finite Larmor radius effect.
- Radiation reaction likely to be important for particle trajectories at the Y-point.

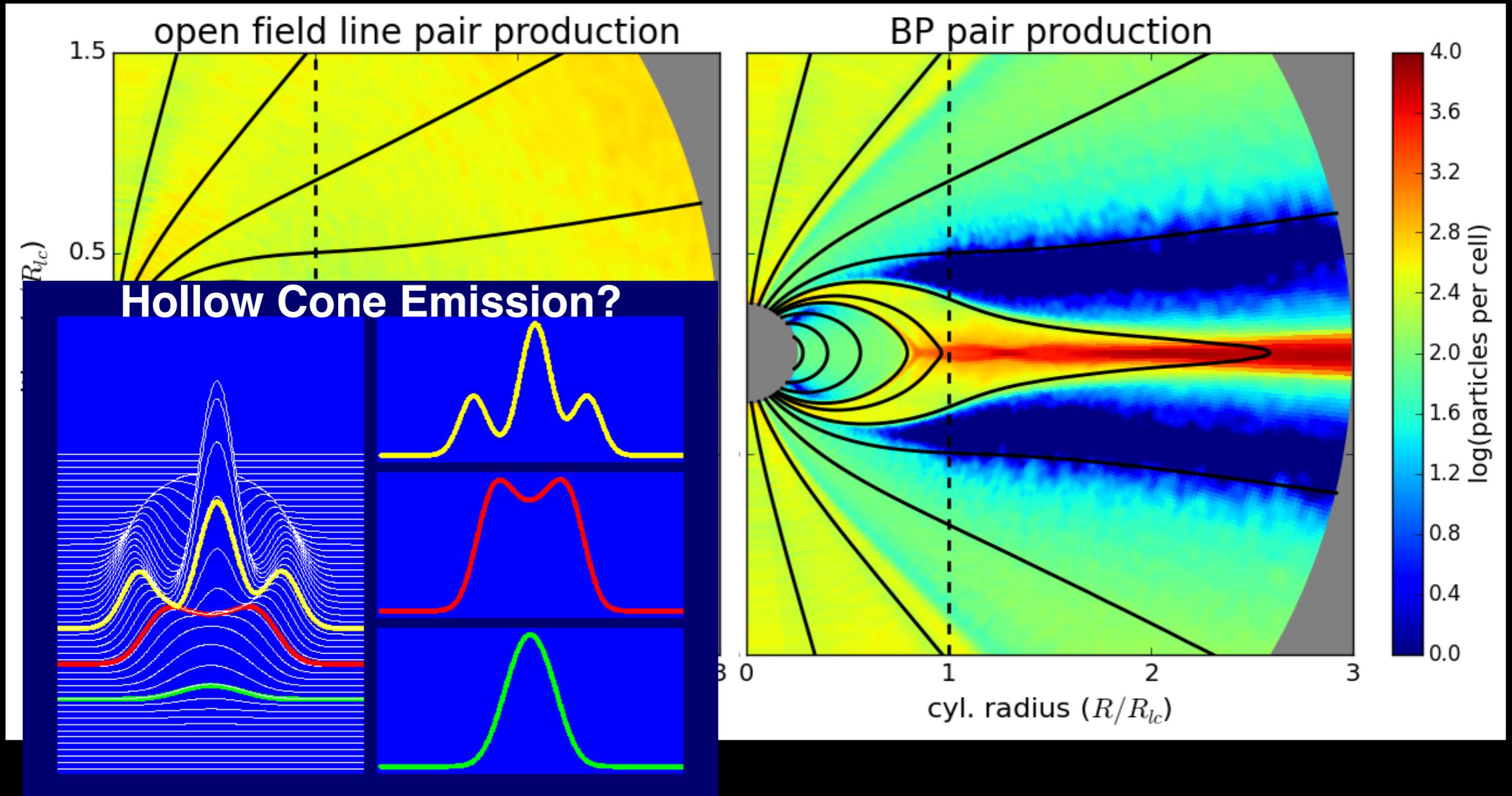
# Magnetospheric Current

*PP Open  
Field lines*

*BP Model*

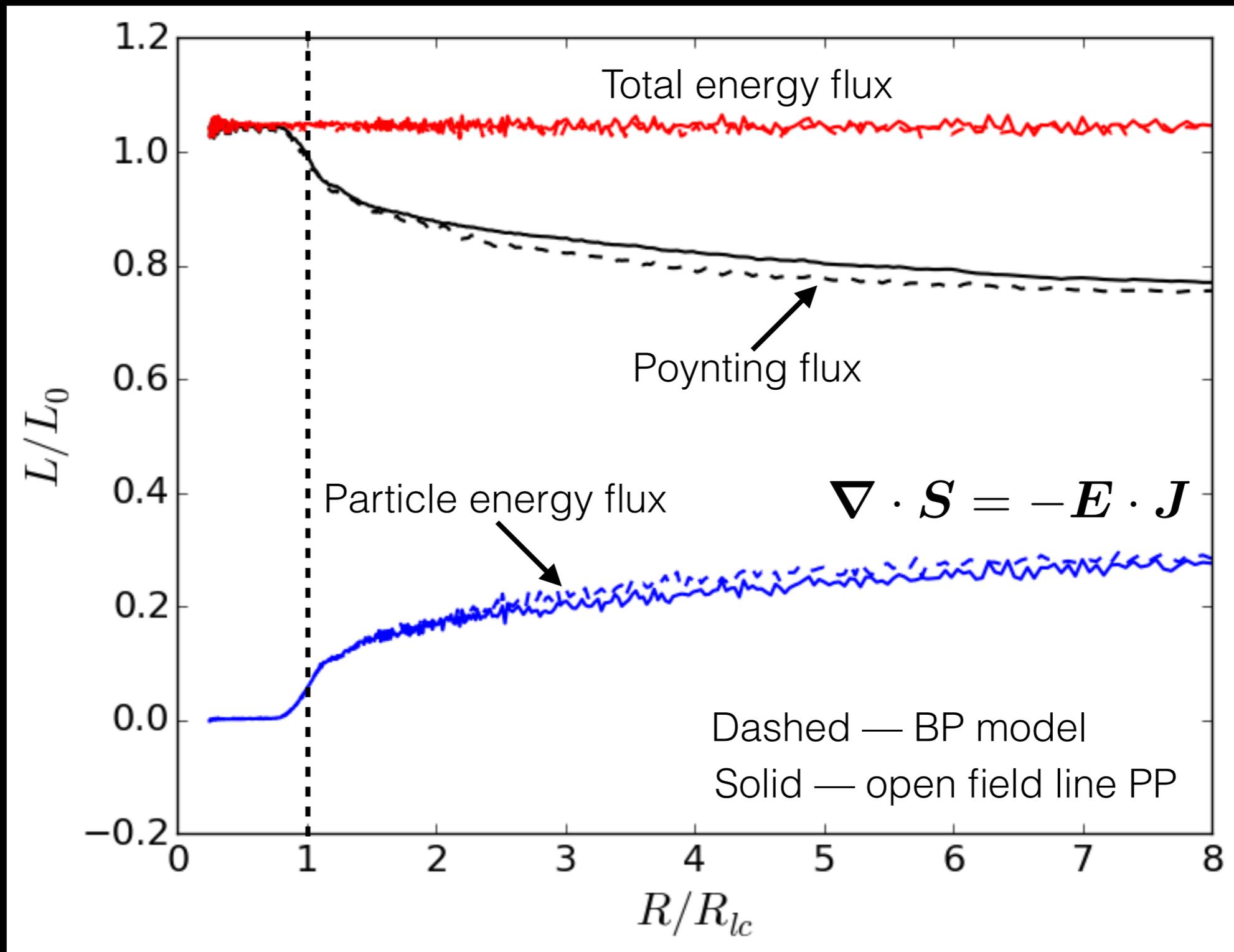


# Mind the Gap!

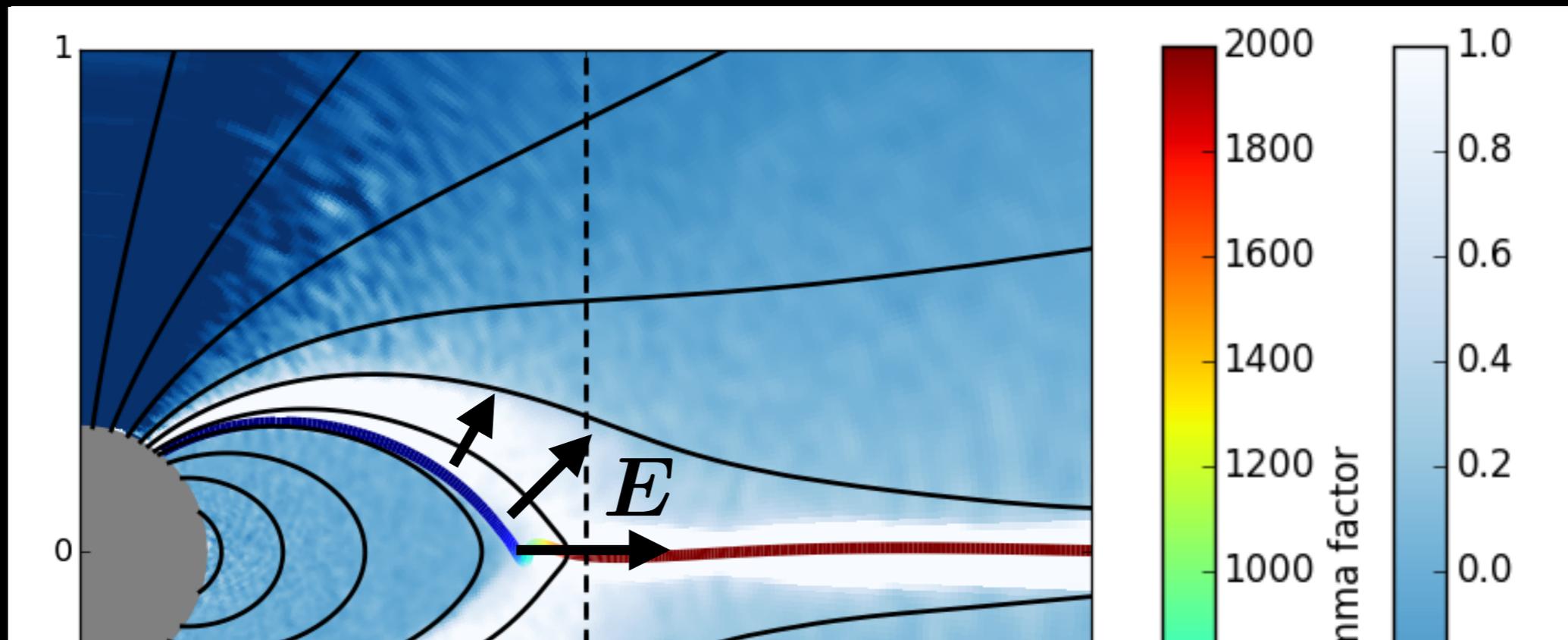


**BP PP model has a large outer gap above the current sheet.**

# Luminosity & Dissipation



# Positron Trajectory

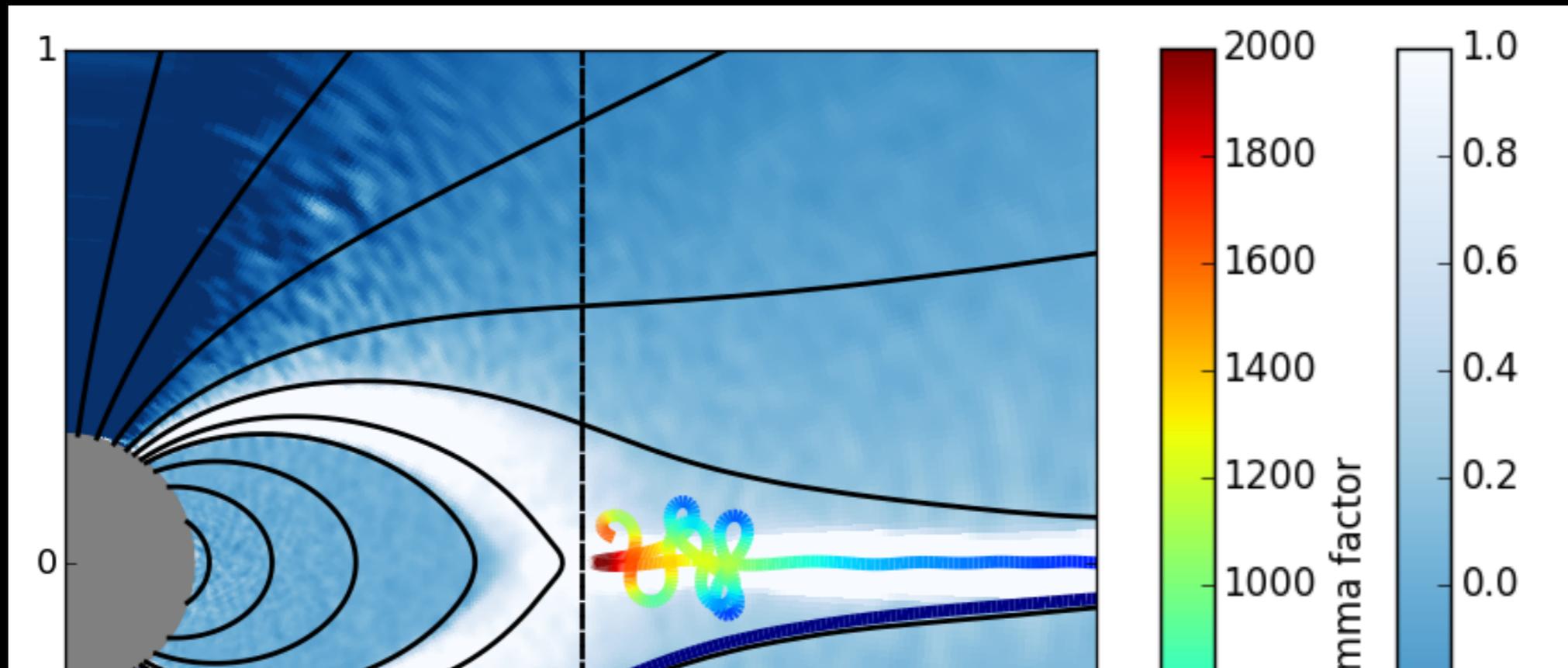


Drift velocity close to speed of light near Y-point:

$$v_{D,\phi} = -E_r / B_z \lesssim c, \quad B'_z = B_z / \gamma_D$$

Particles in closed region accelerate radially across field lines (voltage drop). They cross light cylinder before turning around and escape to infinity in current sheet.

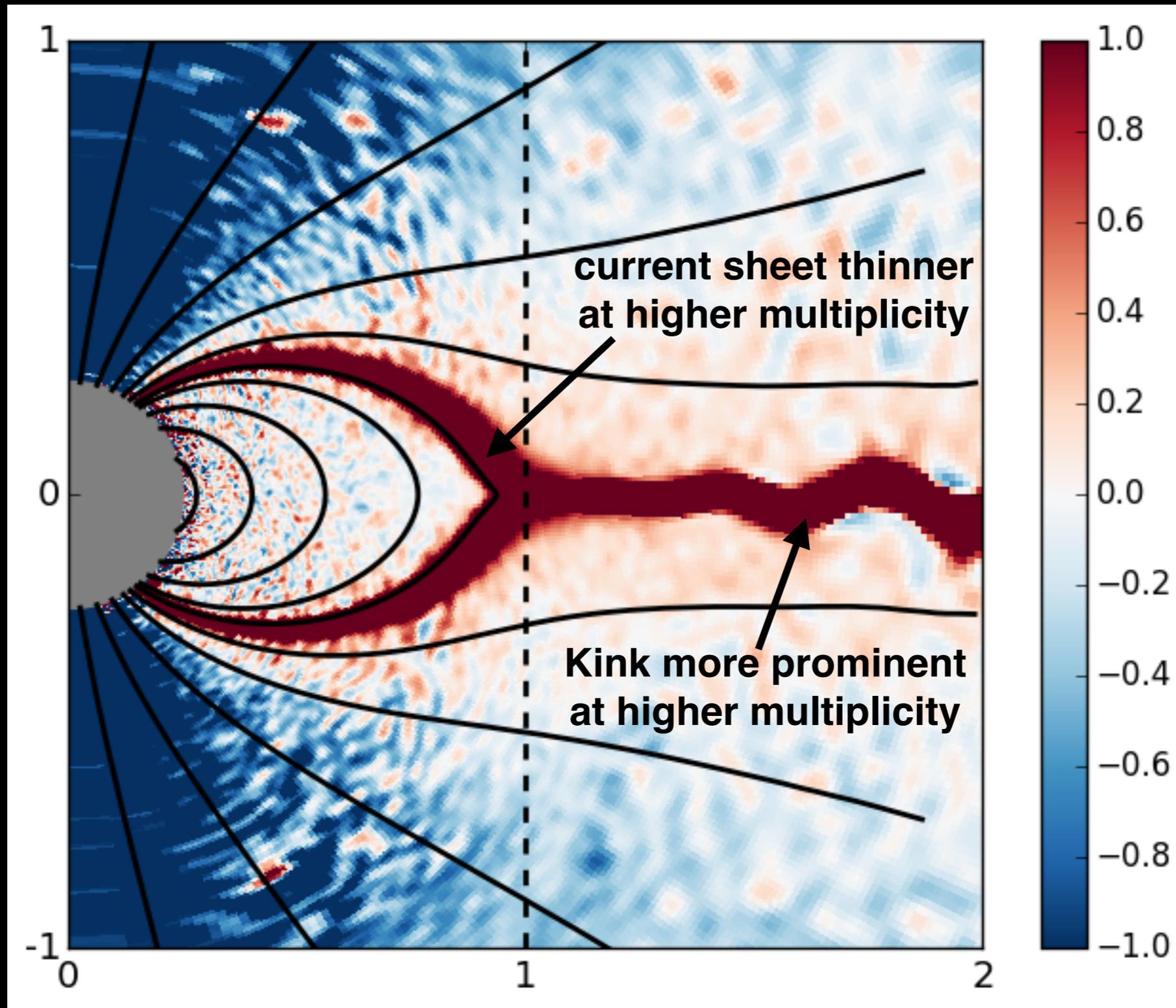
# *Electron Trajectory*



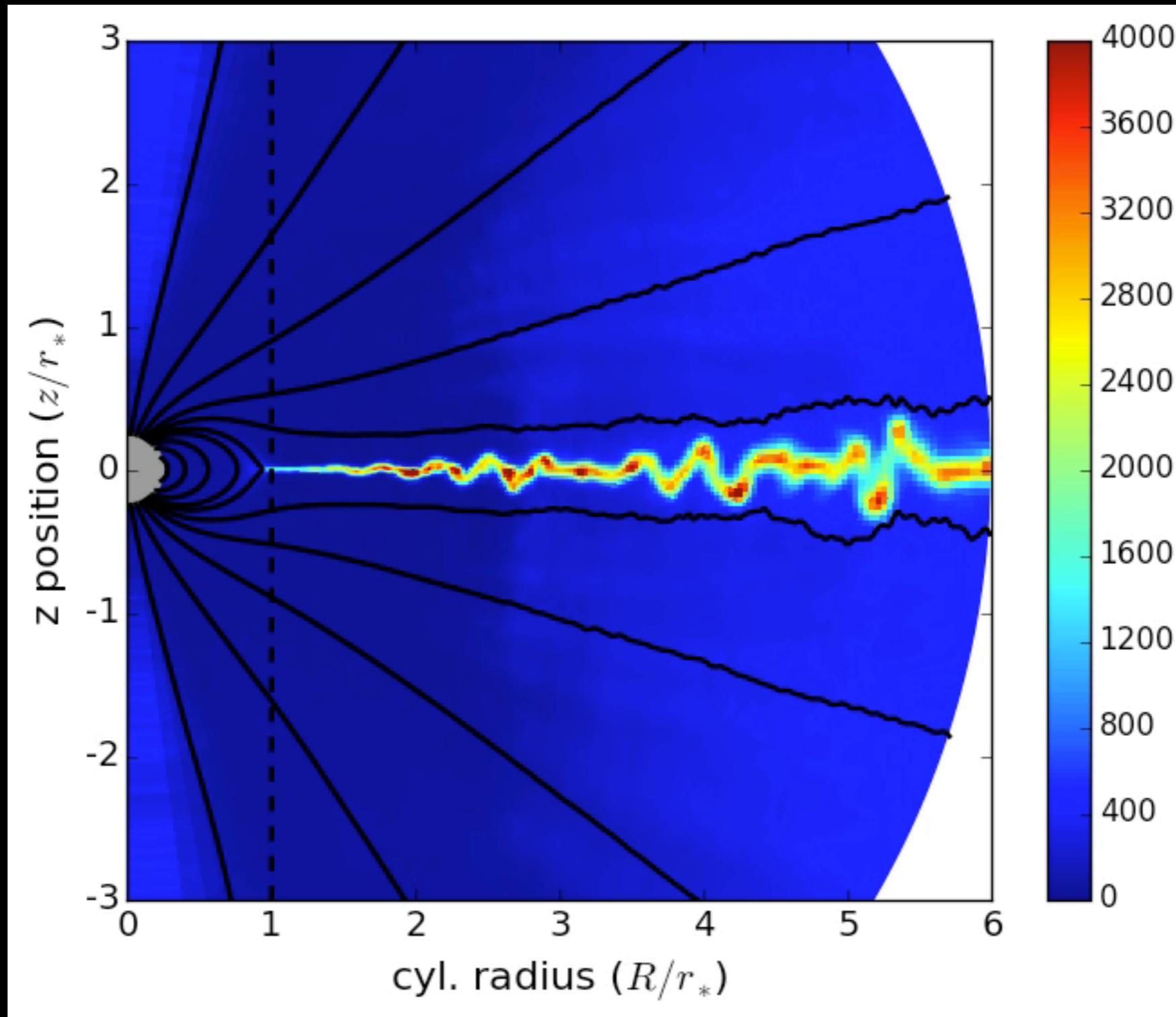
Electrons entering current sheet are sent back towards Y-point  $\rightarrow$  current sheet mostly positive charges

Backflowing electrons cannot cross Y-point due to magnetic mirror effect. With radiation reaction it should be possible for electrons to flow back through Y-point.

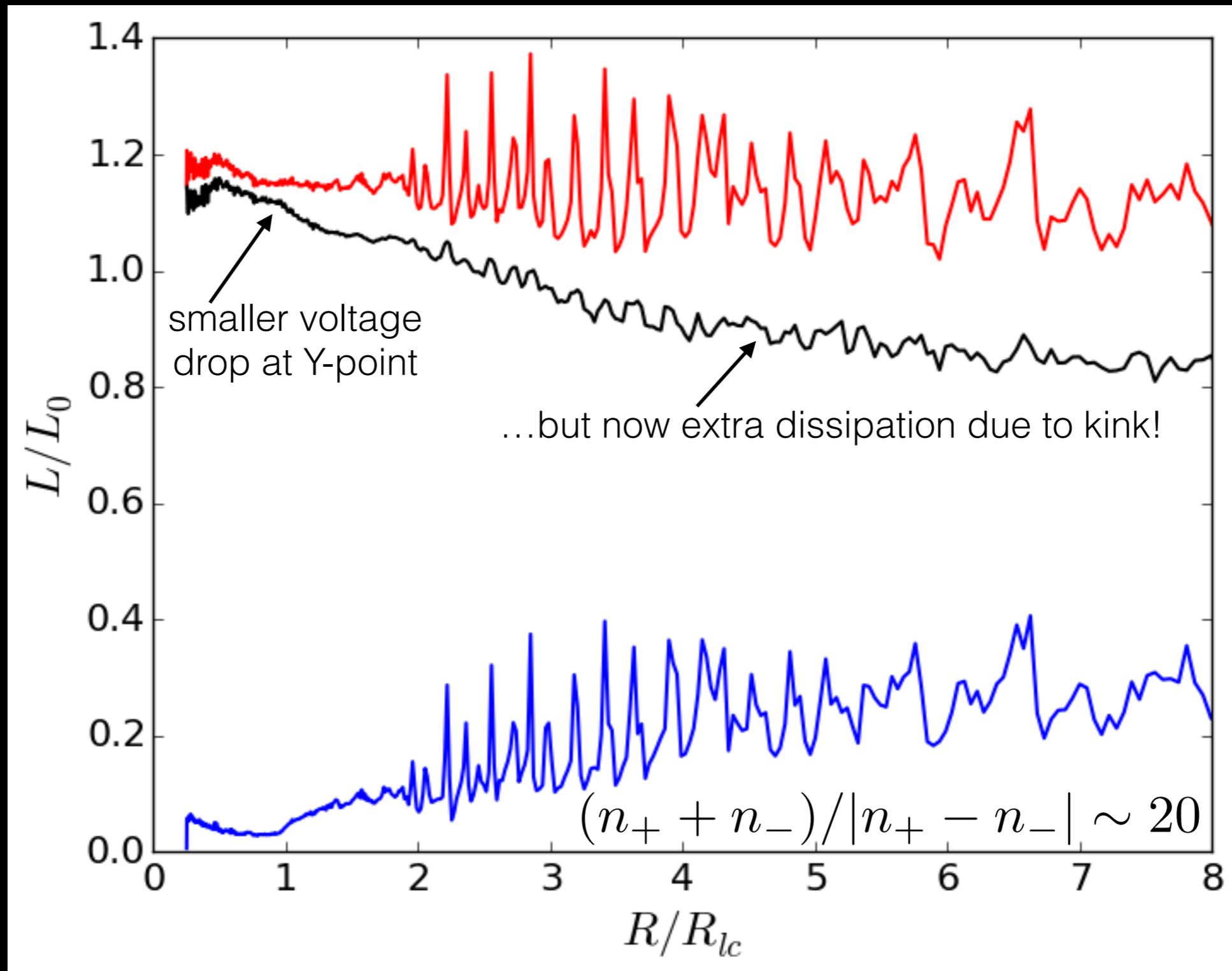
# *Y-Point at Higher Pair Multiplicity*



# *Current Sheet Kink Instability*



# *Dissipation due to Kink*



# *Conclusions*

1. **Targeted studies** of polar cap and Y-point beyond force-free limit.
2. **Polar cap** — computed spatial distribution of pair production with implications for radio & high energy emission as well as for gaps.
3. **Y-point** — studying particle trajectories and dissipation. Around 20% of FF luminosity dissipated in current sheet. Dissipation is inherent to aligned rotator.