Numerical modeling of pulsar magnetospheres: from force-free to particles

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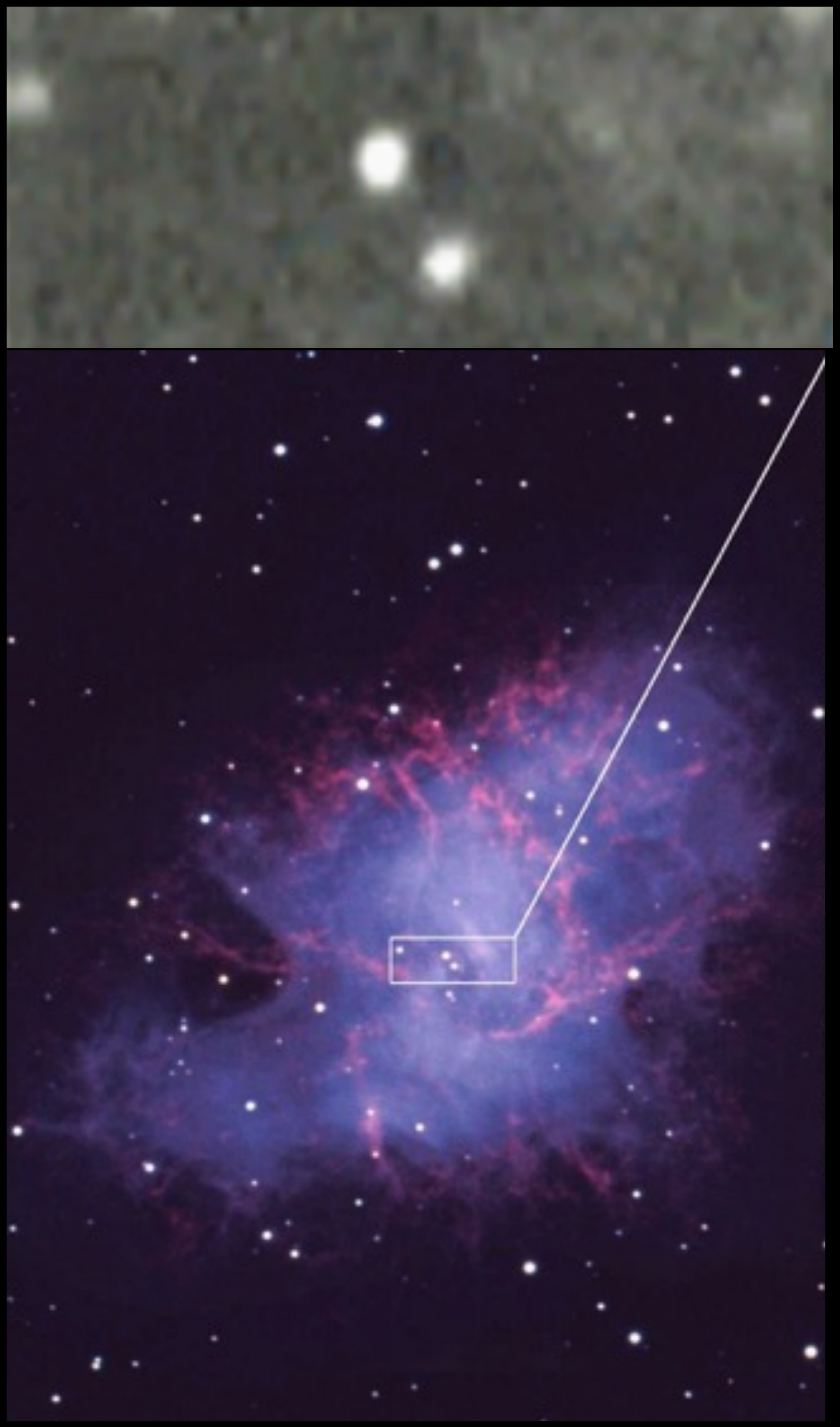
Outline

- Pulsar magnetosphere: background and open questions after 49 years
- Pulsar models: pros, cons and fails
- Plasma filled models
- Kinetic simulations of magnetospheres
- Conclusions and outlook

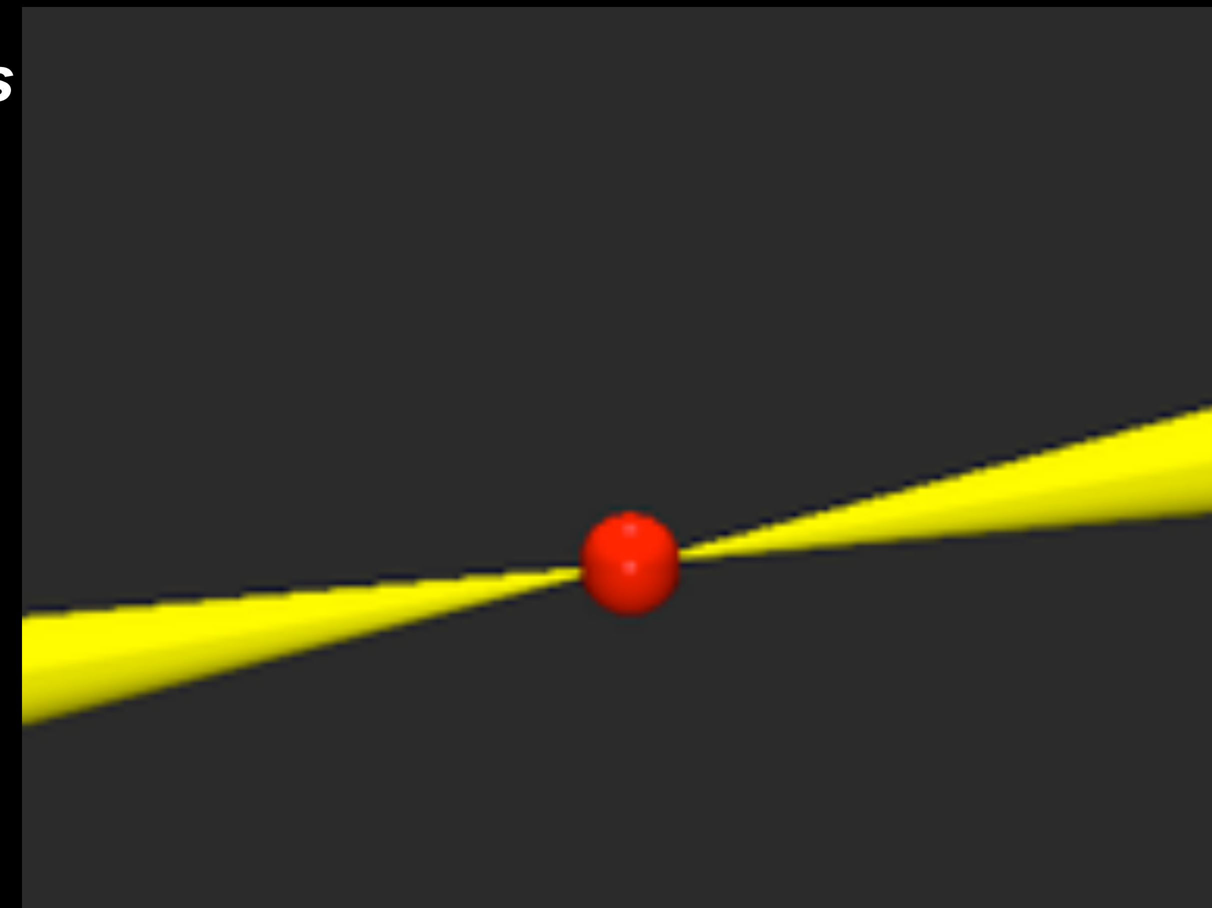
Pulsars

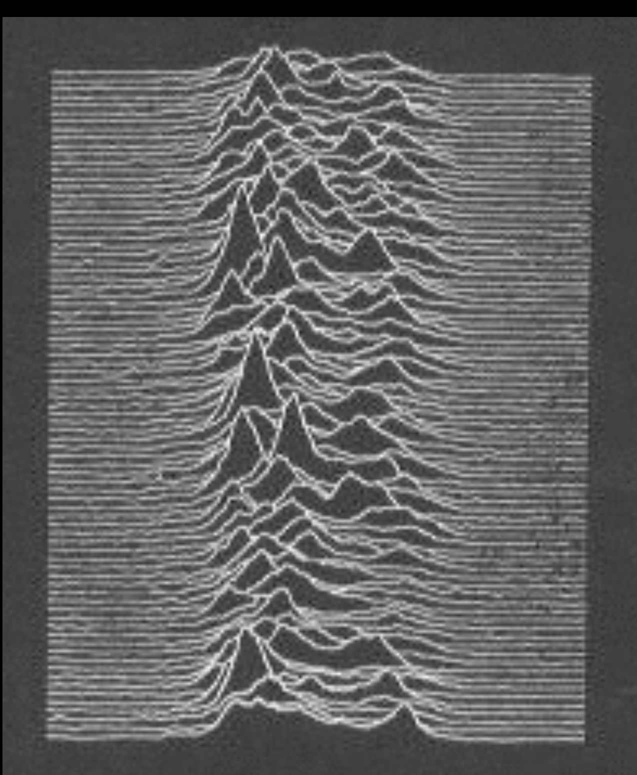
- ***Pulsars are neutron stars, born in supernova explosions***

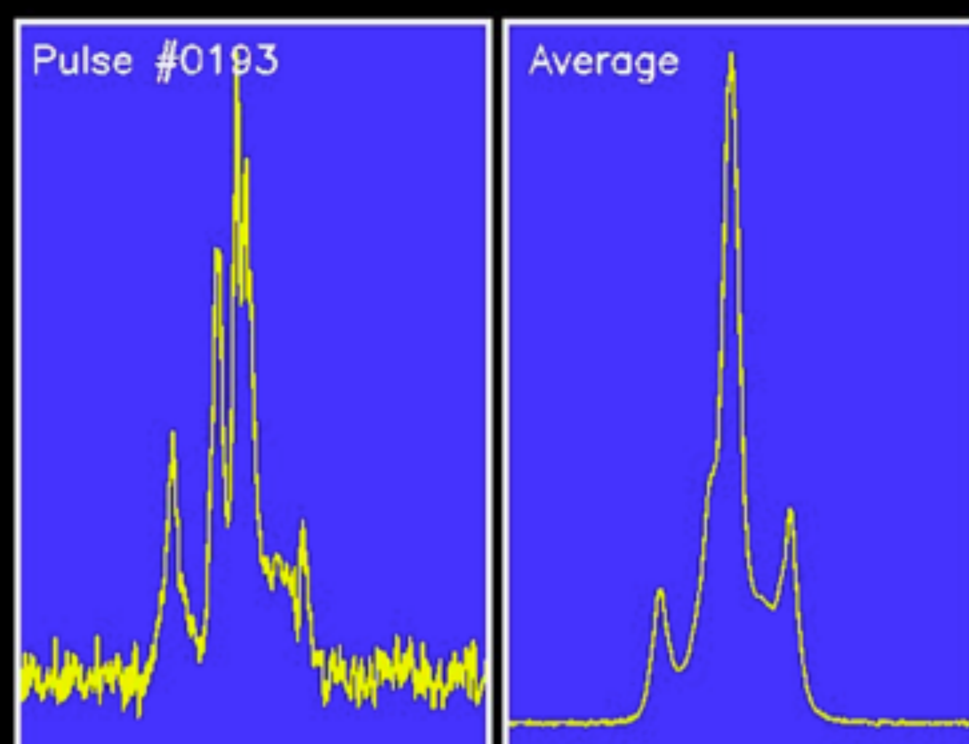


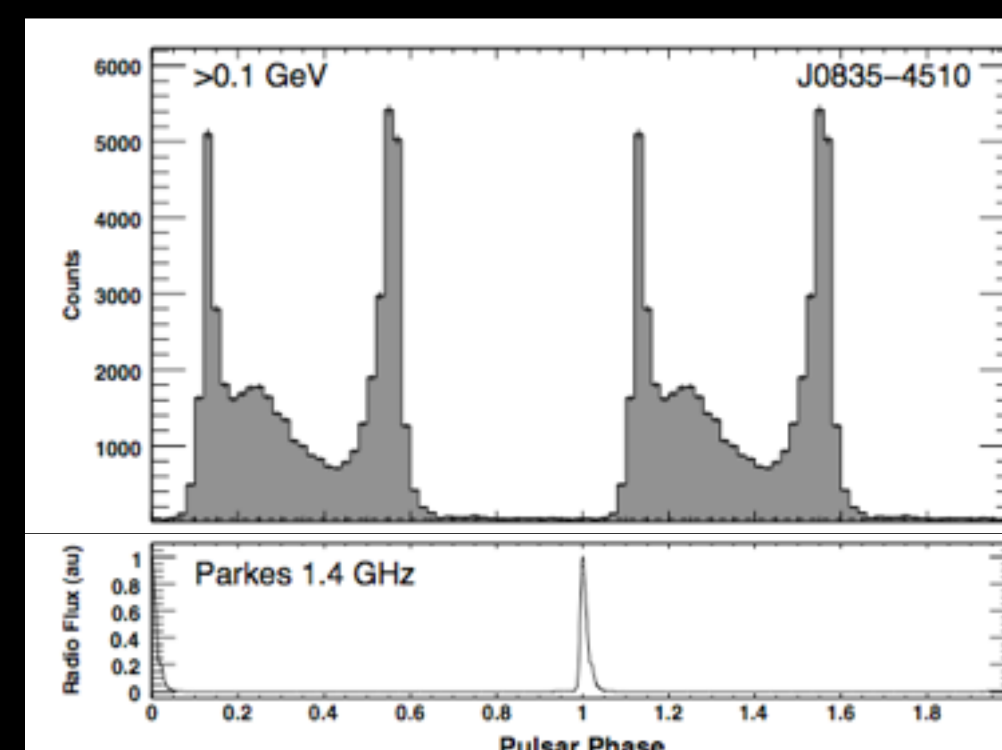
Pulsars: cosmic lighthouses

- *Neutron Star -- 10km in radius, 1.4 Solar Mass*
- *Central densities -- density of nuclei*
- *Gravity is 100 billion times Earth gravity*
- *Pulsars emit from radio to gamma ray*
- *Spin periods -- from 1.5 ms (700 Hz!) to 8 sec*
- *Individual pulses quite different, but average profile is very stable (geometry)*
- *Sweeping dipole magnetic field*
- *Pulsars spin down -- inferred B field 10^{12}G*

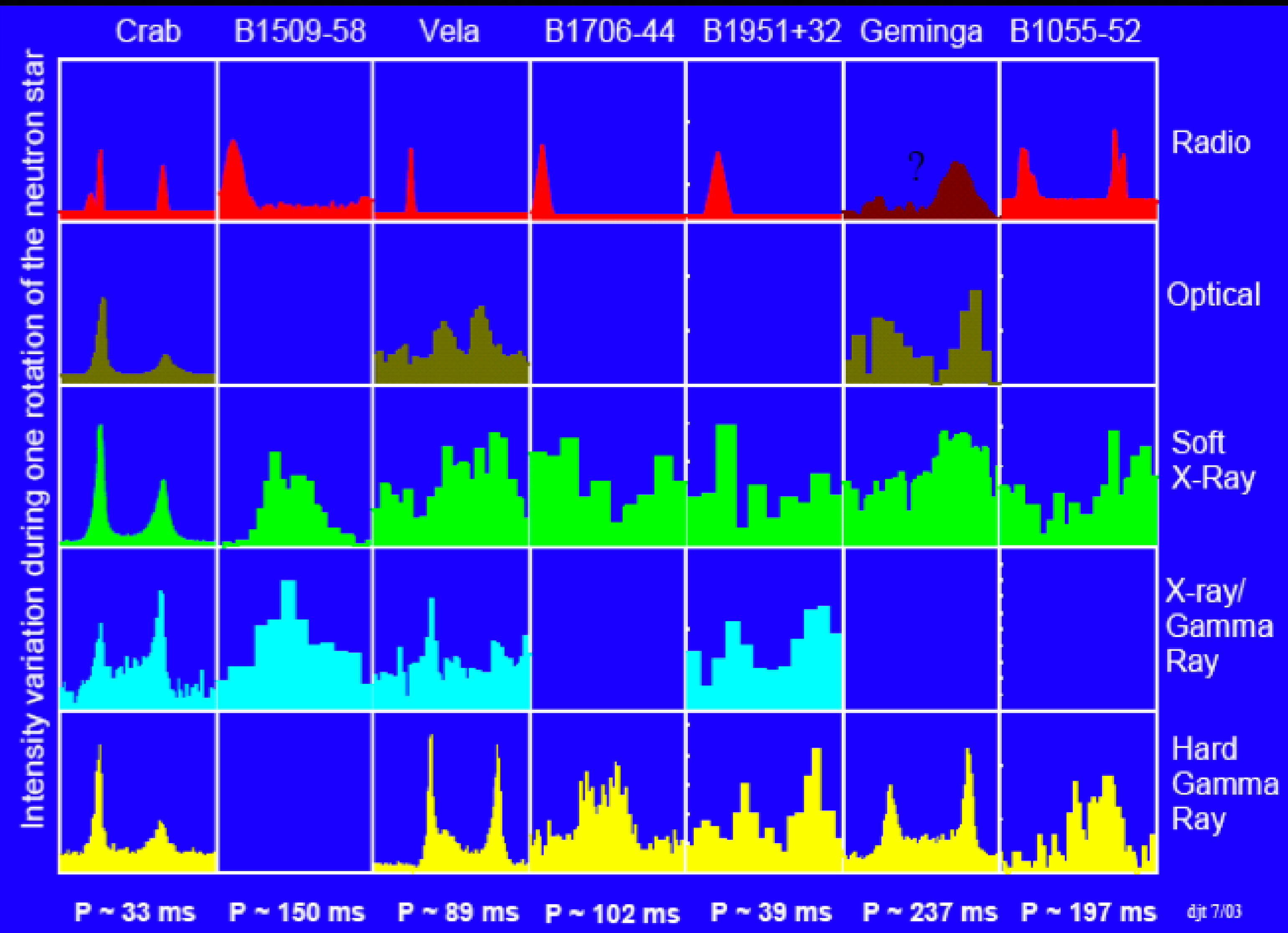


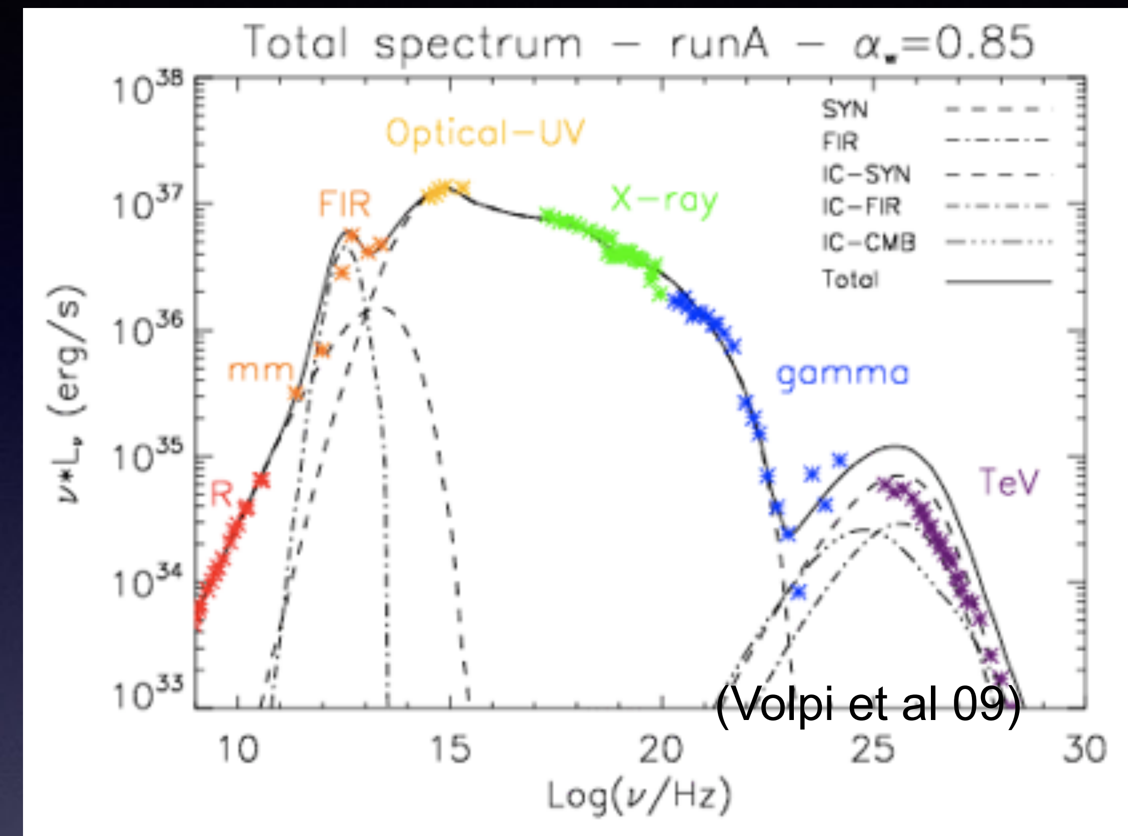


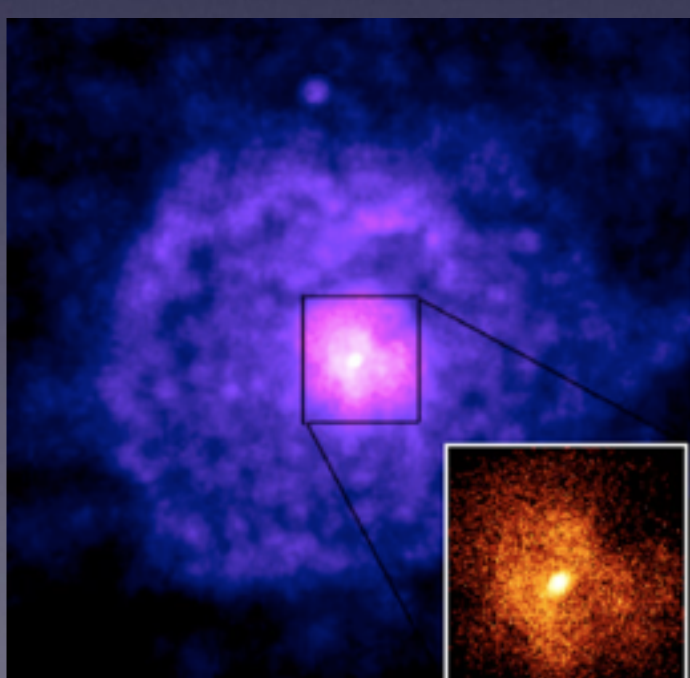




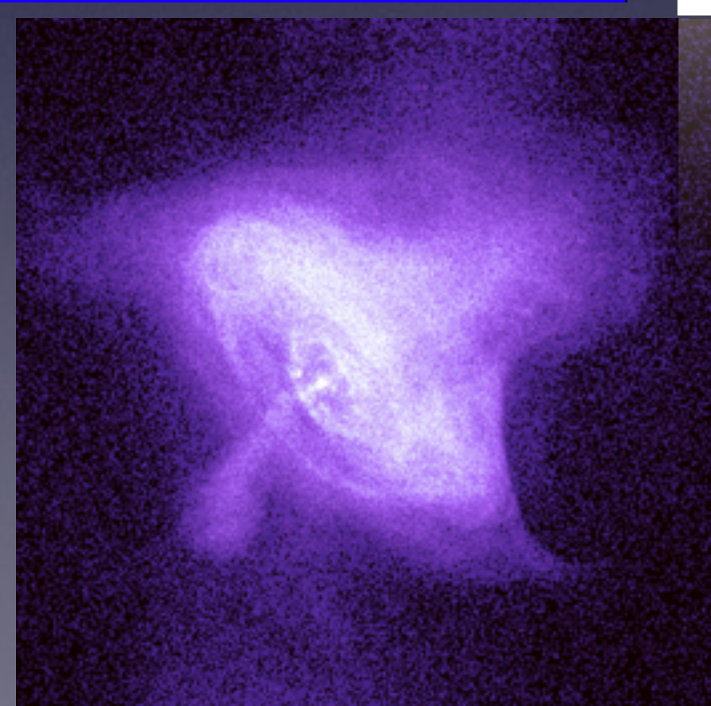
•Broadband pulsed emission, now > 100 GeV (Veritas).
•PWNe: radio-TeV. 10^{40} pairs/sec. Also, flares!



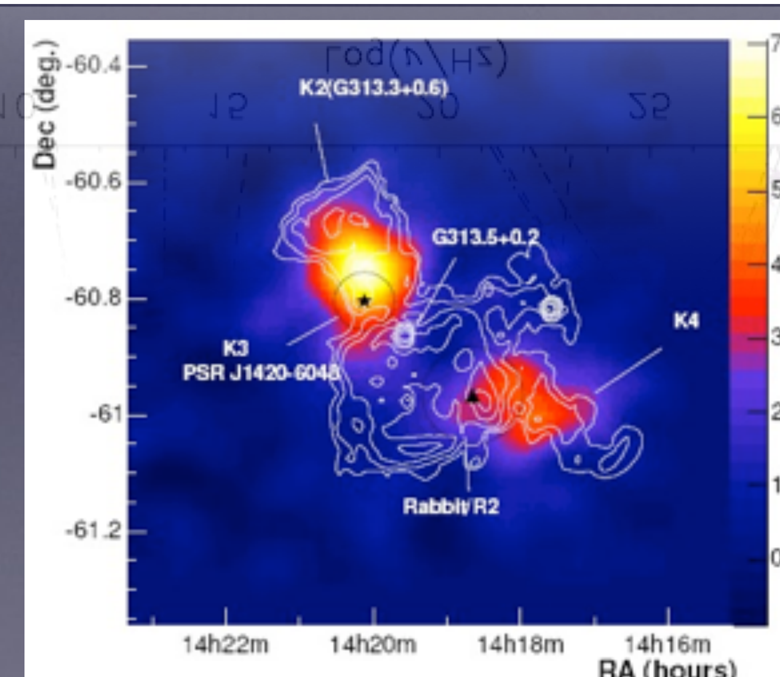




G21.9 (Safi-Harb et al 2004)



Crab (Weisskopf et al 2000)



HESS J1420 (Aharonian et al 2006)

Pulsars: observationally driven

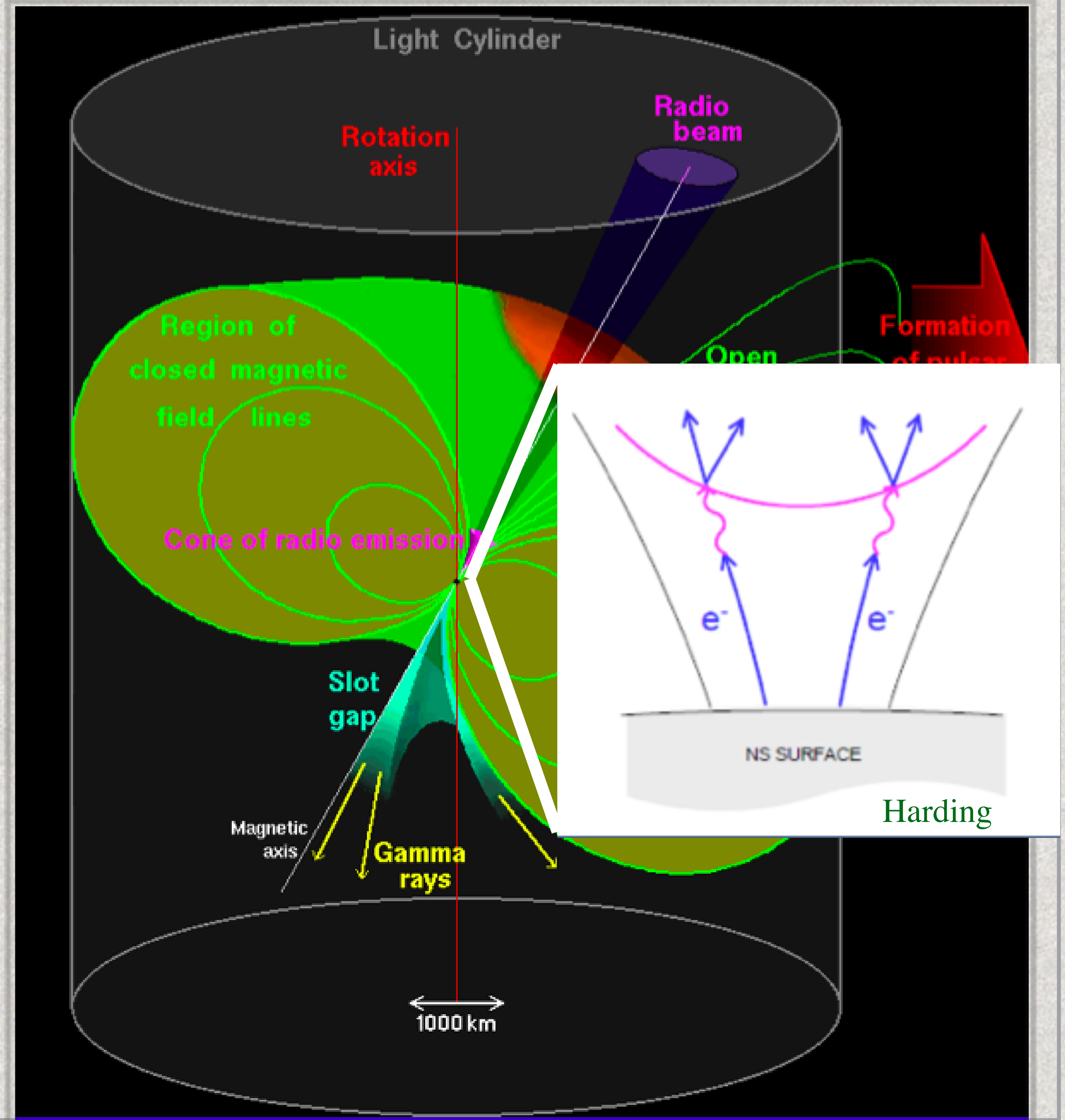
Pulsar theory:

Open questions:

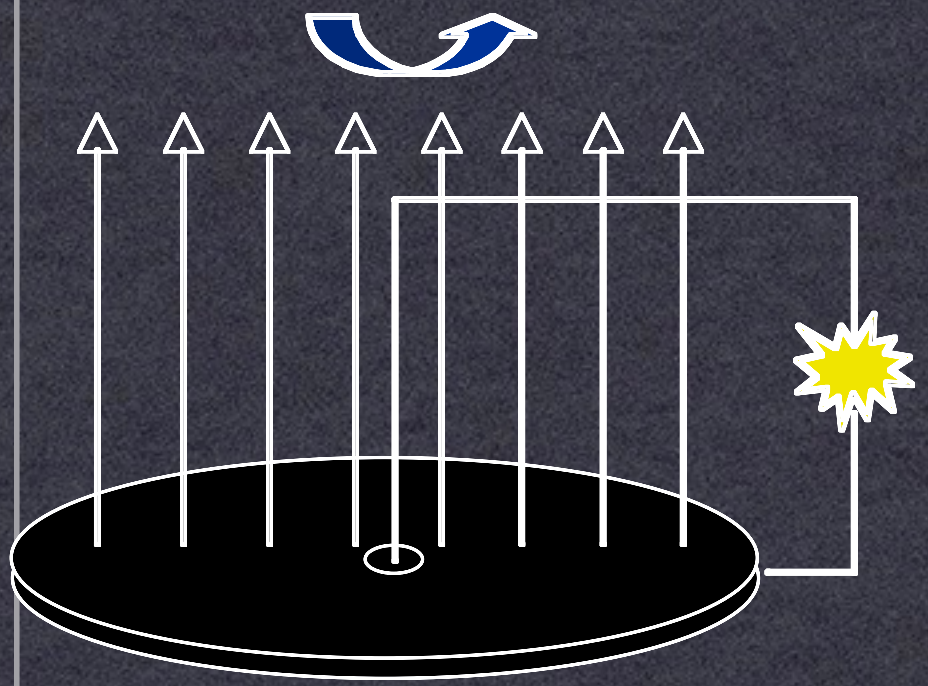
- What is the structure of pulsar magnetosphere and how do pulsars spin down?
- What are the properties of the wind near pulsar? In the nebula?
- What causes pulsed emission?
- How are observed spectra generated? (how particles are accelerated?)

Magnetospheric cartoon

- Open & closed (corotating) zones.
- Light cylinder
- Sweepback
- Plasma is born in discharges
- Minimal (Goldreich-Julian) charge density

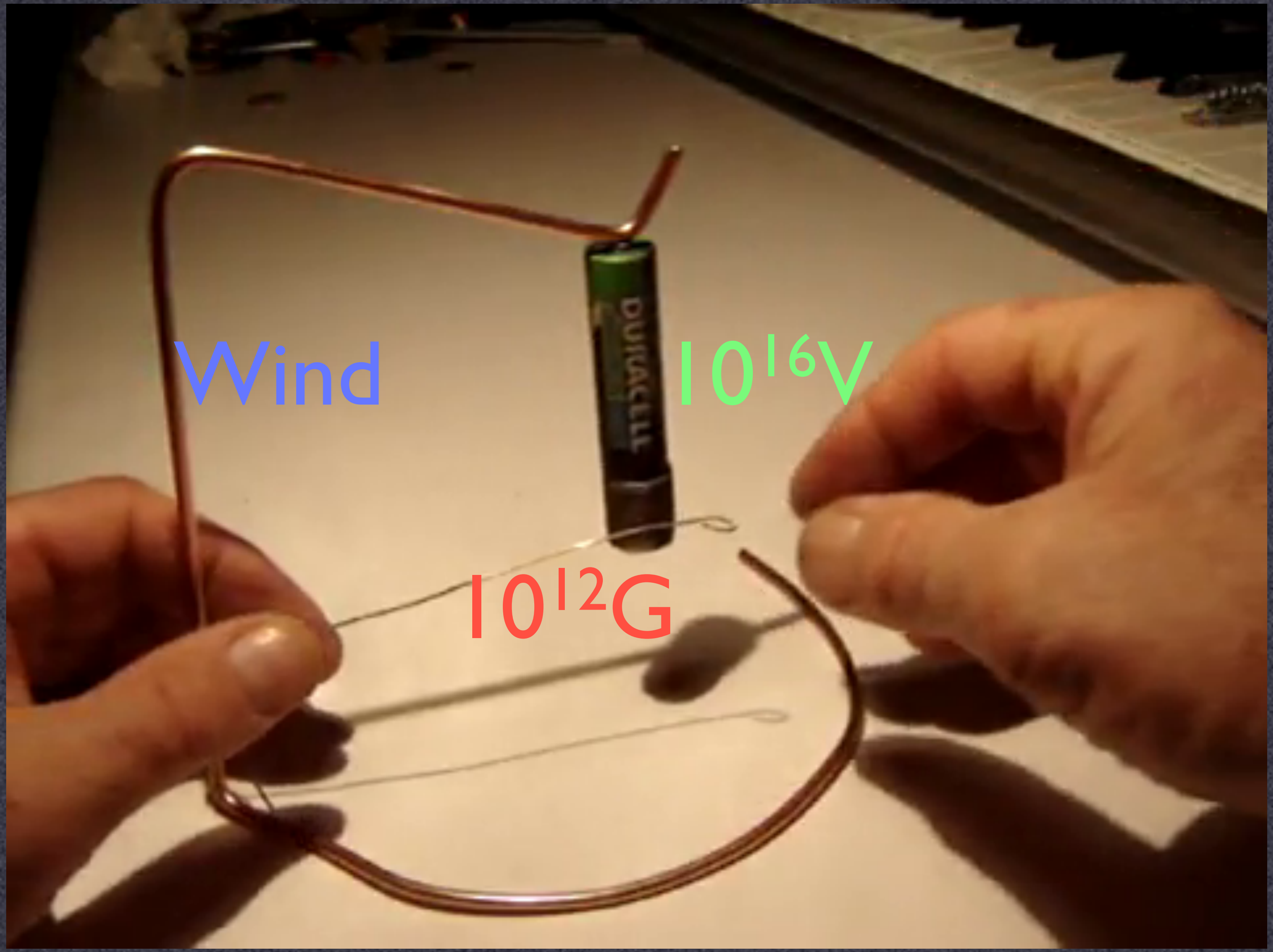


Pulsar physics: unipolar induction



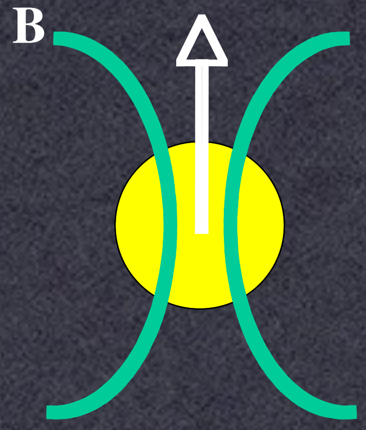
Faraday disk

$$\phi_0 = \Omega B a^2 / c$$



Pulsar "in reverse"

Rule of thumb: $V \sim \Omega\Phi$; $P \sim V^2 / Z_0 = I\,V$



Crab: B ~ 10^{12} G, Ω ~ 200 rad s^{-1}, R ~ 10 km

Voltage ~ 3 x 10^{16} V; I ~ 3 x 10^{14} A; Power ~ 10^{38}erg/s

And yet it spins down...

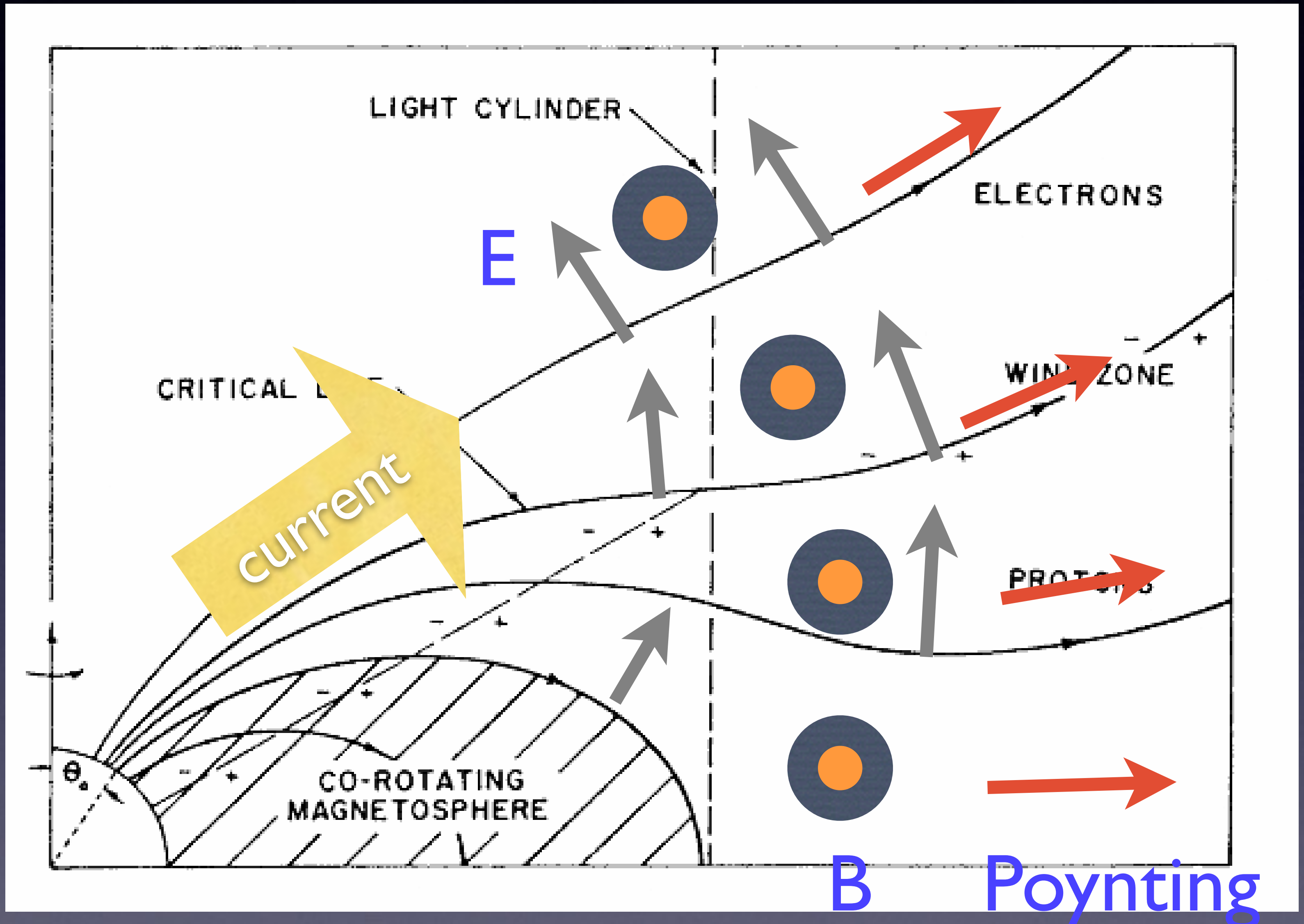
$$j_{GJ} = \rho_{GJ} c = -\frac{\vec{\Omega}\cdot\vec{B}}{2\pi}$$

$$\frac{1}{4\pi}\nabla\cdot\vec{E} = \rho_{GJ} = -\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}$$

$$\rho_{GJ} = -\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}$$

- Corotation electric field
- Sweepback of B field due to poloidal current
- ExB -> Poynting flux

- Electromagnetic energy loss



Goldreich & Julian 1969

MODELING: TWO PATHS

Is there dense ($n \gg n_{GJ}$) plasma in the magnetosphere?

No!

Charge separated magnetosphere

as in Golderich & Julian '69

Michel et al 1980s+

Yes, but not everywhere, and not always

Gapology

(Ruderman et al, Cheng et al, Romani, Harding)

Yes!

MHD / force-free

Contopoulos et al 1999, AS 06

+ many others

Plasma-filled models

- Abundant supply of highly magnetized plasma: force-free model

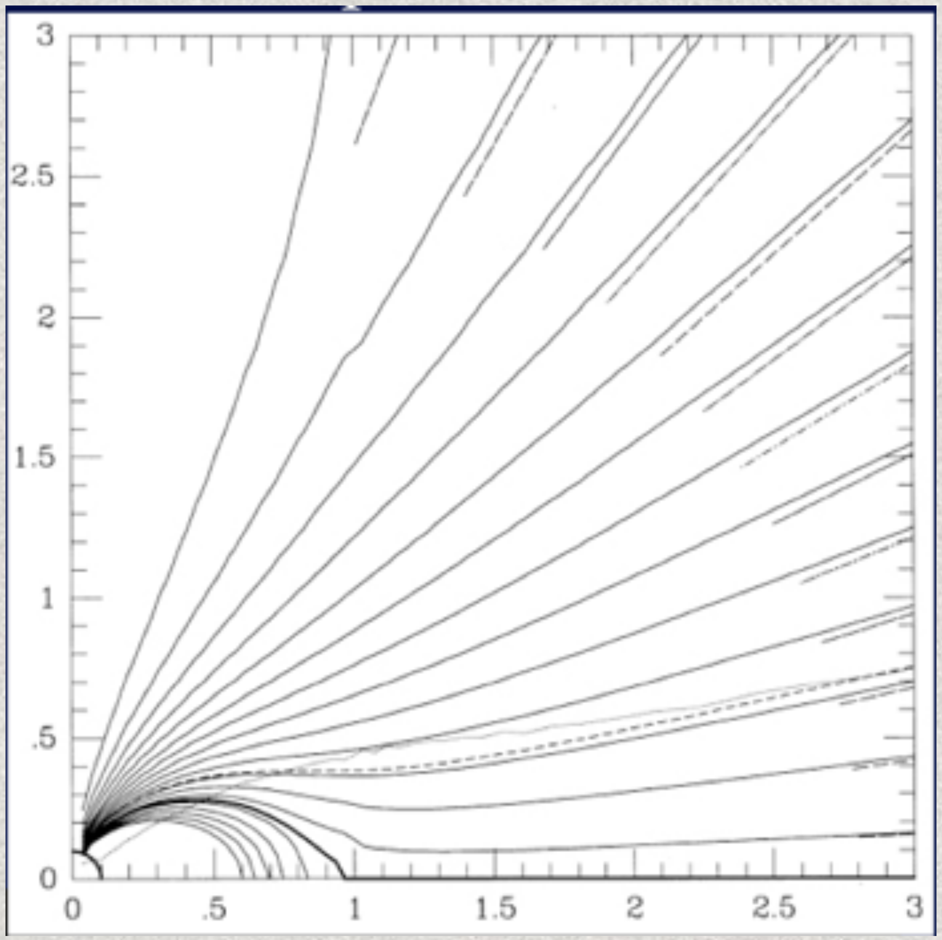
NS is immersed in massless conducting fluid with no inertia.

$$\rho \boldsymbol{E} + (1/c)\boldsymbol{j} \times \boldsymbol{B} = 0$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{j}, \quad \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E},$$

$$\boldsymbol{j} = \frac{c}{4\pi}\nabla \cdot \boldsymbol{E}\frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} + \frac{c}{4\pi}\frac{(\boldsymbol{B} \cdot \nabla \times \boldsymbol{B} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{E})\boldsymbol{B}}{B^2}$$

Gruzinov 99, Blandford 02



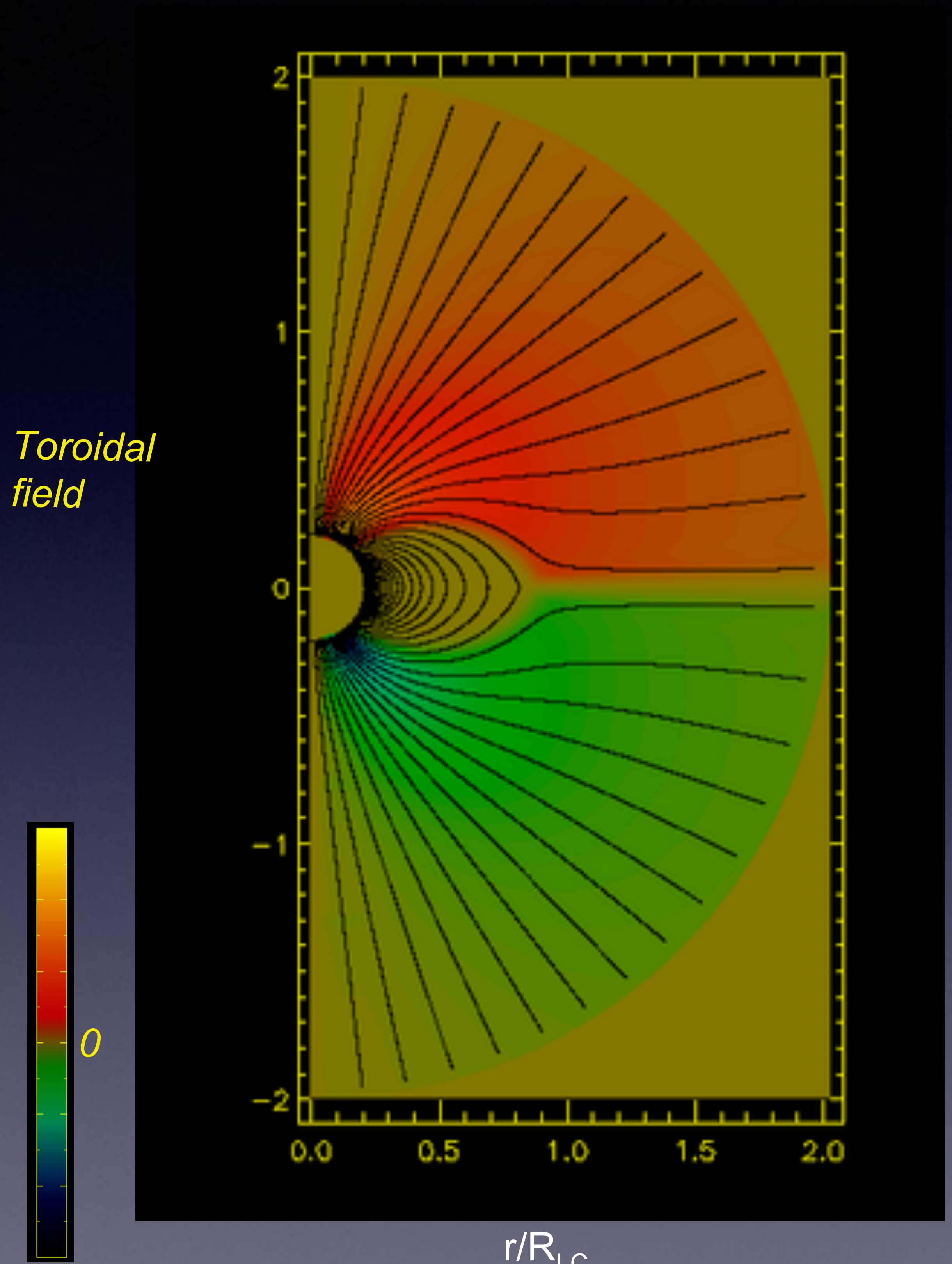
Contopoulos, Kazanas & Fendt 1999

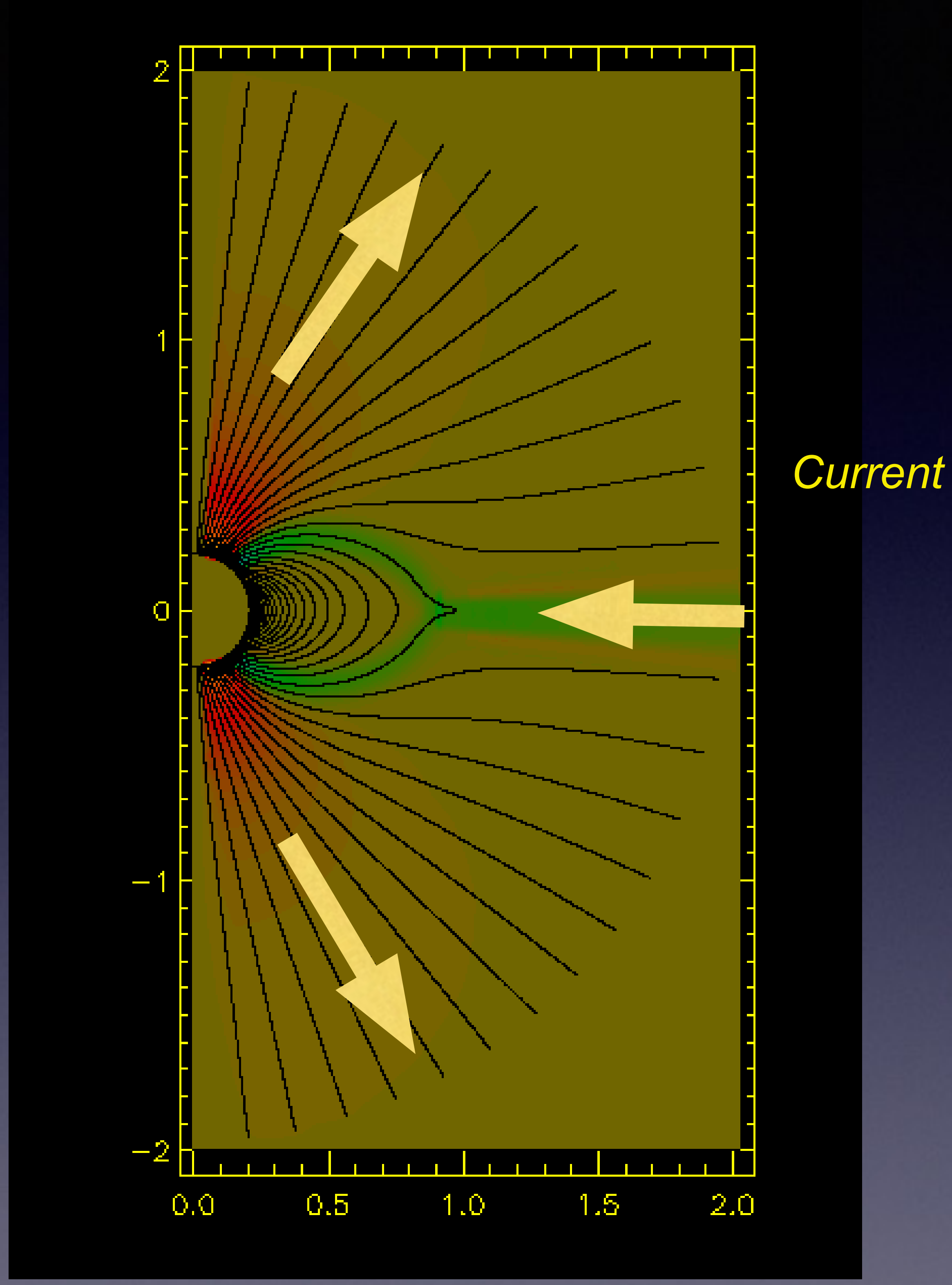
Time-independent version -- pulsar equation (Scharleman & Wagoner 73, Michel 73)

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial z^2} - \frac{1+x^2}{x(1-x^2)}\frac{\partial\Psi}{\partial x} = -\frac{I(\Psi)I'(\Psi)}{R_L^2(1-x^2)}$$

Closed-open geometry is recovered for aligned rotators

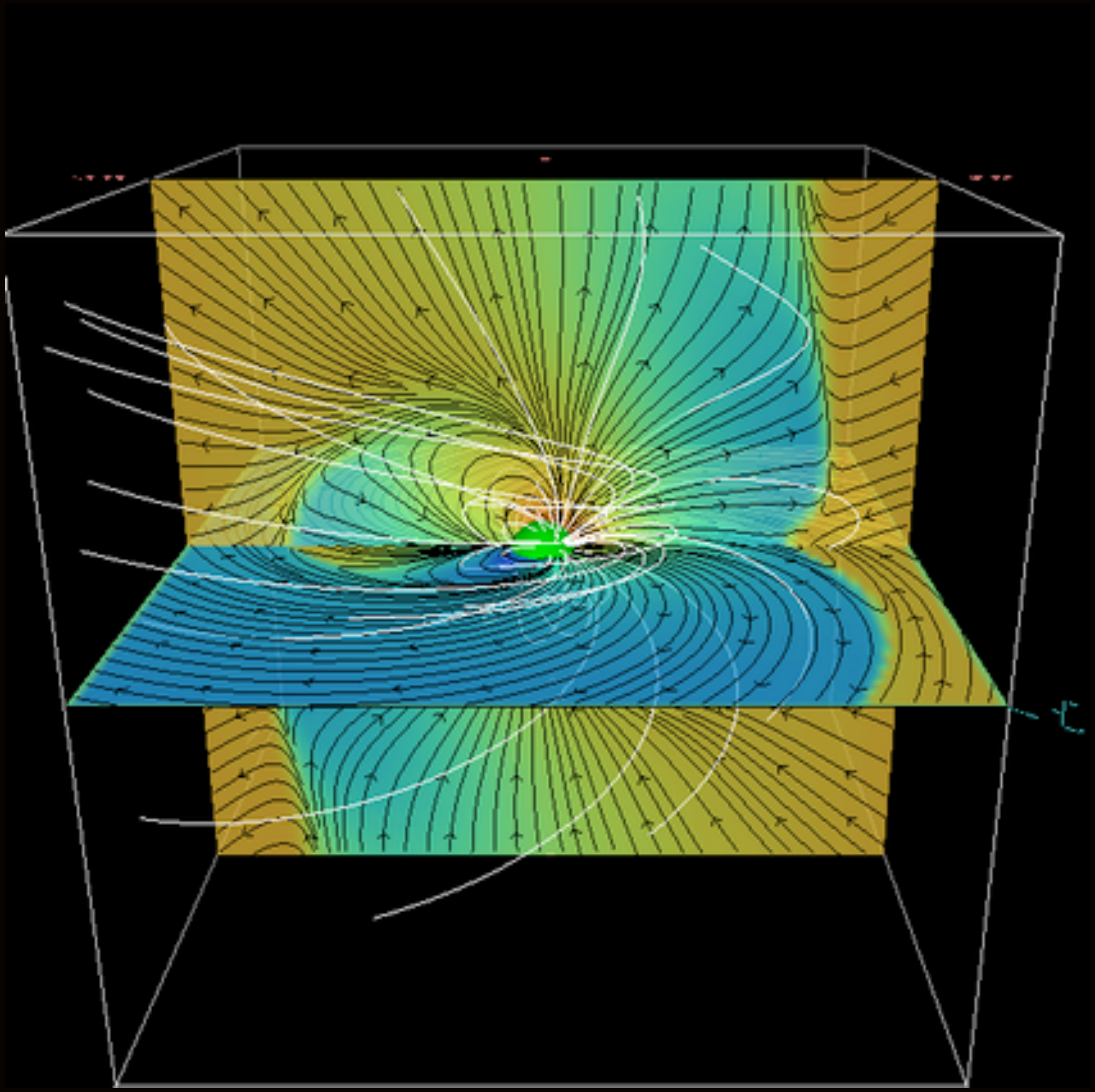
Aligned rotator: plasma magnetosphere





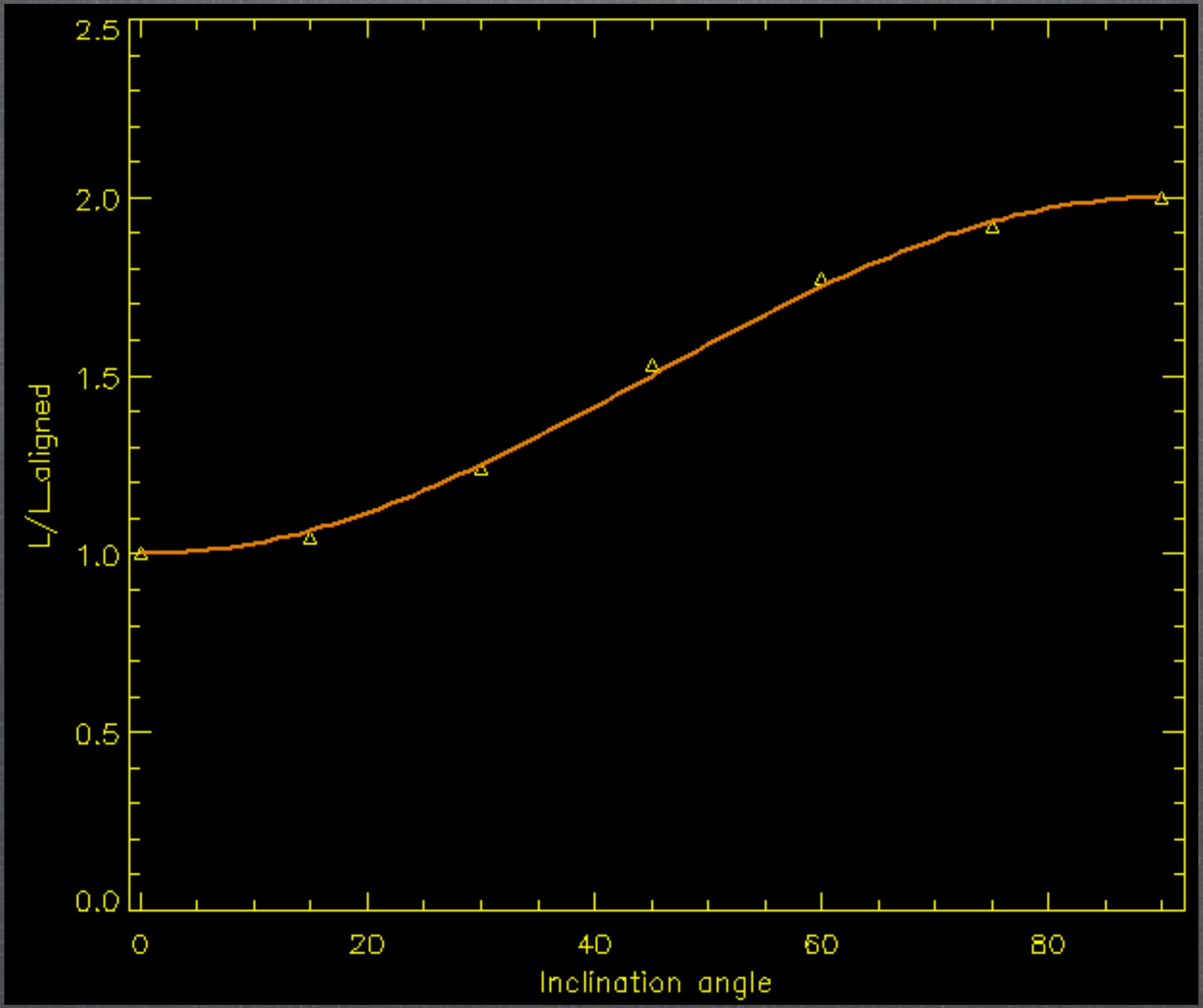
Properties: current sheet, split-monpolar asymptotics; closed-open lines; Y-point; (AS 2006). Now at least 5 groups can do this (also, Yu 11, Parfrey 11+, Petri 12+, Palenzuela 12 in addition to McKinney 06, Kalapotharakis 09)

Oblique rotator: force-free



A.S. 2006

SPIN-DOWN POWER



Spin-down of oblique rotator

NB: this is a fit!

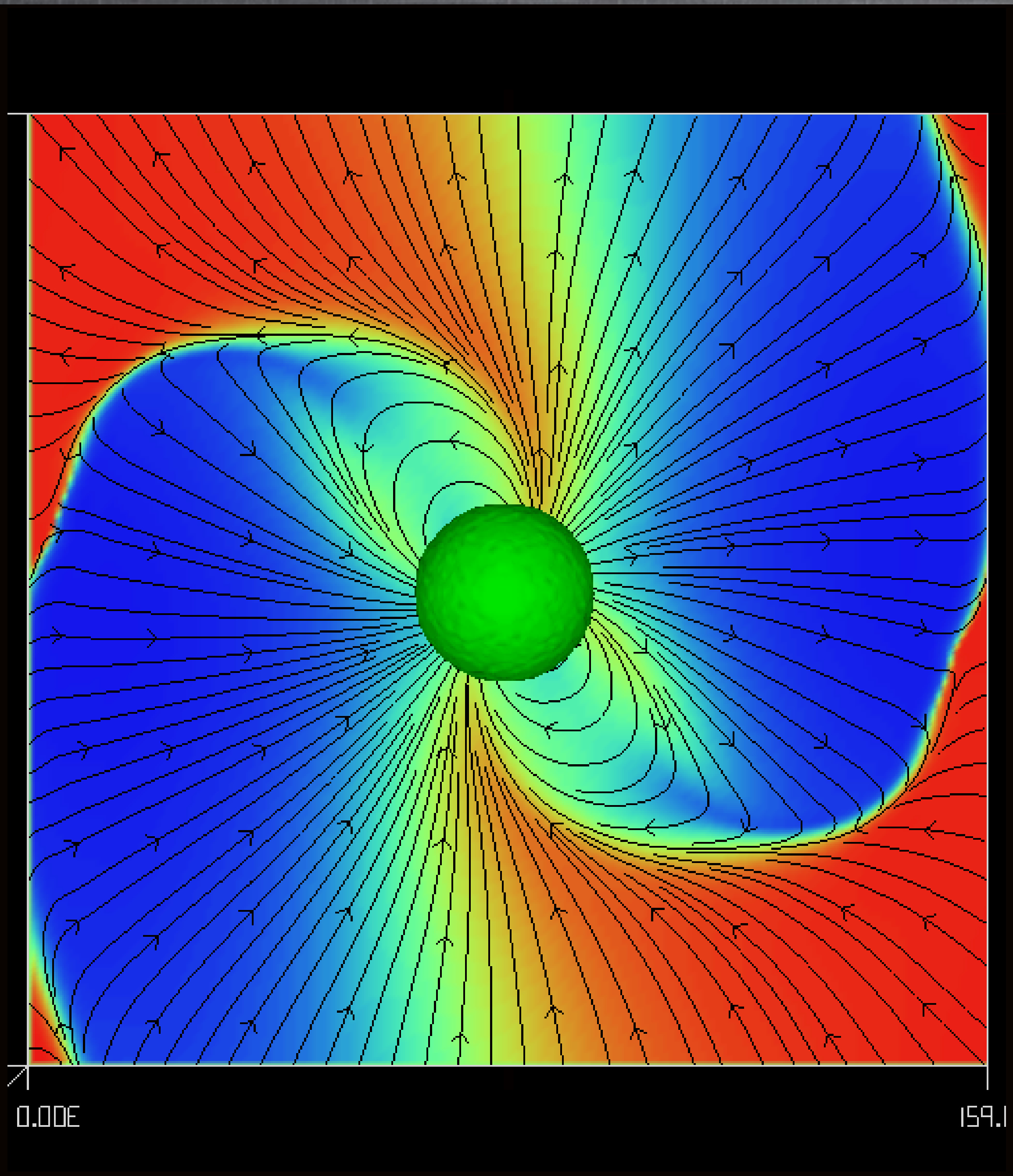
$$\dot{E} = \frac{\mu^2 \Omega^4}{c^3}(1 + \sin^2\theta)$$

$$\dot{E}_{vac} = \frac{2}{3}\frac{\mu^2 \Omega^4}{c^3}\sin^2\theta$$

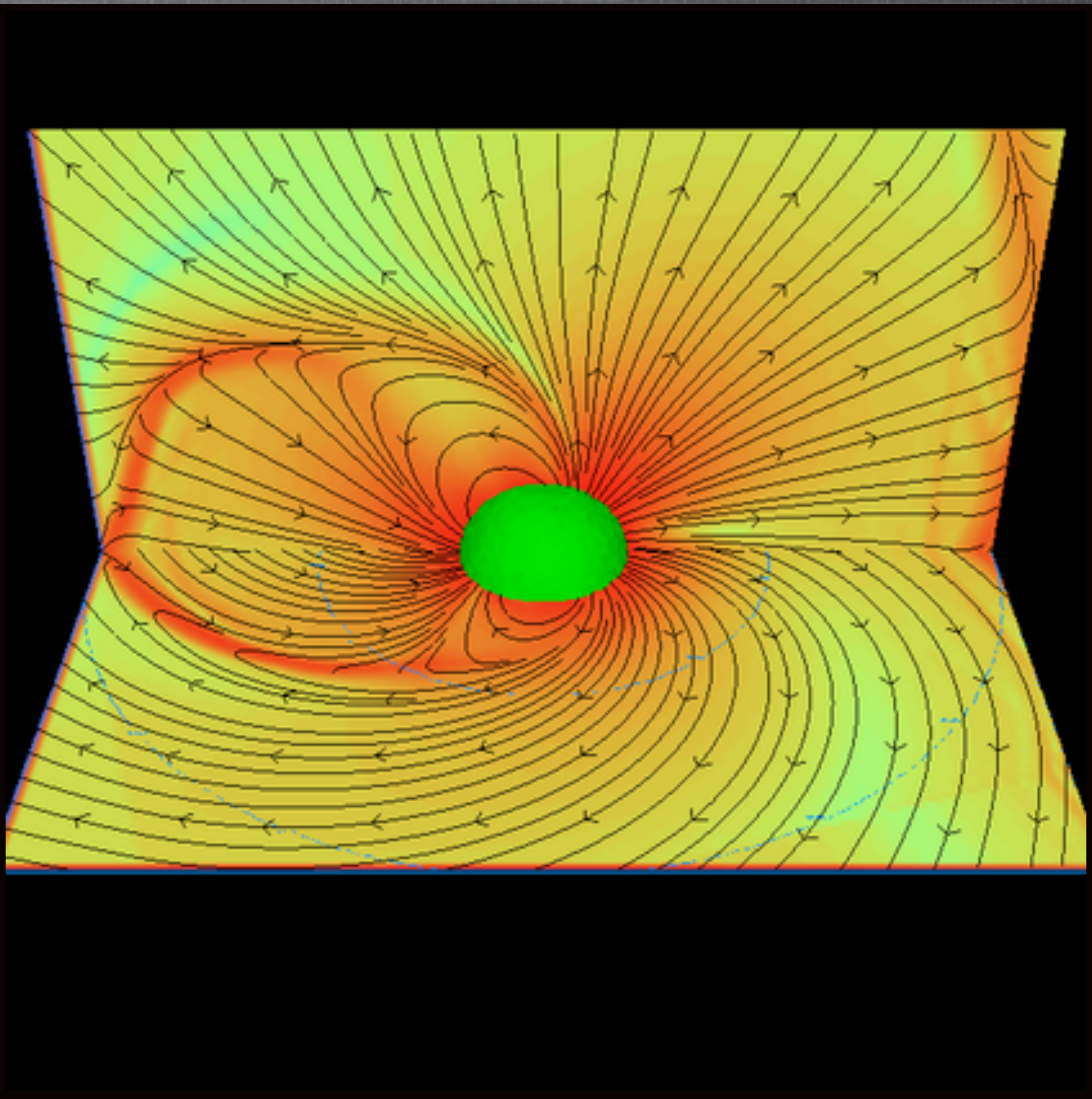
A.S.'06; also confirmed by Kalapotharakos & Contopoulus 09

IN COROTATING FRAME

60 degree inclination

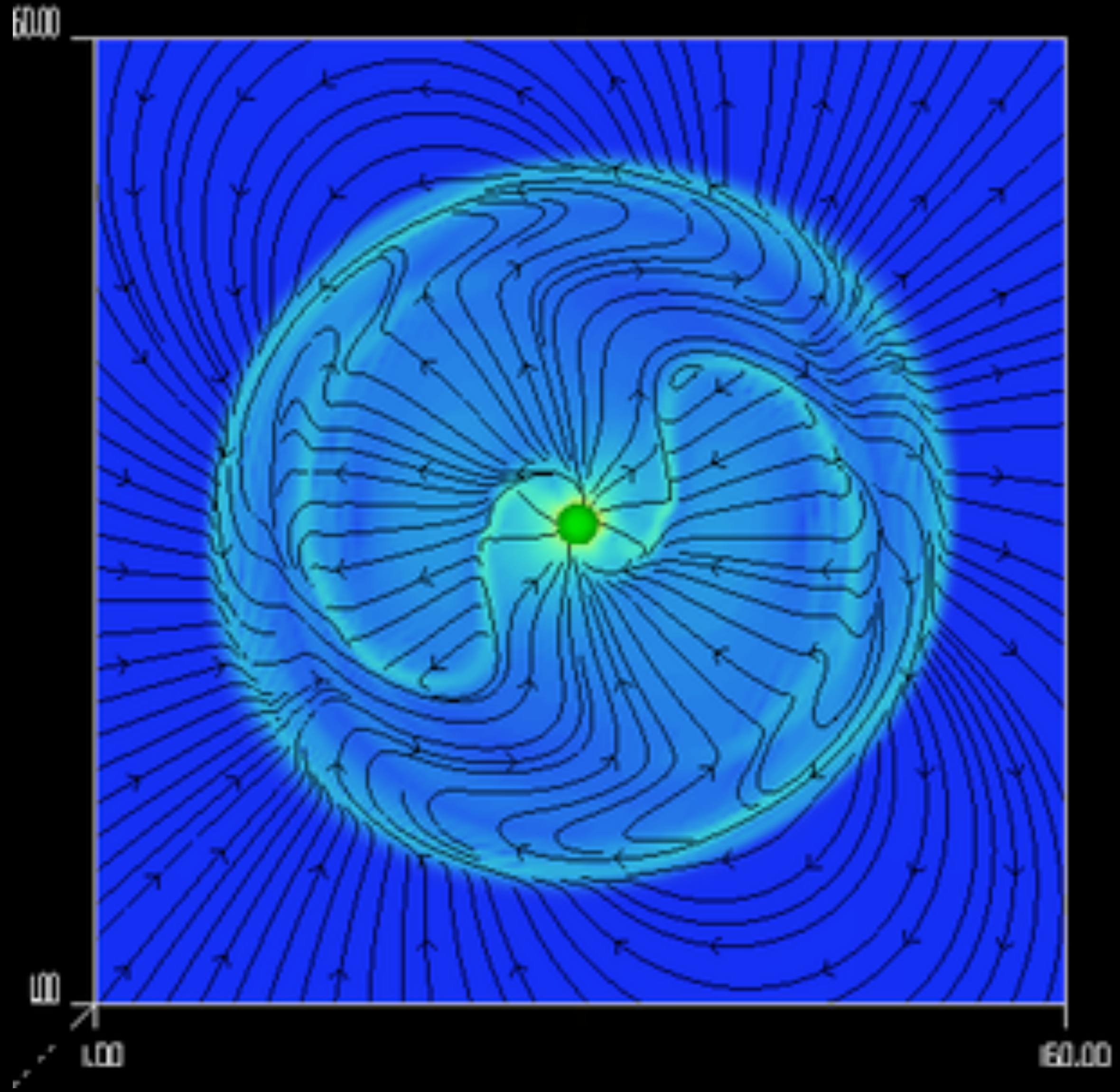


Force-free



Force-free current density

3D force-free magnetosphere: 60 degrees inclination

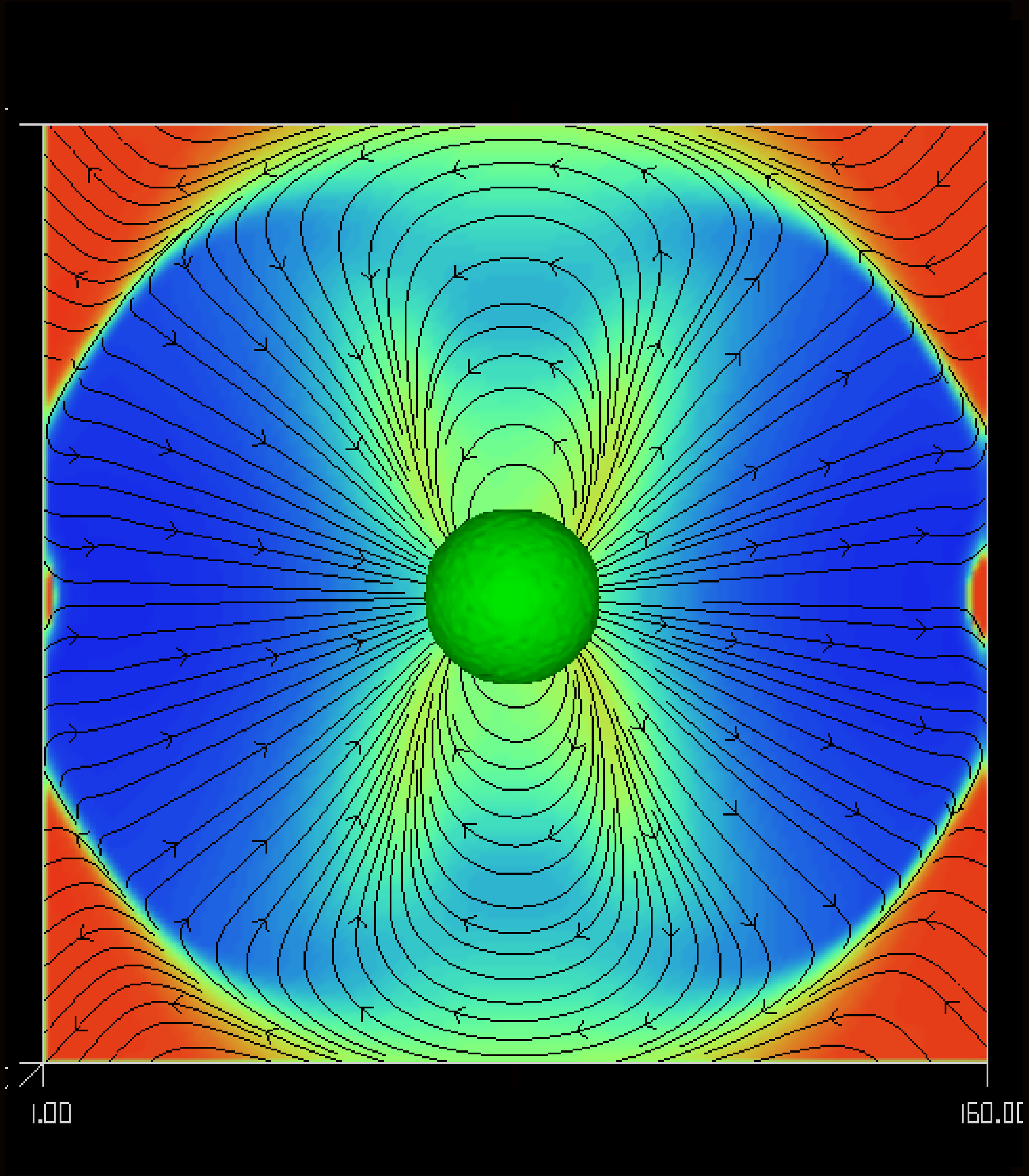


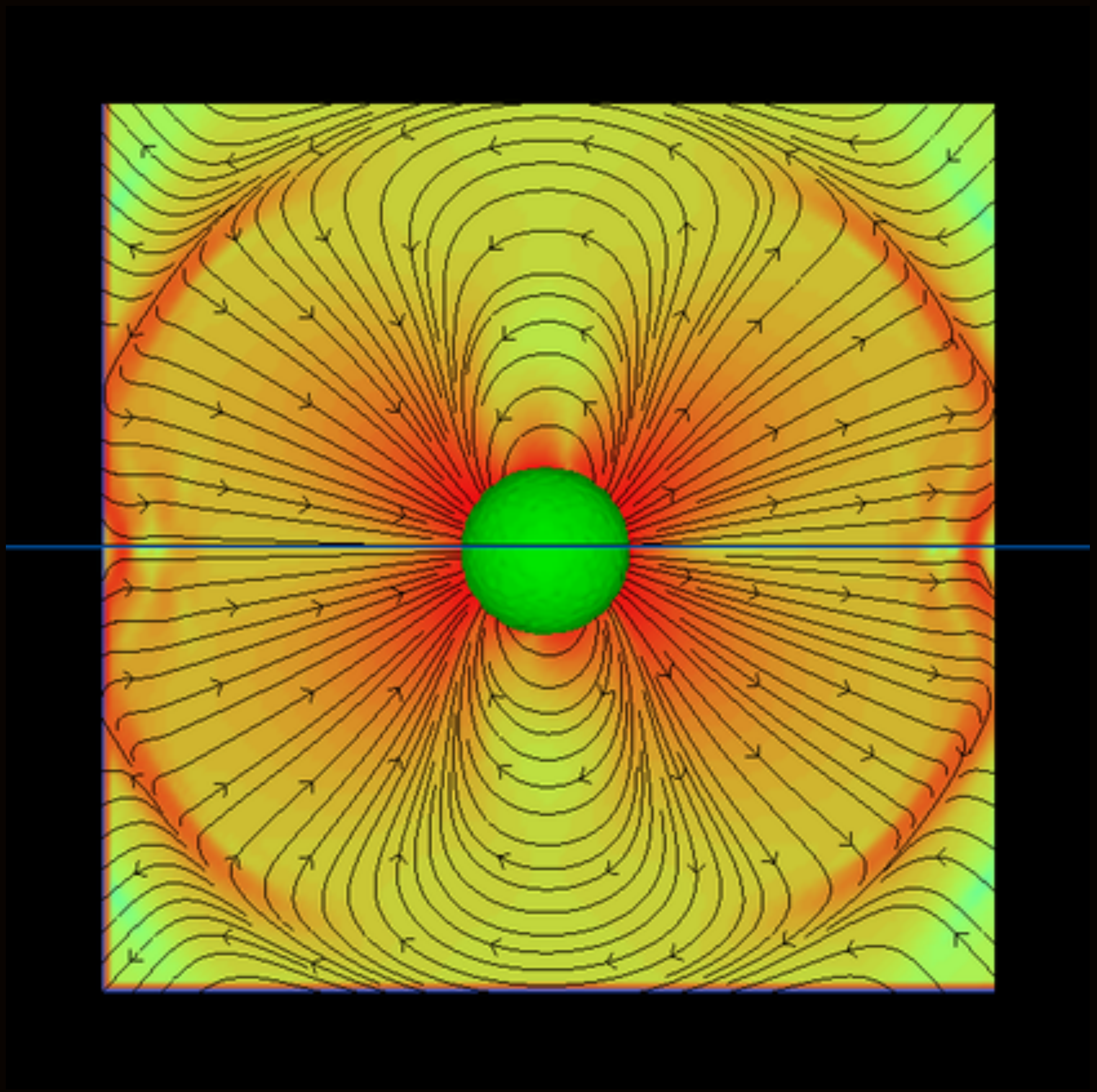
60 degrees force-free current

Similar to heliospheric current sheet

IN COROTATING FRAME

90 degree inclination



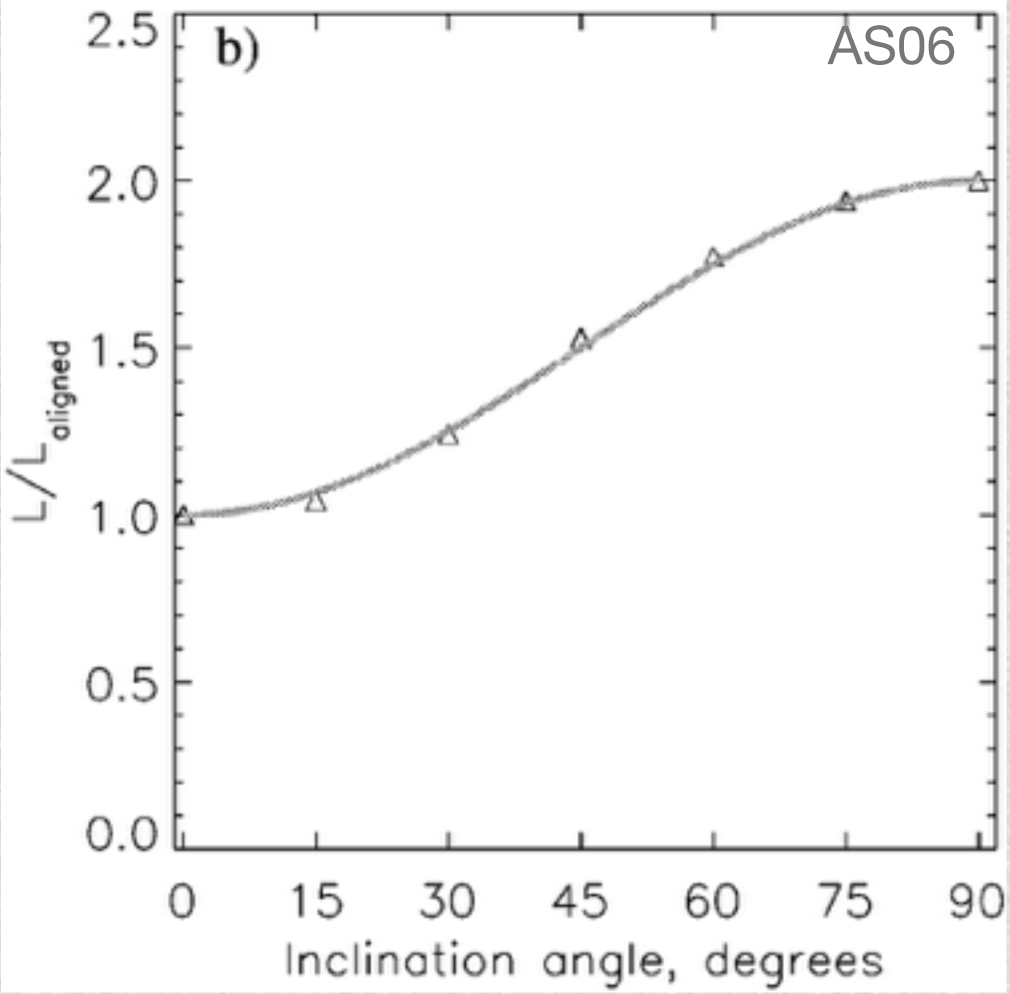


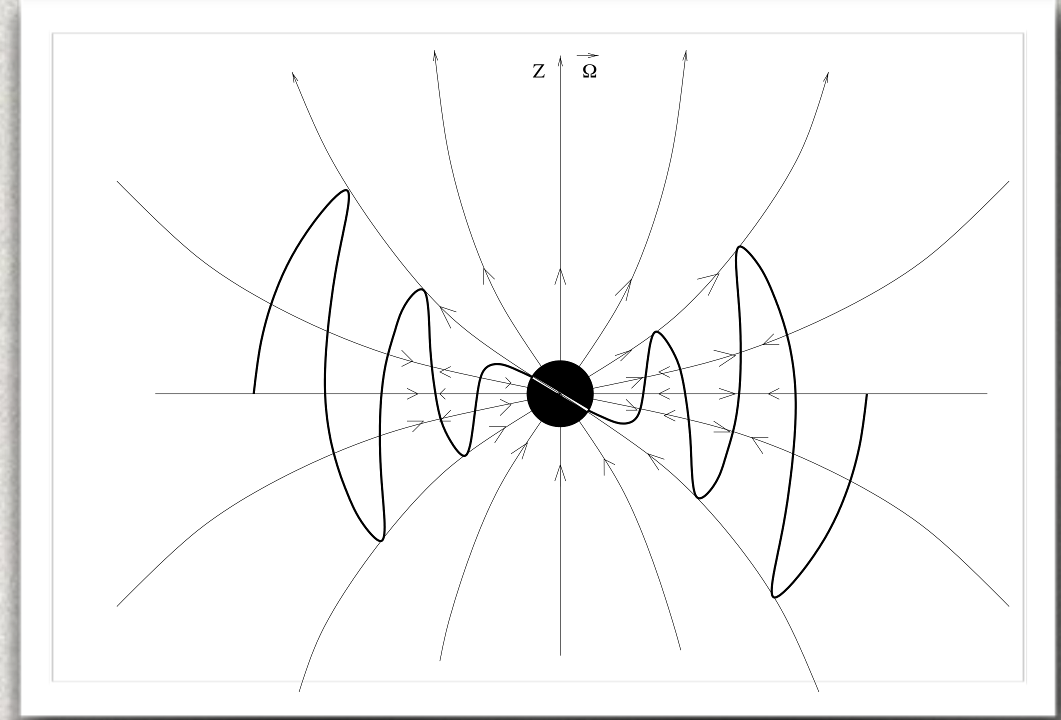
Force-free

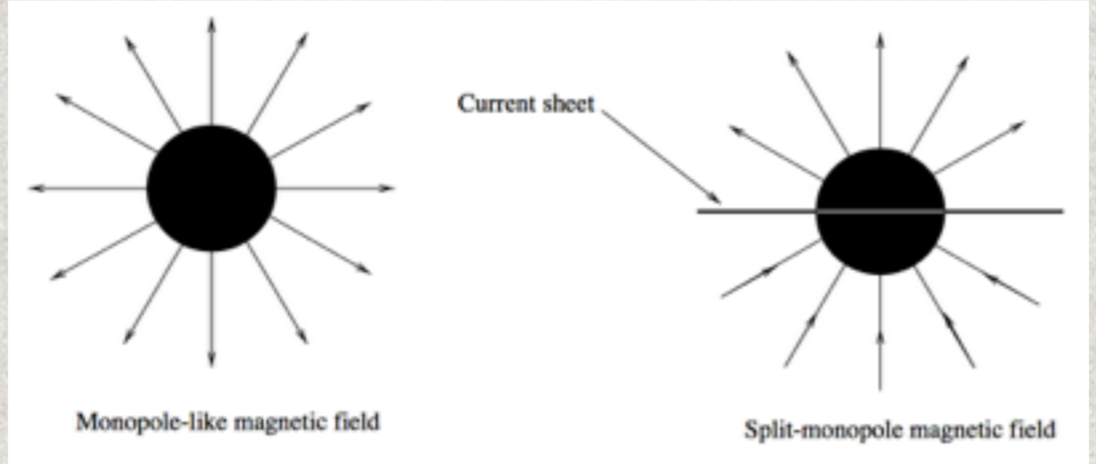
Force-free current density

More on the magnetosphere

- Can we understand $1+\sin^2\alpha$ dependence of spin-down?
- Bogovalov 1999 split monopole: spin-down constant with angle!

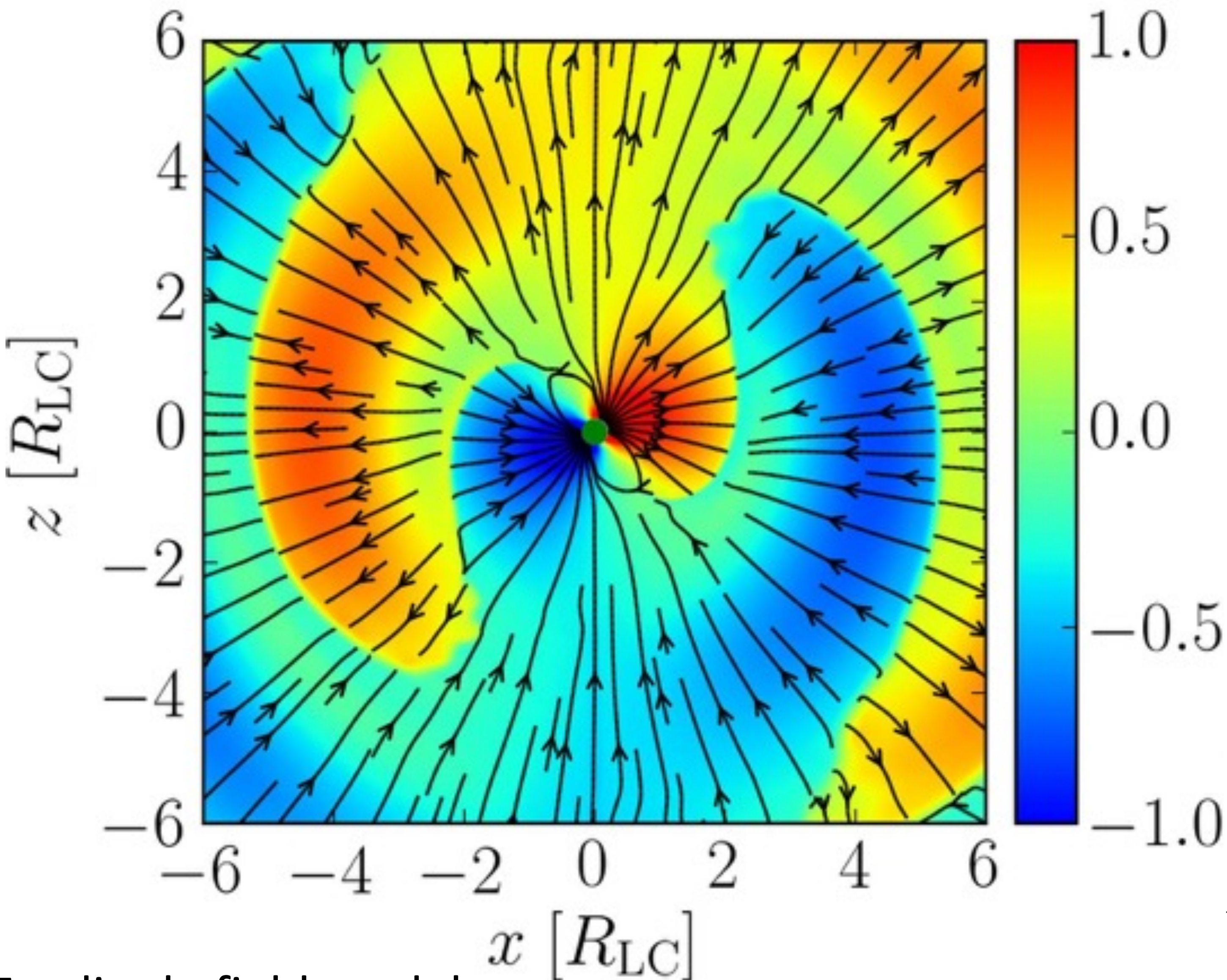


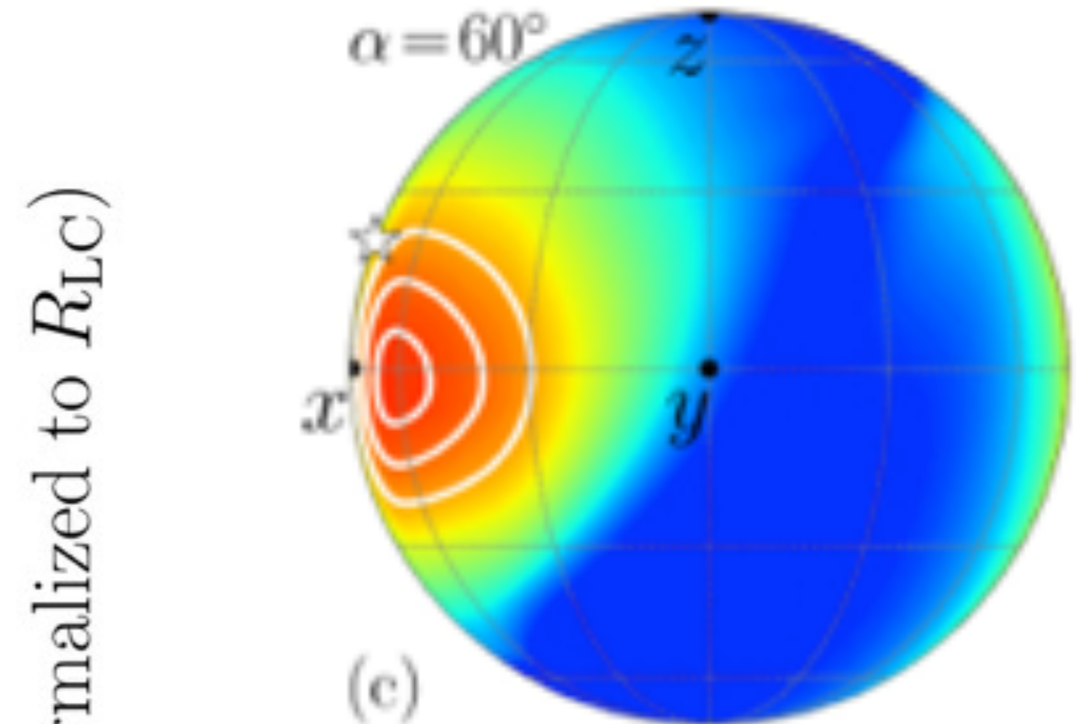




Are asymptotic field lines like split-monopole?

Not exactly a split-monopole!



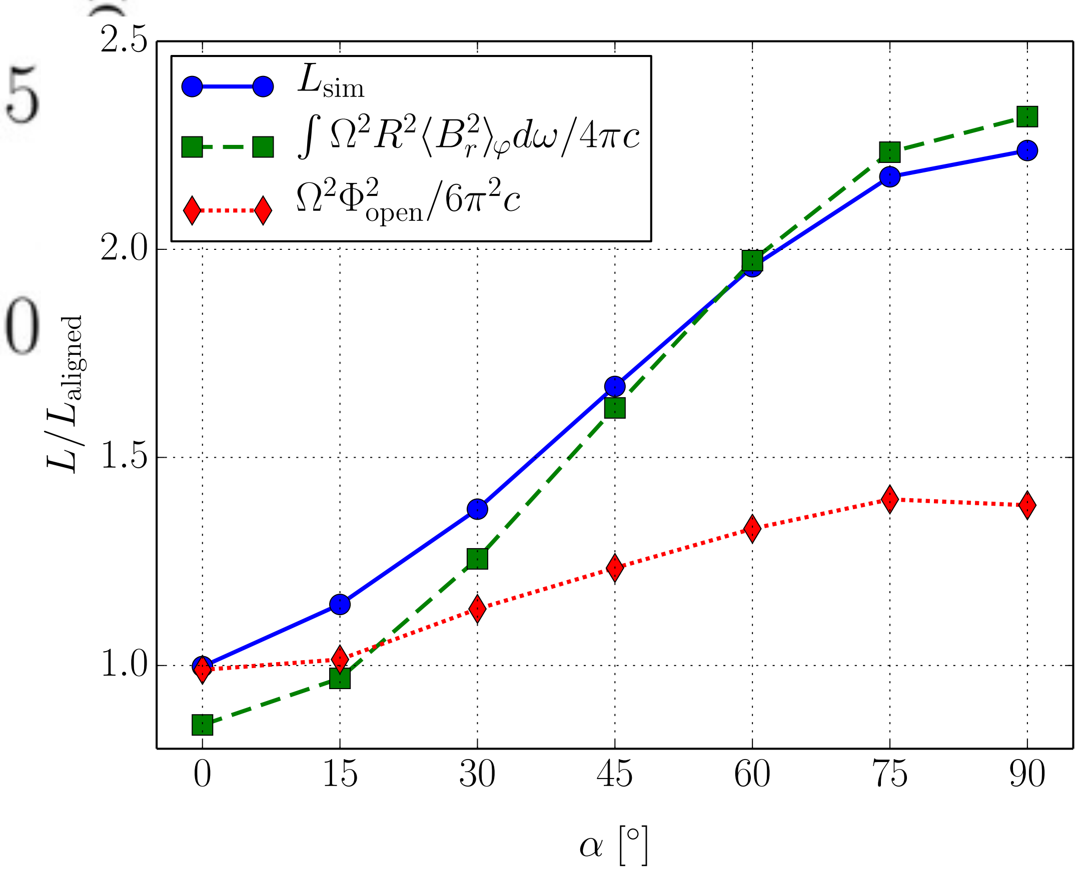


Try dipole field model:

$$B_r = B_0(r/r_0)^{-2}\cos\theta_{\mathrm{m}},$$

$$\langle B_r^2\rangle_\varphi = B_0^2(r/r_0)^{-4}(0.5\sin^2\alpha\sin^2\theta + \cos^2\theta\cos^2\alpha).$$

$$L_{\mathrm{dipole}}(\alpha) = \frac{\Omega^2\Phi_{\mathrm{open}}^2}{7.5\pi^2 c}(1+\sin^2\alpha).$$

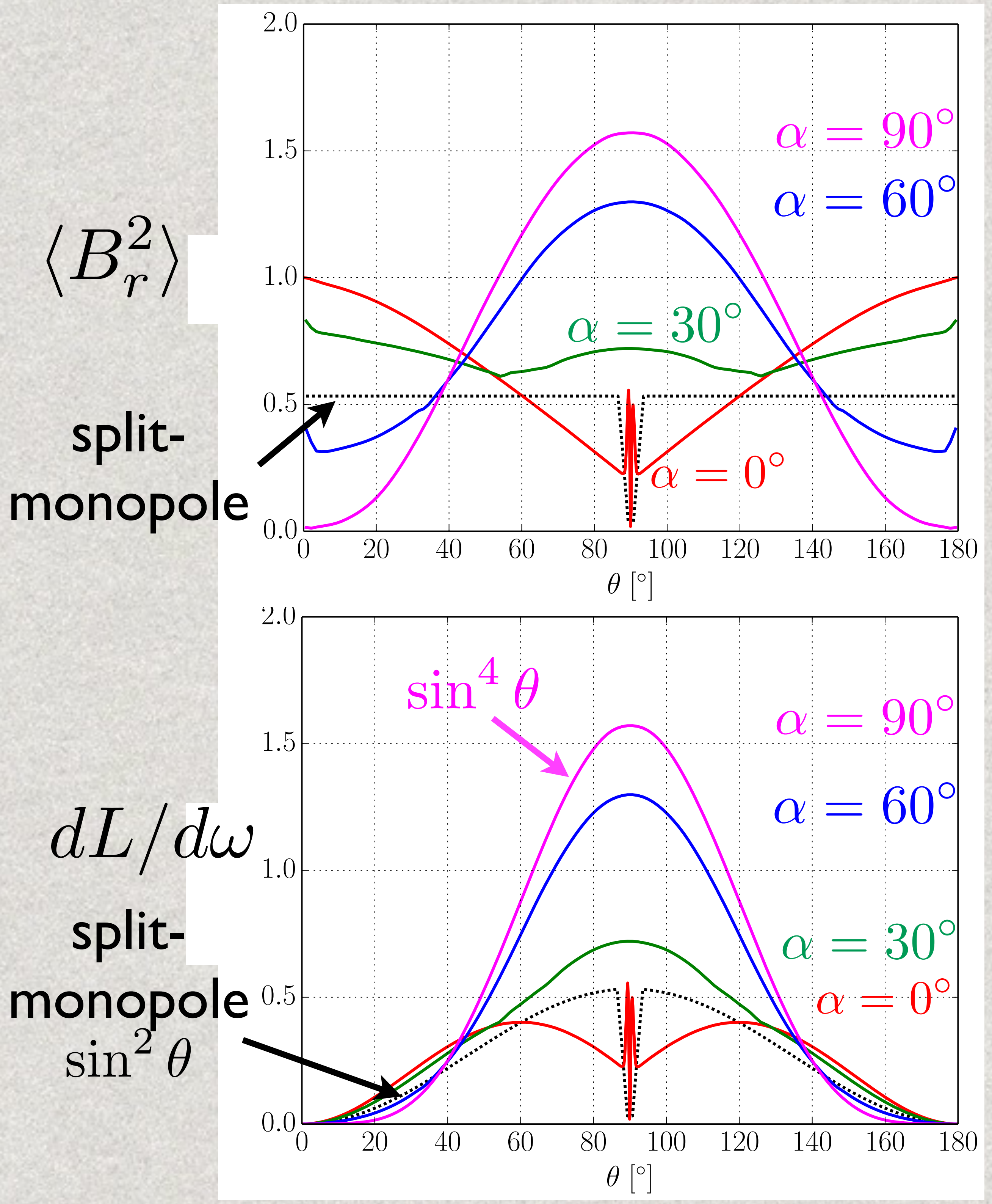


$$L = \iint S_r d\omega \approx \frac{\Omega^2}{2c}\int_0^\pi \langle B_r^2\rangle_\varphi r^4 \sin^3\theta d\theta$$

Tchekhovskoy, Philippov, AS (2016)

More on the magnetosphere

- B-field is equatorially-concentrated
- Wind luminosity is more equatorially concentrated than monopole
- This effect needs to be included for gamma-ray emission light curve calculation and PWN models.



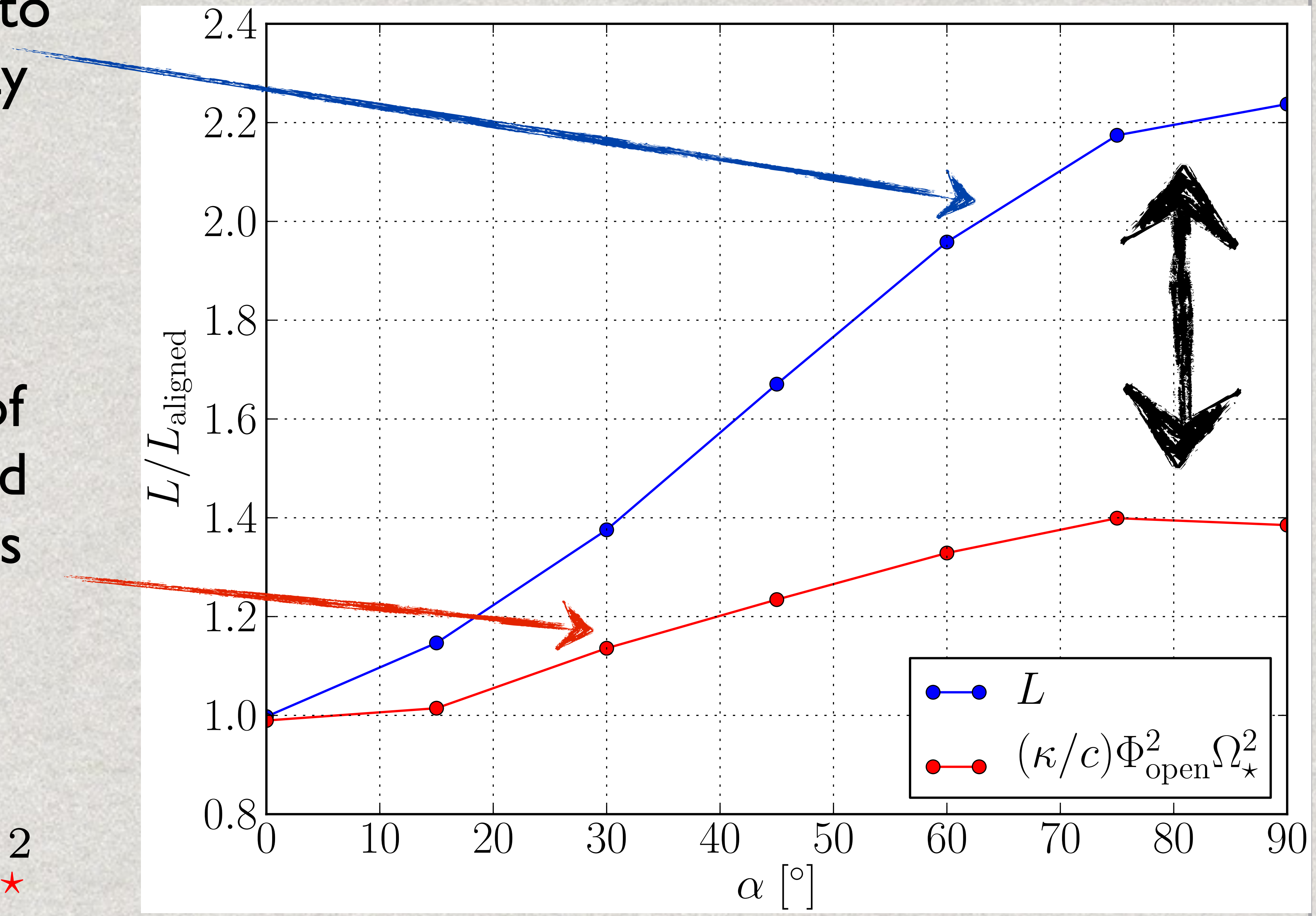
Tchekhovskoy, Philippov, Spitkovsky 2016.

Field Non-uniformity Explains Enhanced Spindown of Oblique Pulsars

Enhanced spindown due to non-uniformity of B-field?

Assumption of uniform B-field under-predicts spindown

$$P_{\rm NS} = \frac{\kappa}{c}\Phi_{\rm open}^2 \Omega_\star^2$$



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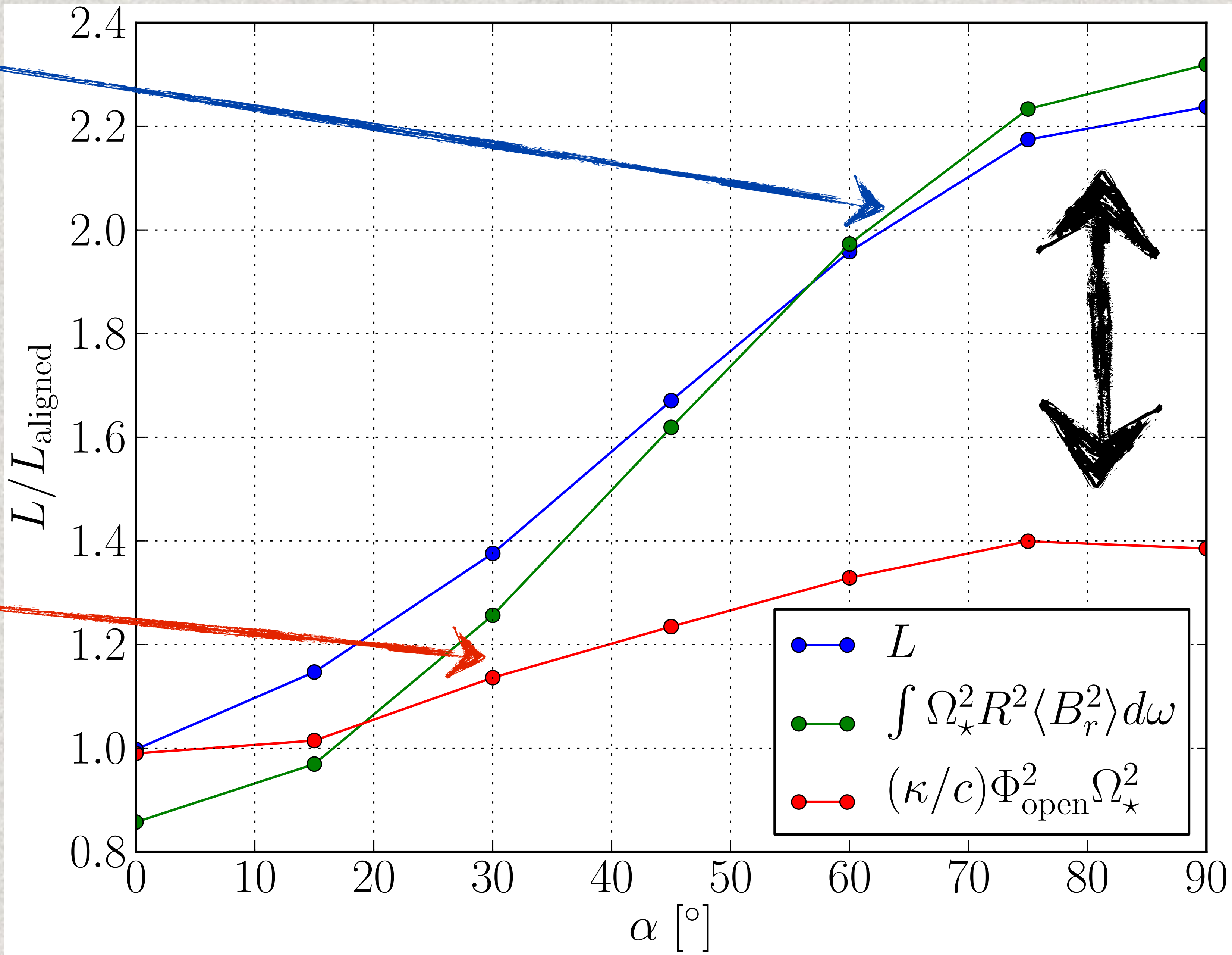
(Spitkovsky'06, Petri'12, AT, Spitkovsky, Li'13)

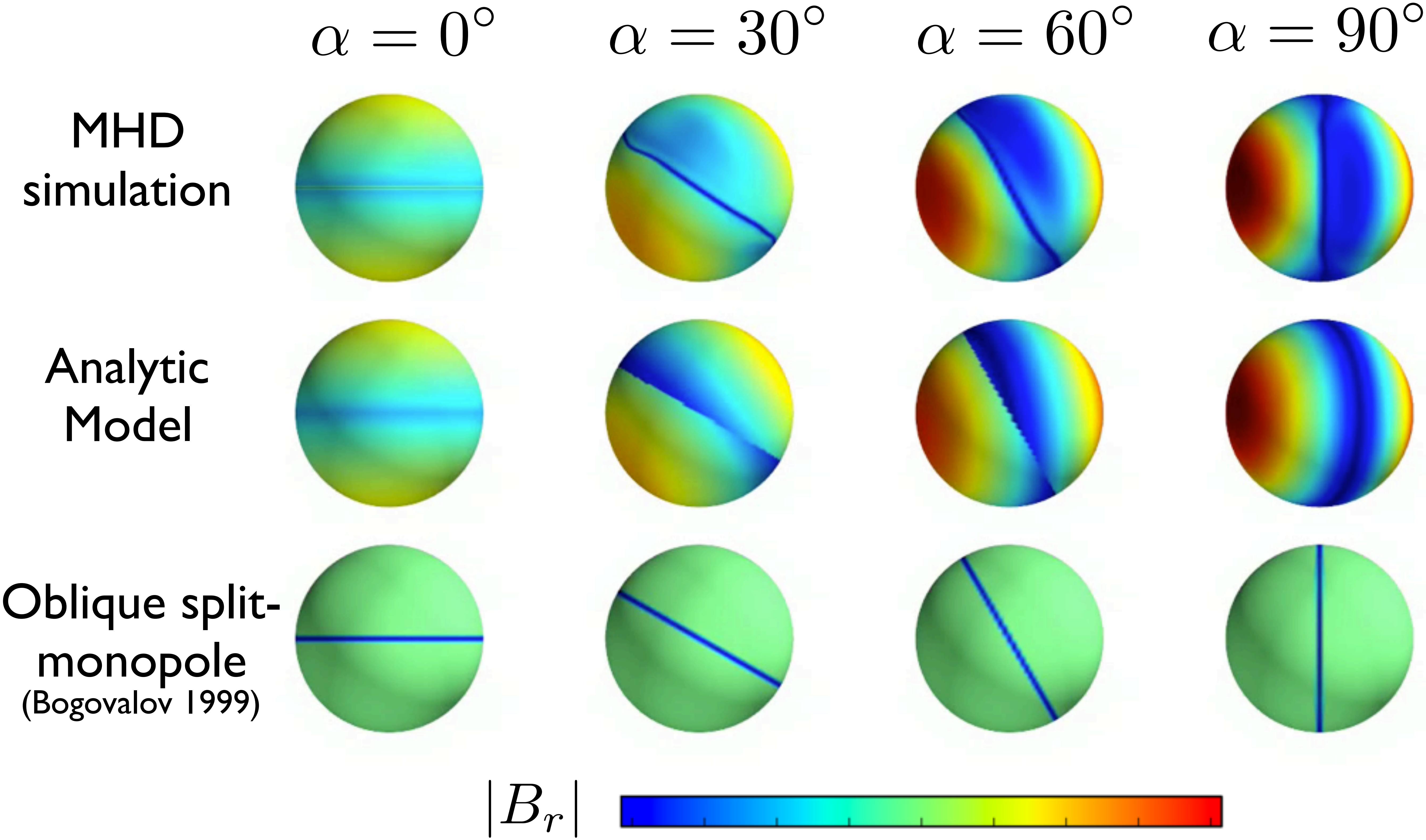


Tchekhovskoy, Philippov, AS 2016.

just B_r variation from inclined dipolar field gives $1+\sin^2\alpha$

Analytic fitting model of 3D pulsar wind

Superposition of aligned Br + vacuum 90 deg

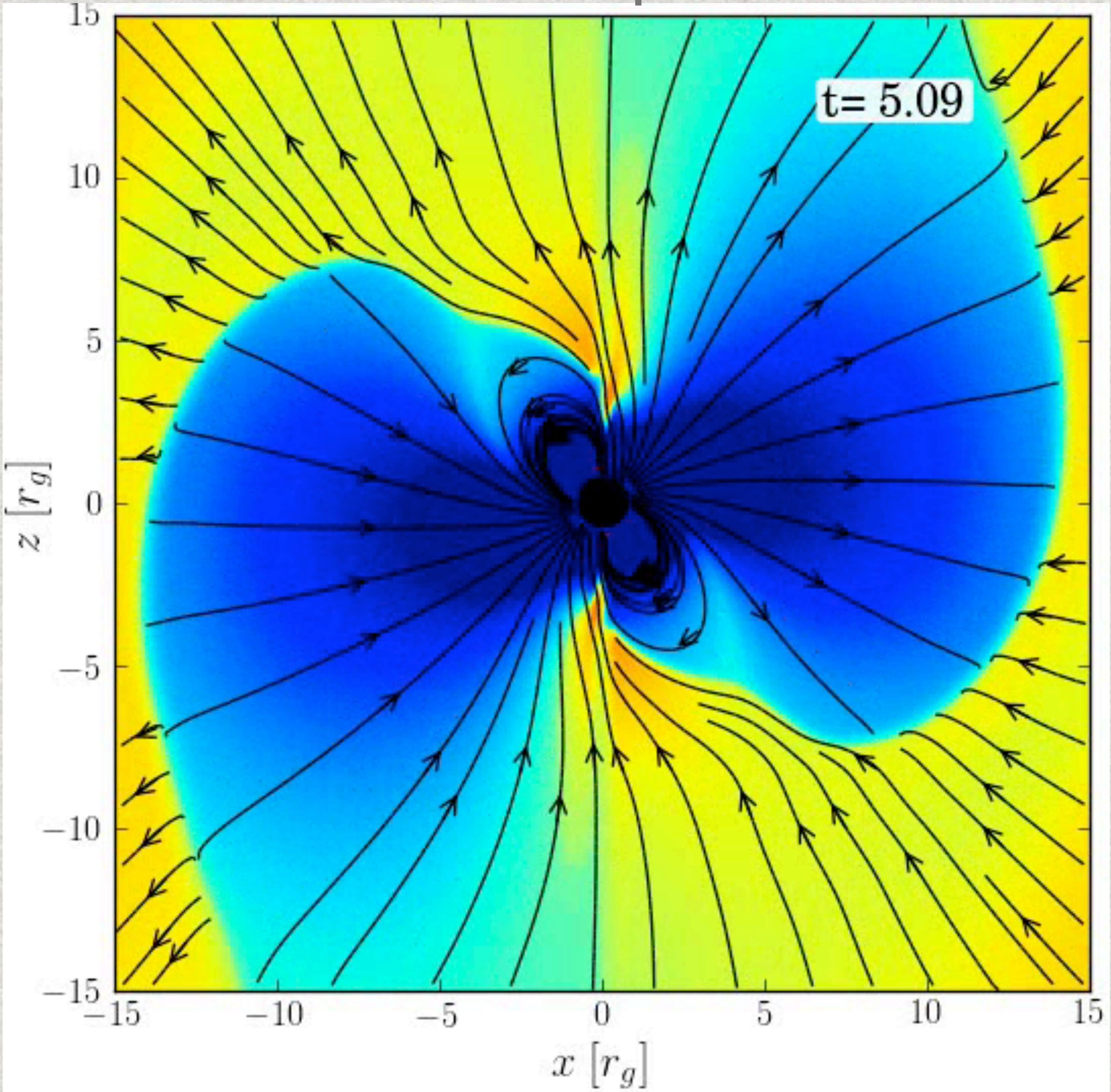


Fitting model for oblique pulsar wind is now available

MHD advances:

- Full RMHD is now in 3D!
- Oblique rotator can now be studied in ideal MHD (Tchekhovskoy, AS, Li 2013)
- Spherical grid which allows non-axisymmetric solutions. Magnetization > 100. Fixed magnetization inside 0.7 LC

color: out of plane B field



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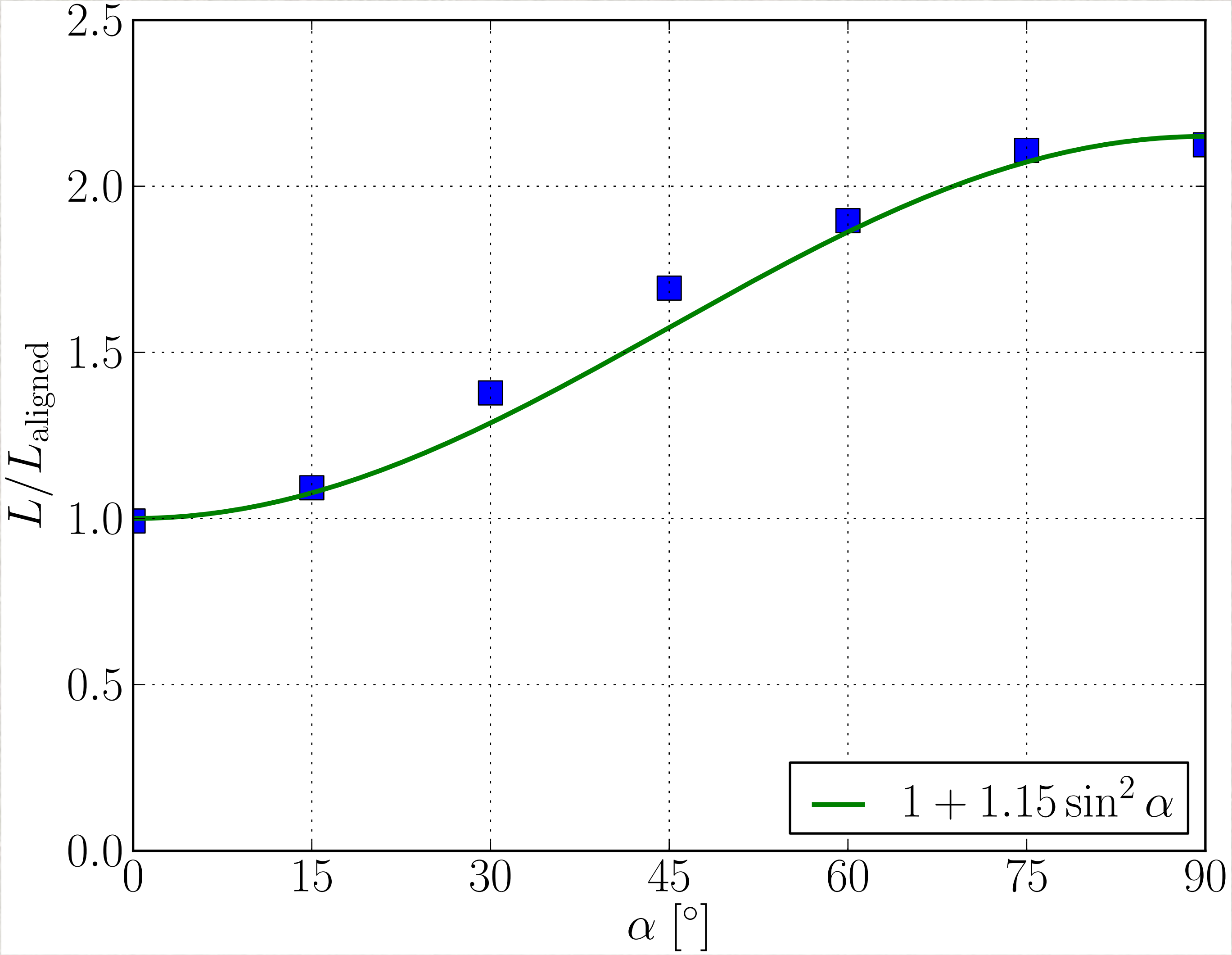
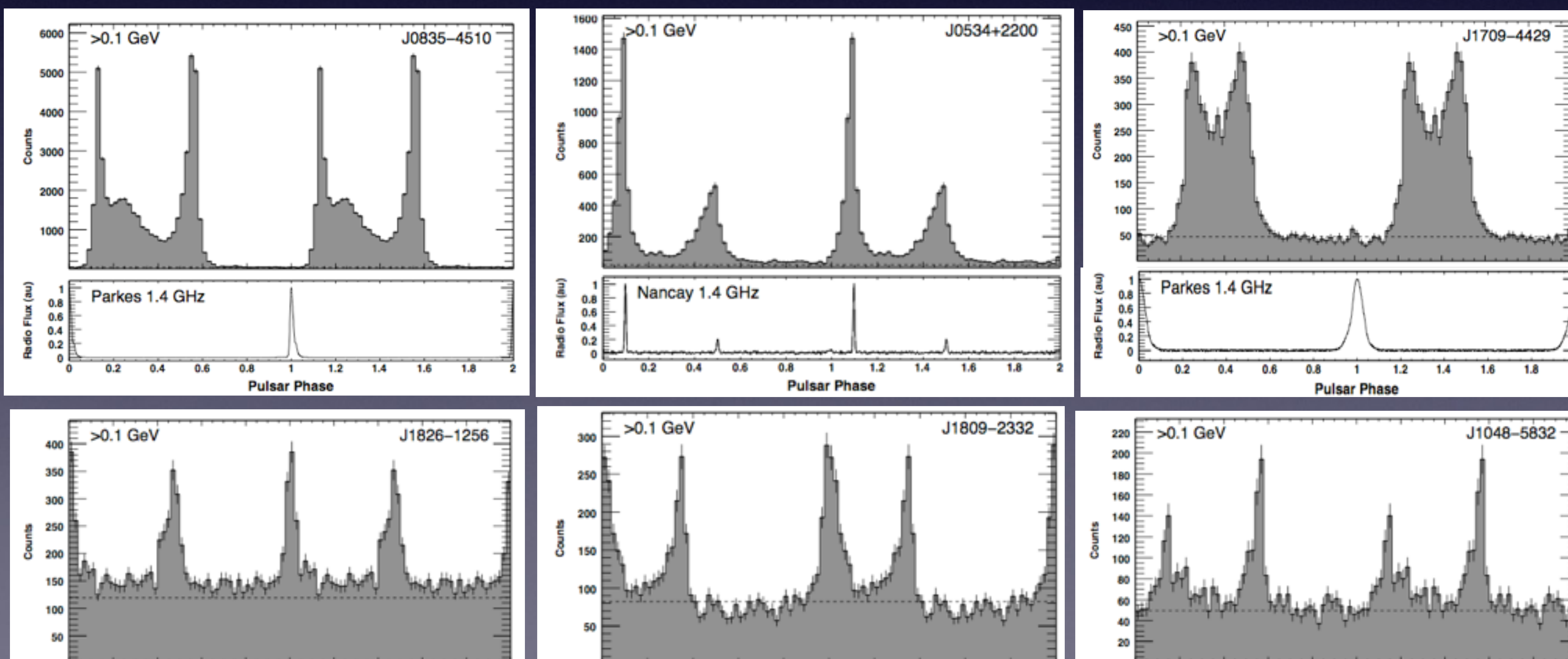
Spin down luminosity



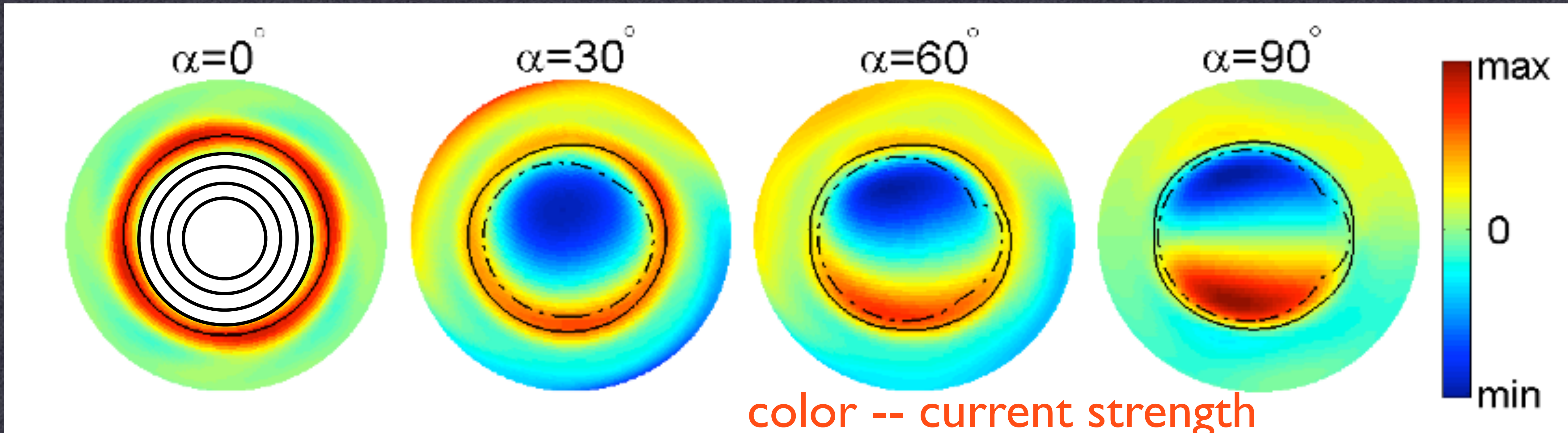
Obliqueness

Variation with angle is similar to force-free

Gamma-ray emission from pulsars



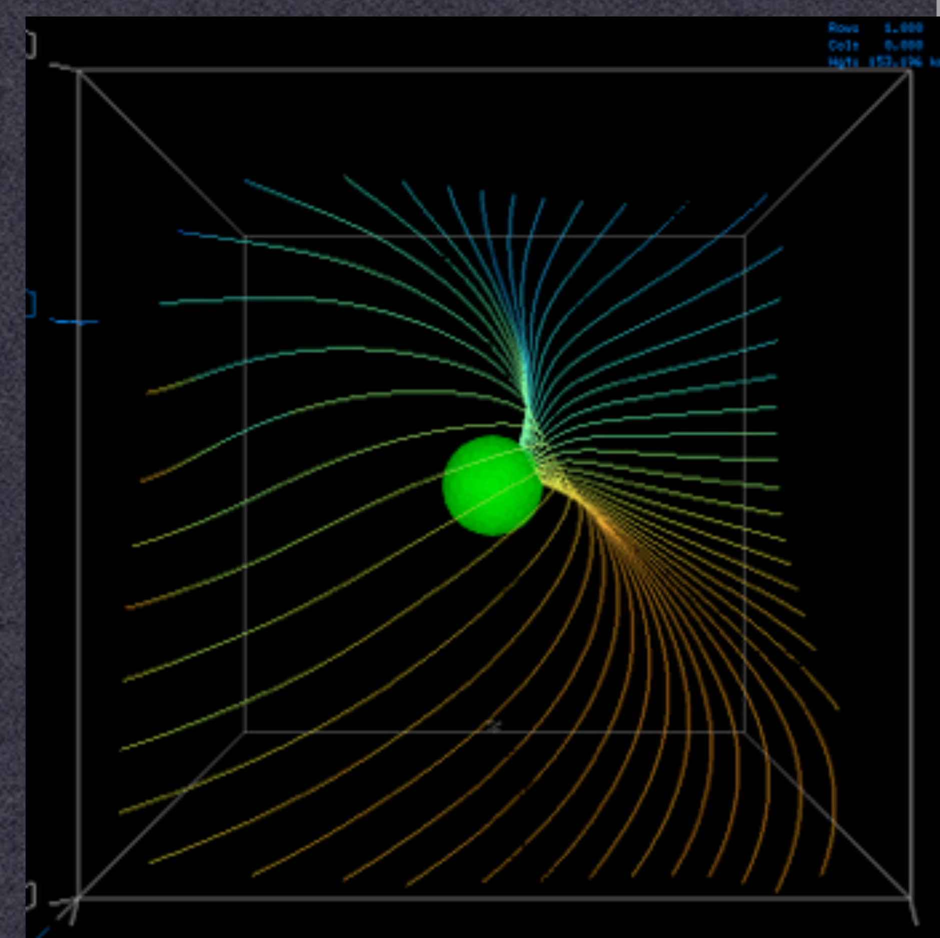
Where does emission come from?



- Select flux tubes that map into rings on the polar caps. The rings are congruent to the edge of the polar cap.

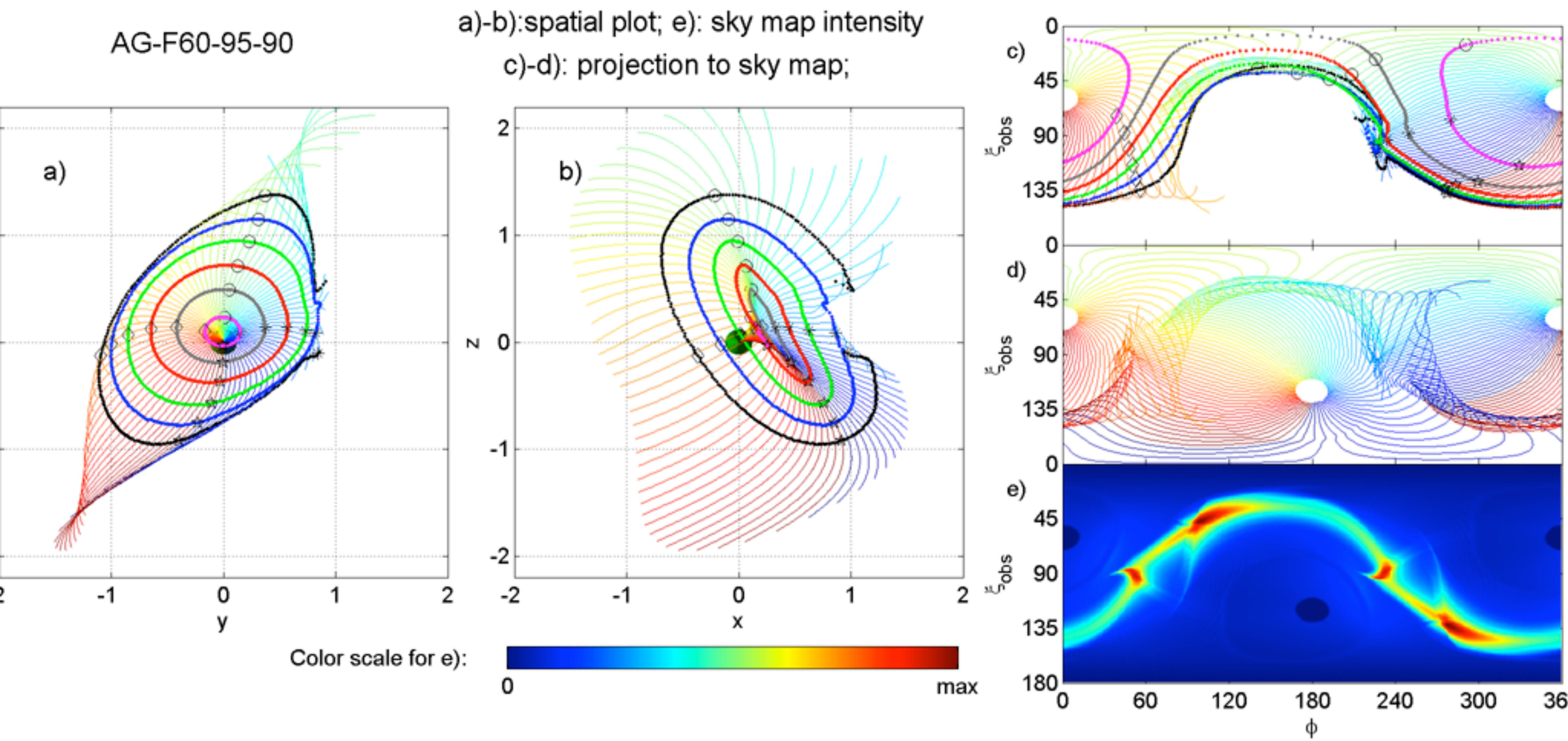
- While ad-hoc, the point is to study the geometry of the possible emission zone.

- Emission is along field lines, with aberration and time delay added



Emission from different flux tubes

Bai & A. S. 2010



Emission from two poles merges on some flux tubes: what's special about them?

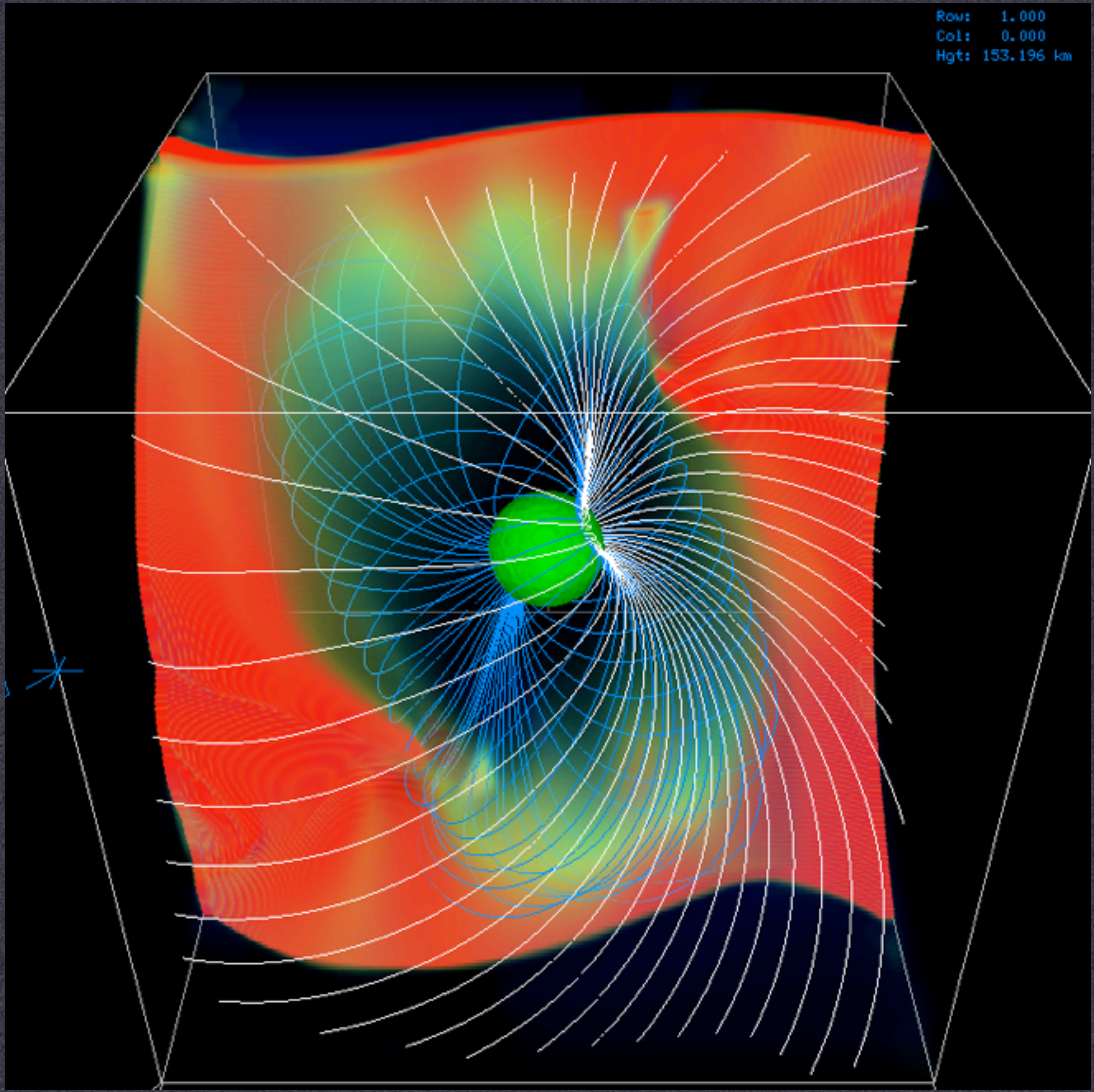
Association with the current sheet

Color -> current

Field lines that produce best force-free caustics seem to "hug" the current sheet at and beyond the LC.

Significant fraction of emission comes from beyond the light cylinder.

Best place to put a resistor in the circuit!



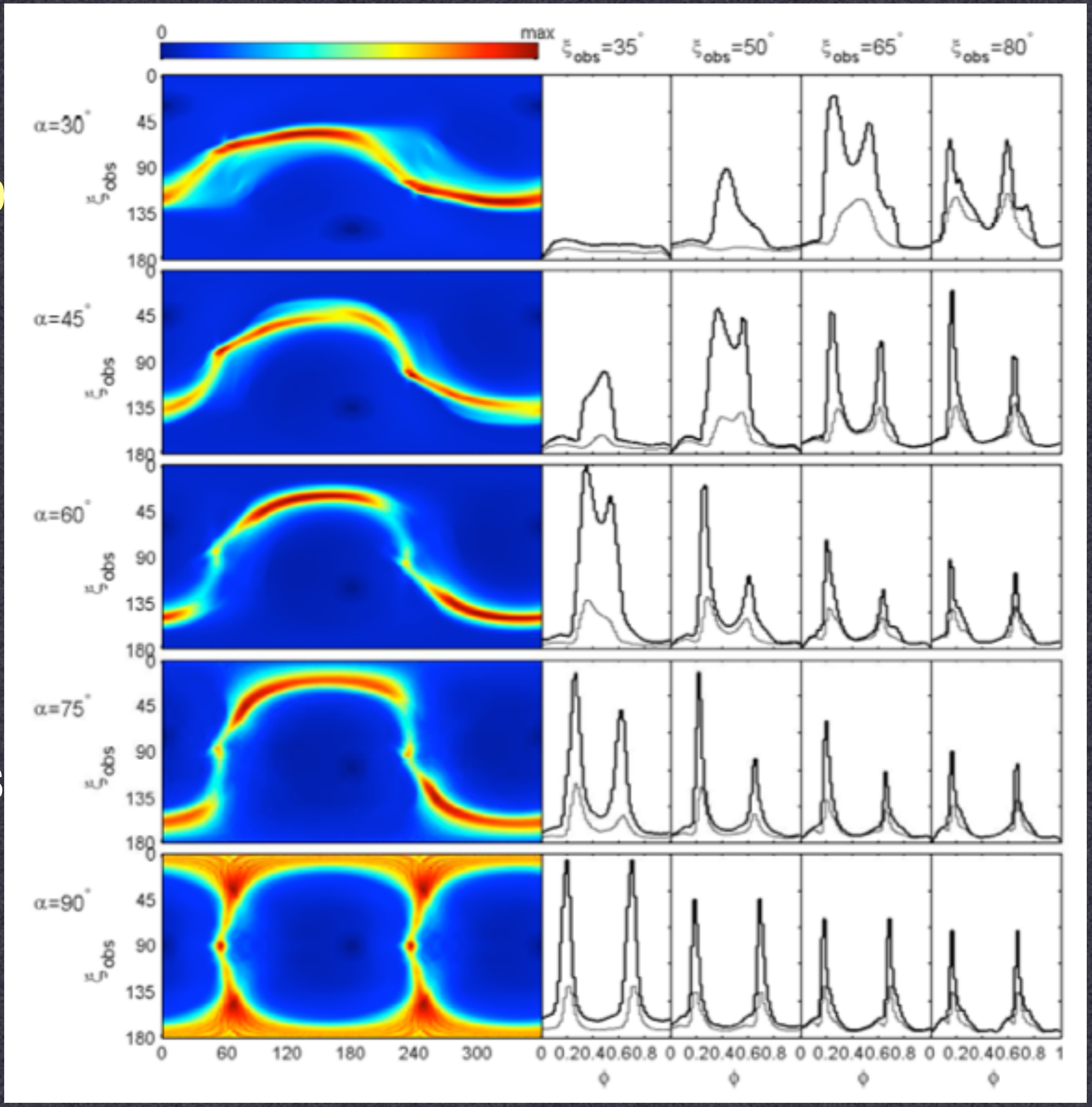
Light curves from the current sheet

Viewing angle

Current sheet emission is a strong contender to explain light curve morphology in 3D

Inclination angle

Most of the emission in FF model accumulates beyond 0.9 Rlc



Double peak profiles very common.

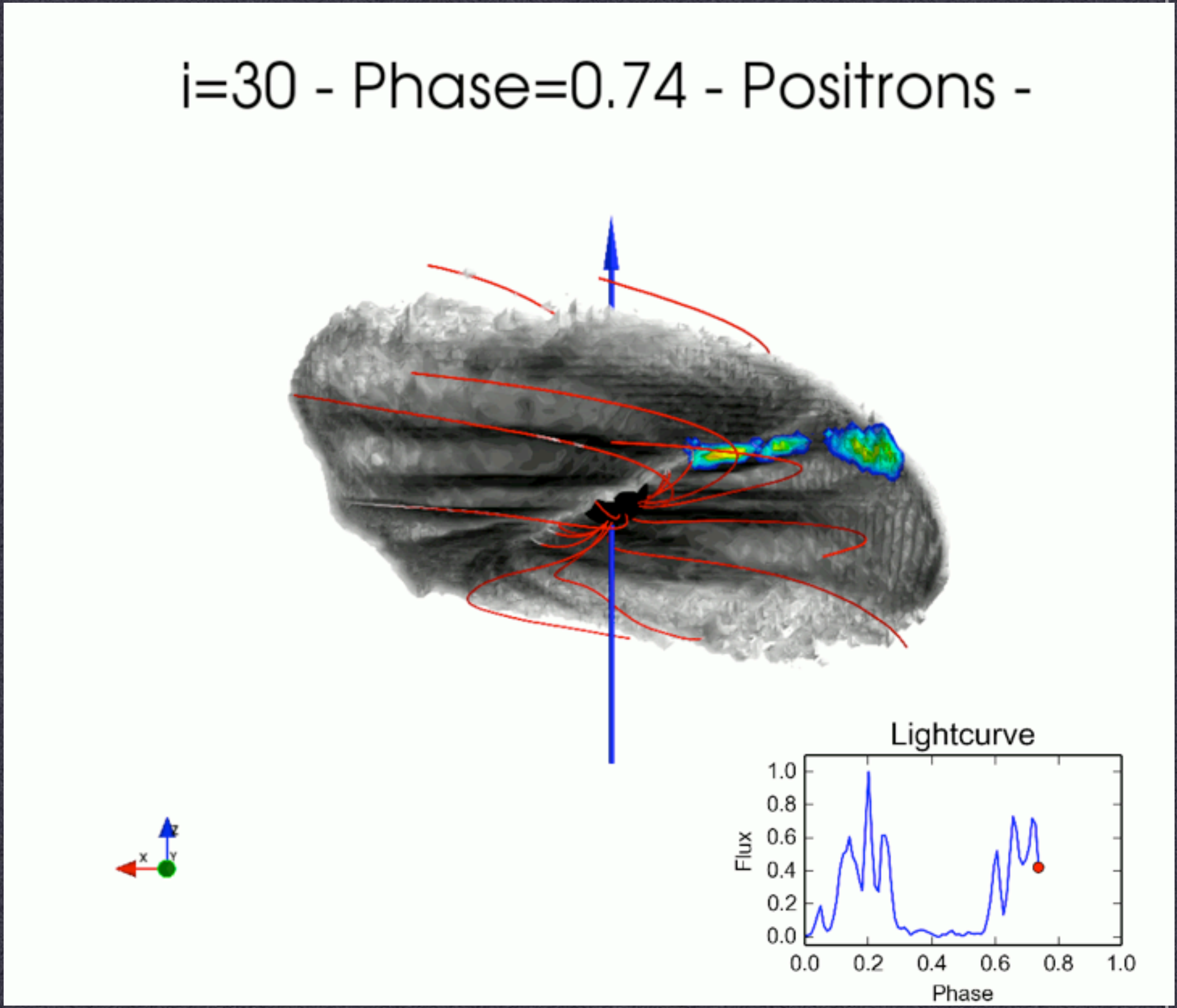
Bai & AS, 2010

Light curves from the current sheet

Particle acceleration is mainly in the sheet: reconnection

Light curve from kinetic simulation

Spectra to come



Cerutti, Philippov, AS 2016

Abundant plasma models

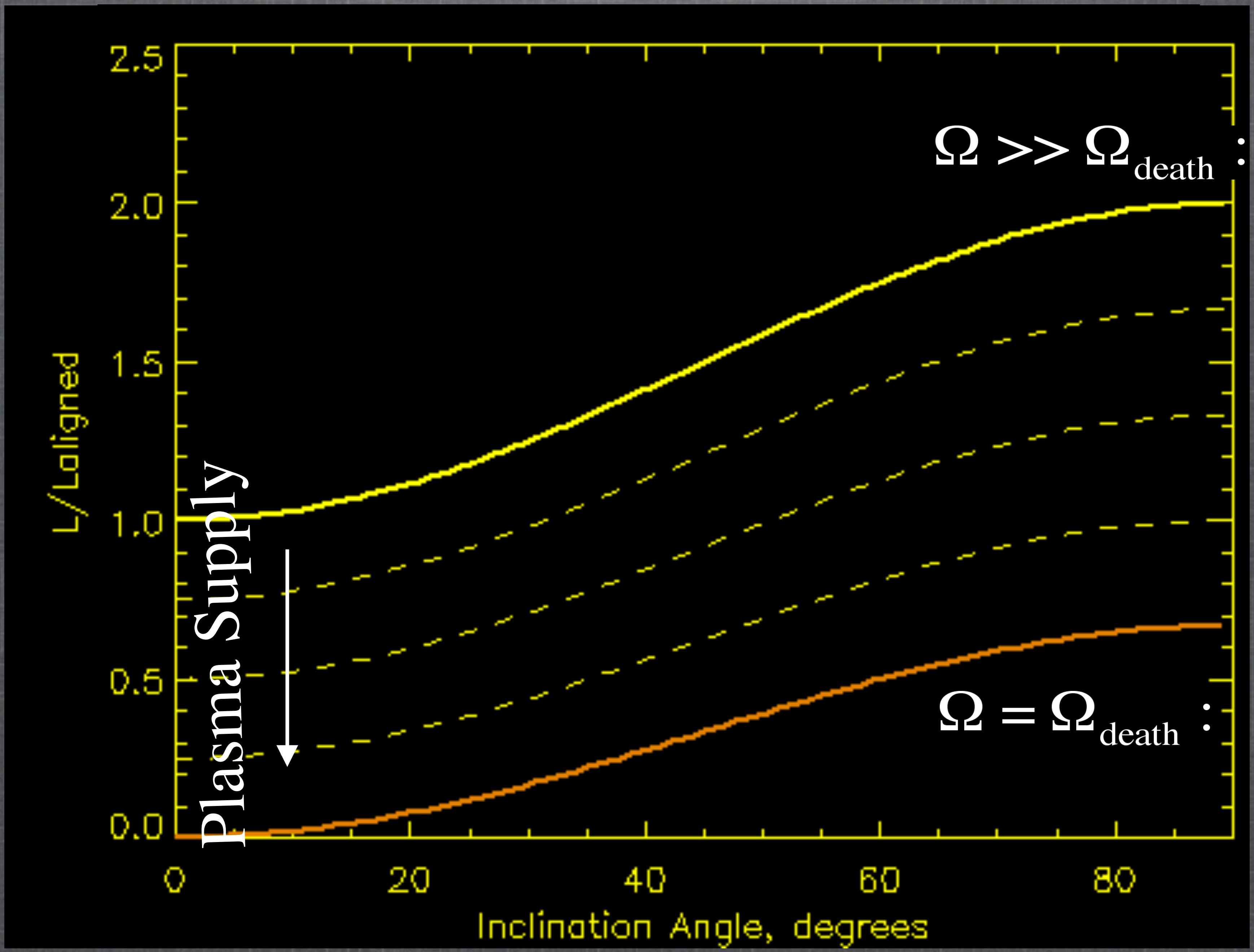
Pros:

- Allow us to compute global structure of the magnetosphere
- Spin-down power
- Geometry of emission

Cons:

- No acceleration; dissipation is artificial
- No radiation; have to beam radiation along B field in sheets
- Are these solutions unique?

SPIN-DOWN POWER



There is a continuum of solutions depending on plasma supply. These can be characterized by the presence of accelerating E field, or resistivity.

Resistive force-free

- There is a continuum of solutions between vacuum and ideal conducting force-free magnetosphere if plasma is not perfect everywhere.
- Can parameterize these with resistivity in the proper frame.
- Nice feature: re-emergence of parallel E field.

Ohm's law in the proper frame:

$$\vec{j}_{\rm fluid} = \sigma \vec{E}_{\rm fluid}$$

In lab frame:

$$\vec{j} = \frac{\rho c \vec{E}\times\vec{B}}{B^2+E_0^2} + \frac{\left(-\beta_{||}\rho c + \sqrt{\frac{B^2+E_0^2}{B_0^2+E_0^2}}(1-\beta_{||}^2)\sigma E_0\right)(B_0\vec{B}+E_0\vec{E})}{B^2+E_0^2}$$

$$B_0^2 = \frac{\vec{B}^2-\vec{E}^2+\sqrt{(\vec{B}^2-\vec{E}^2)^2+4(\vec{E}\cdot\vec{B})^2}}{2},$$

$$E_0 = \sqrt{B_0^2-\vec{B}^2+\vec{E}^2},$$

$$B_0 = \mathrm{sign}(\vec{E}\cdot\vec{B})\sqrt{B_0^2},$$

cf. Lyutikov 03
Gruzinov 07-11

Li, AS, Tchekhovskoy, 2011

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Minimal || velocity frame:

$$\vec{j} = \frac{\rho c \vec{E} \times \vec{B} + \sqrt{\frac{B^2+E_0^2}{B_0^2+E_0^2}}\,\sigma E_0 (B_0 \vec{B} + E_0 \vec{E})}{B^2 + E_0^2}.$$

$$B_0^2 = \frac{\vec{B}^2 - \vec{E}^2 + \sqrt{(\vec{B}^2 - \vec{E}^2)^2 + 4(\vec{E}\cdot\vec{B})^2}}{2},$$

$$E_0 = \sqrt{B_0^2 - \vec{B}^2 + \vec{E}^2},$$

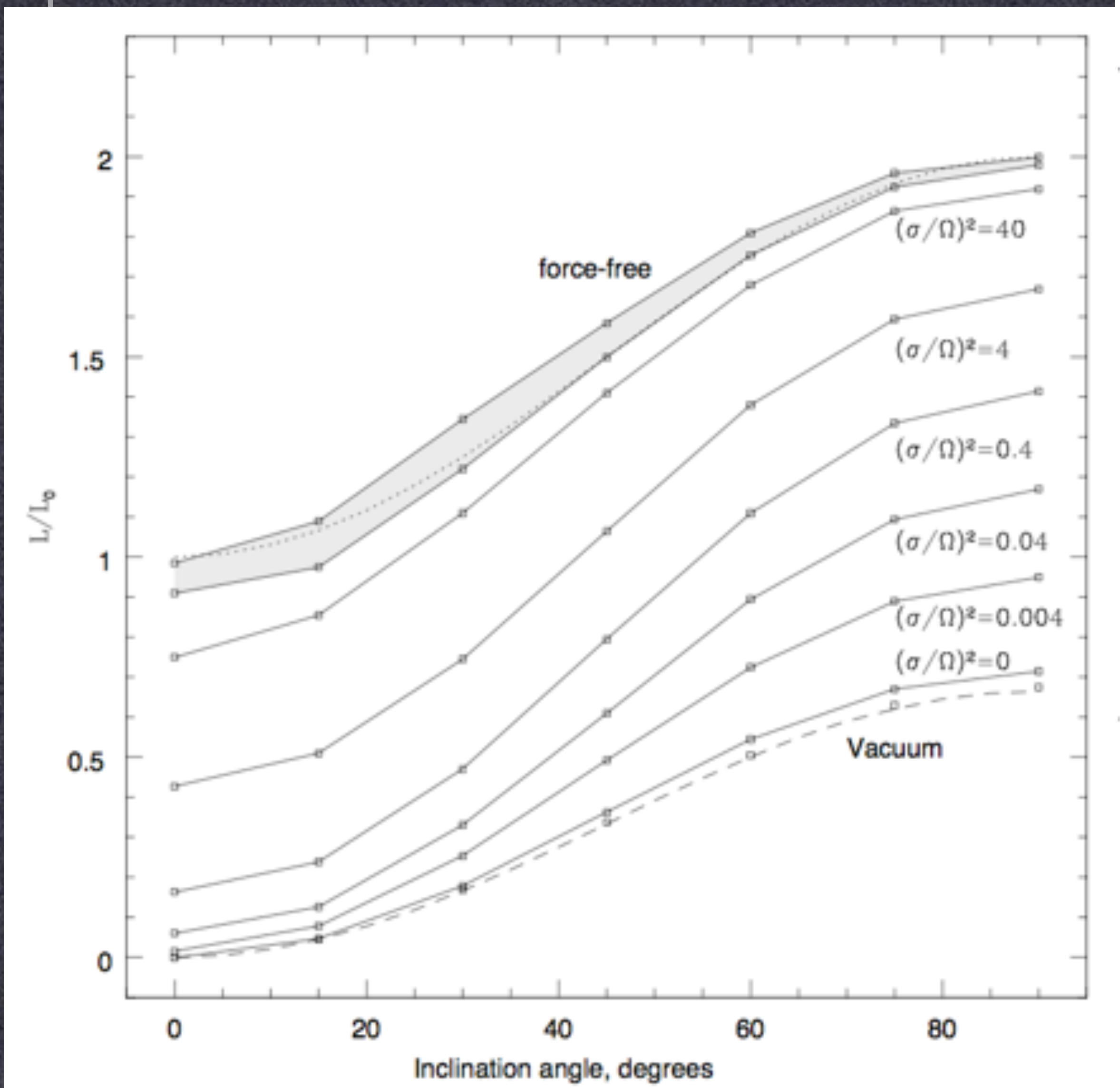
$$B_0 = \mathrm{sign}(\vec{E}\cdot\vec{B})\sqrt{B_0^2},$$

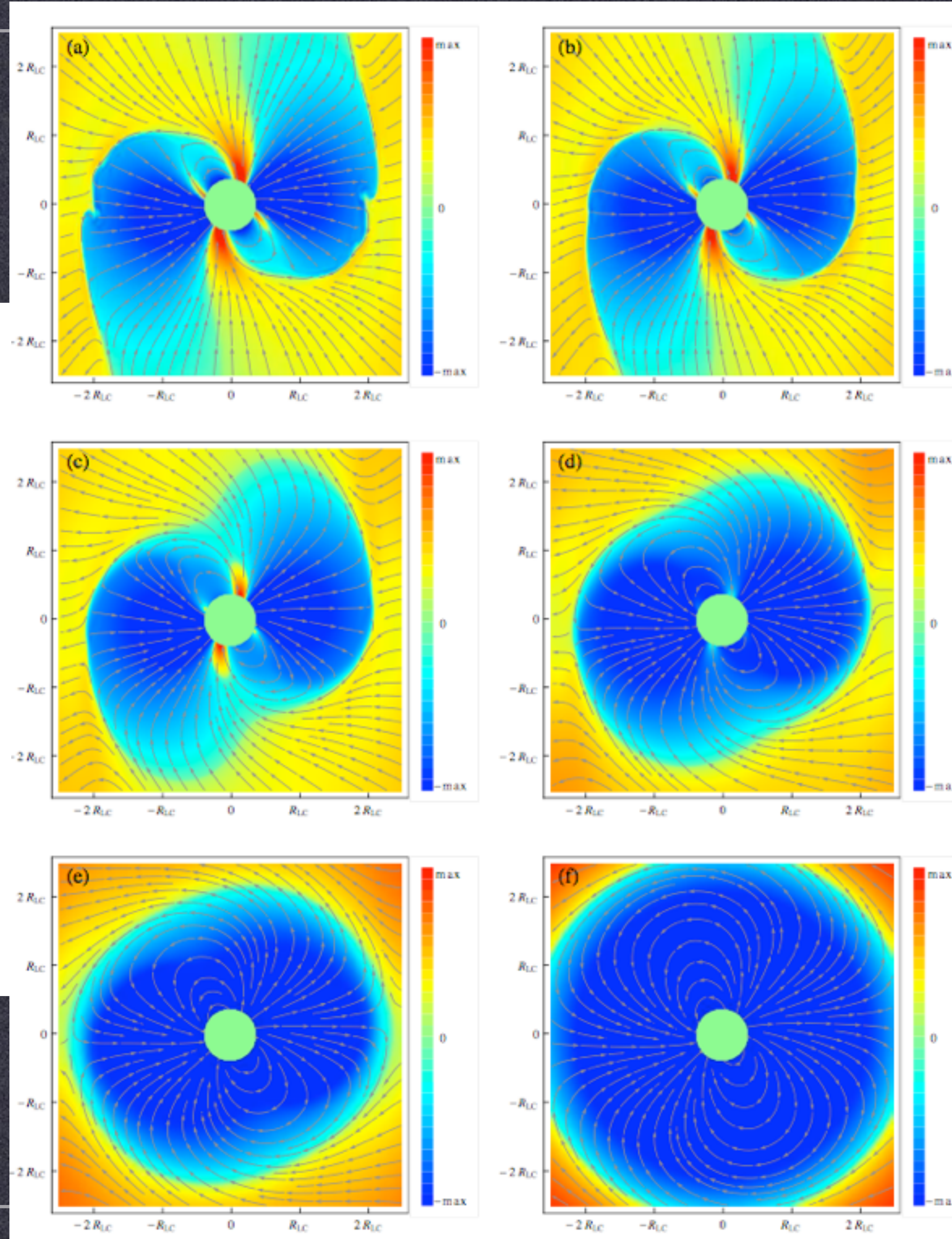
cf. Lyutikov 03
Gruzinov 07-11

Li, AS, Tchekhovskoy, 2011
also, Kalapotharakos et al 11

Resistive:

Vary σ/Ω

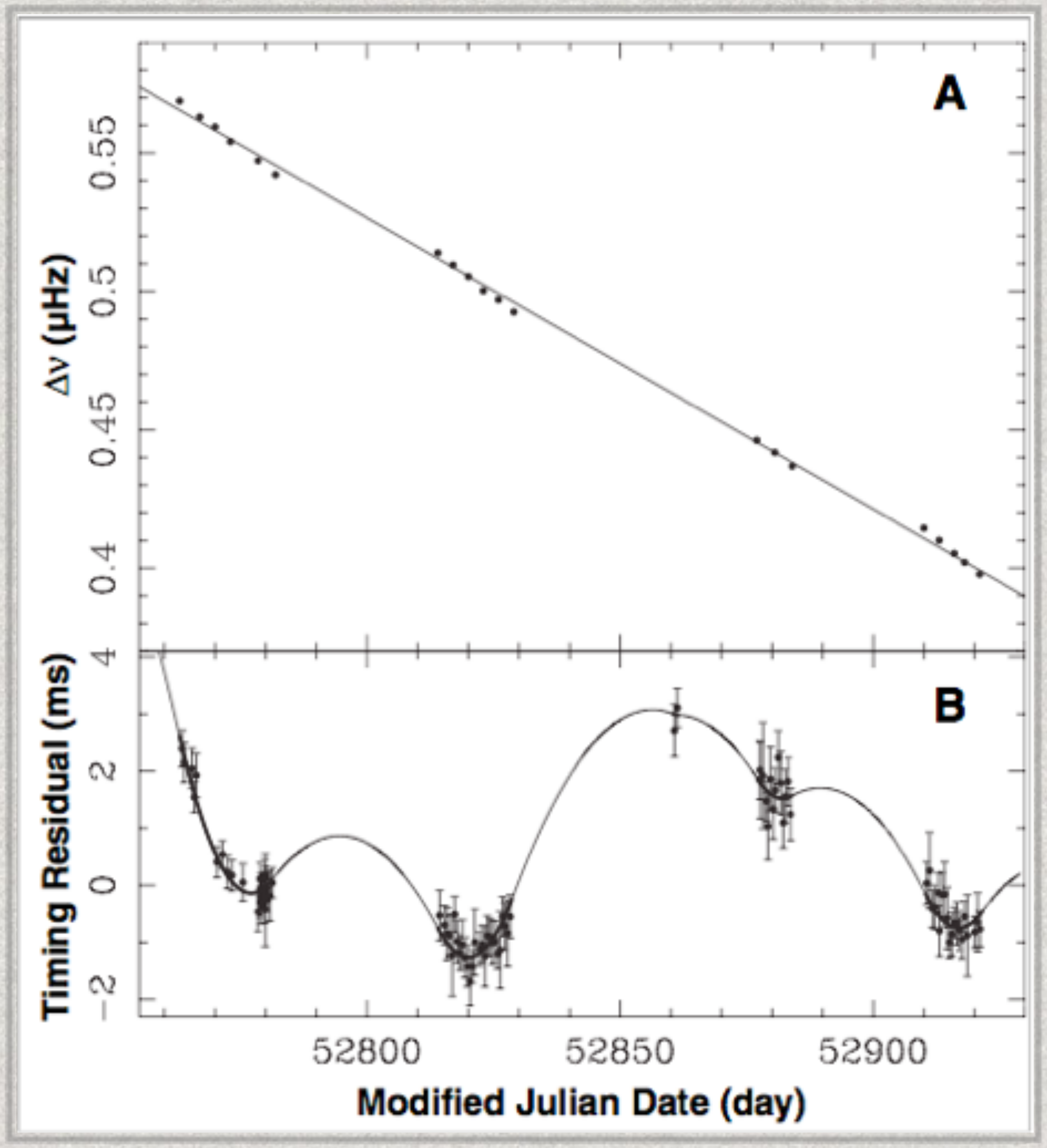




Spin-down power

Application: intermittent pulsars

- Intermittent pulsars display changes in spin-down power when they are ON and OFF in radio by factor >1.5
- One possibility: conducting closed zone, vacuum-like open zone; Interrupted plasma production



Kramer et al 06

Application: intermittent pulsars

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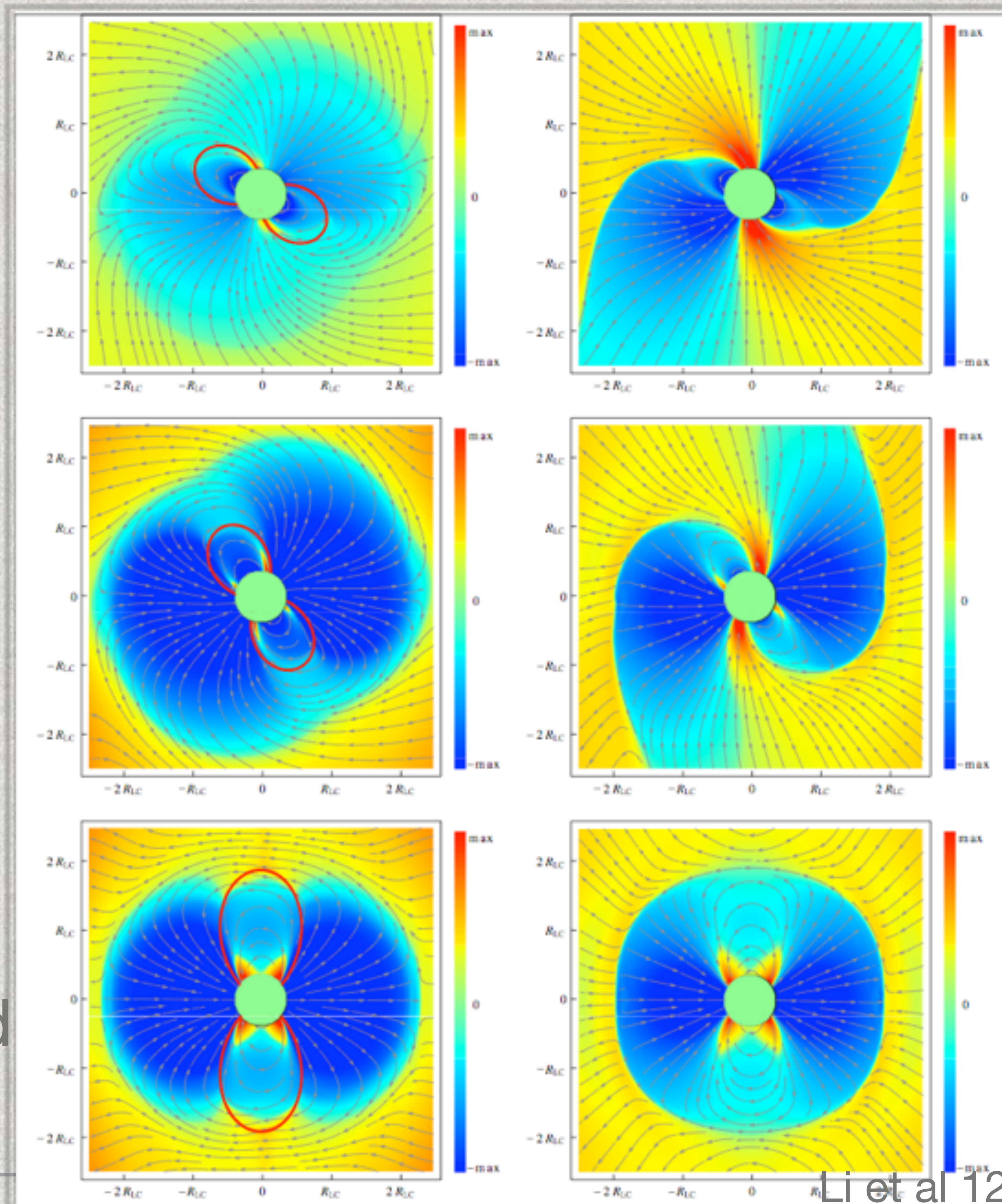
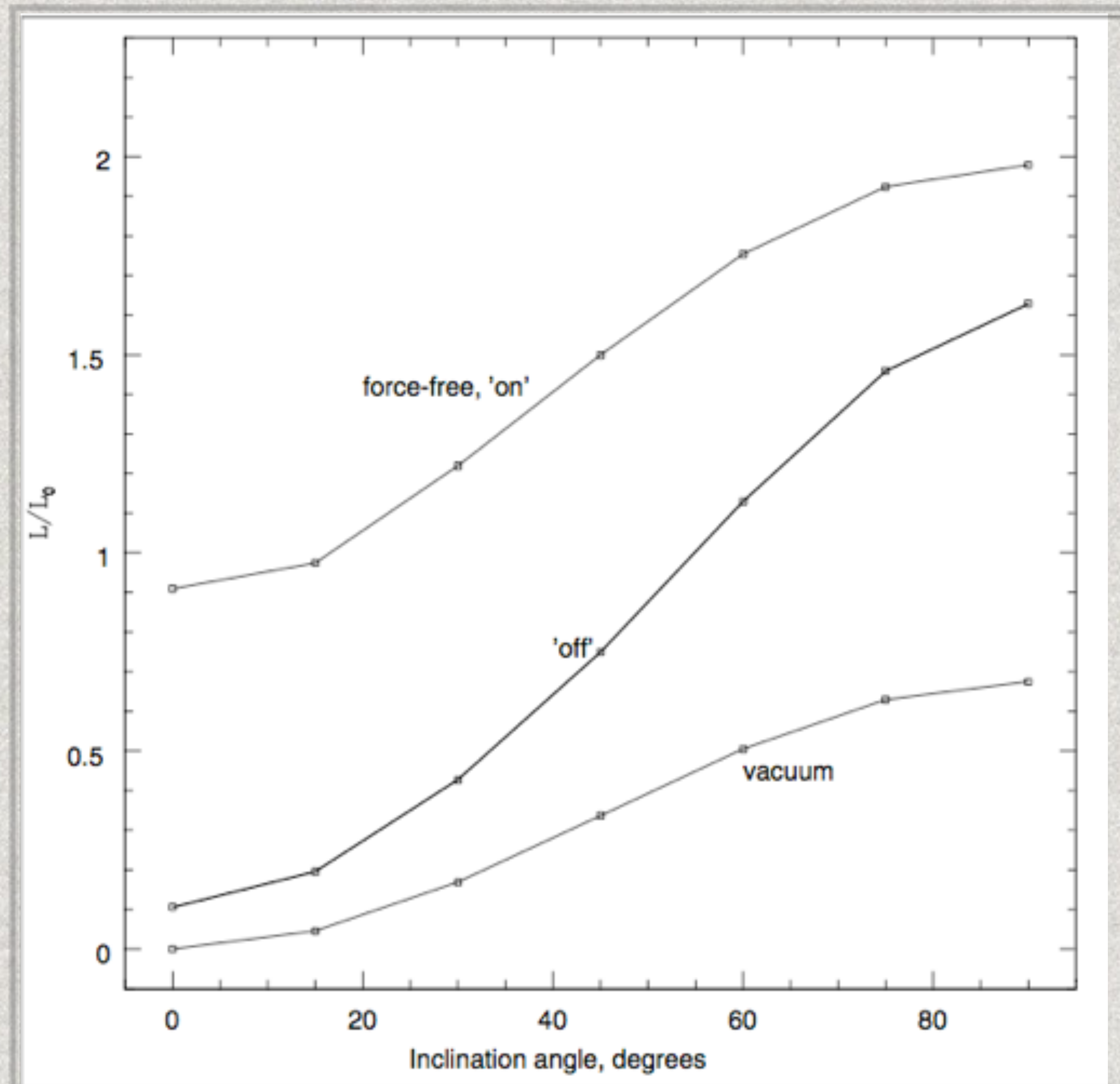


Li et al 12

Application: intermittent pulsars

- Factor of > 1.5 can be explained with "hybrid" vacuum-conducting magnetosphere.
- The physical origin of switch is completely unclear.

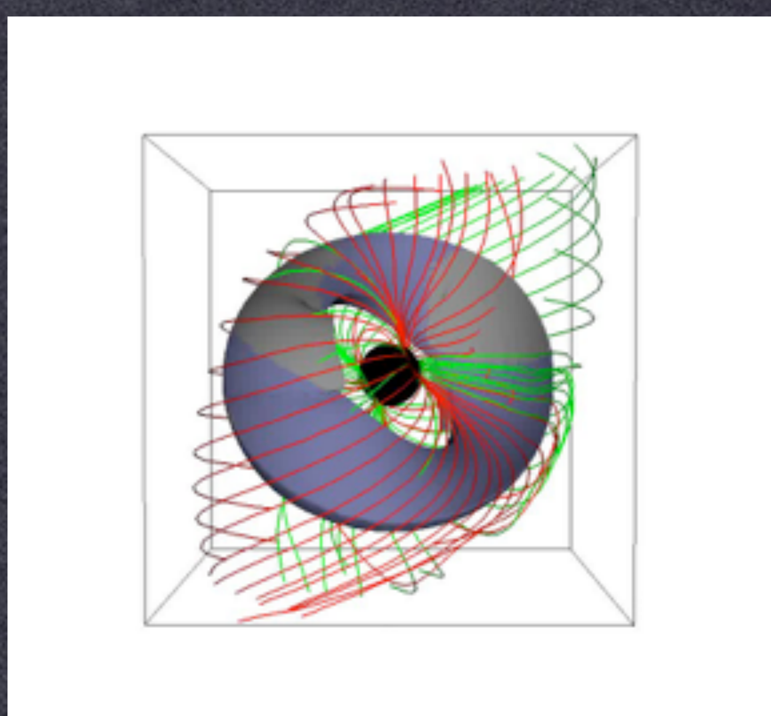


Li et al 12

Resistive Force-free light curves

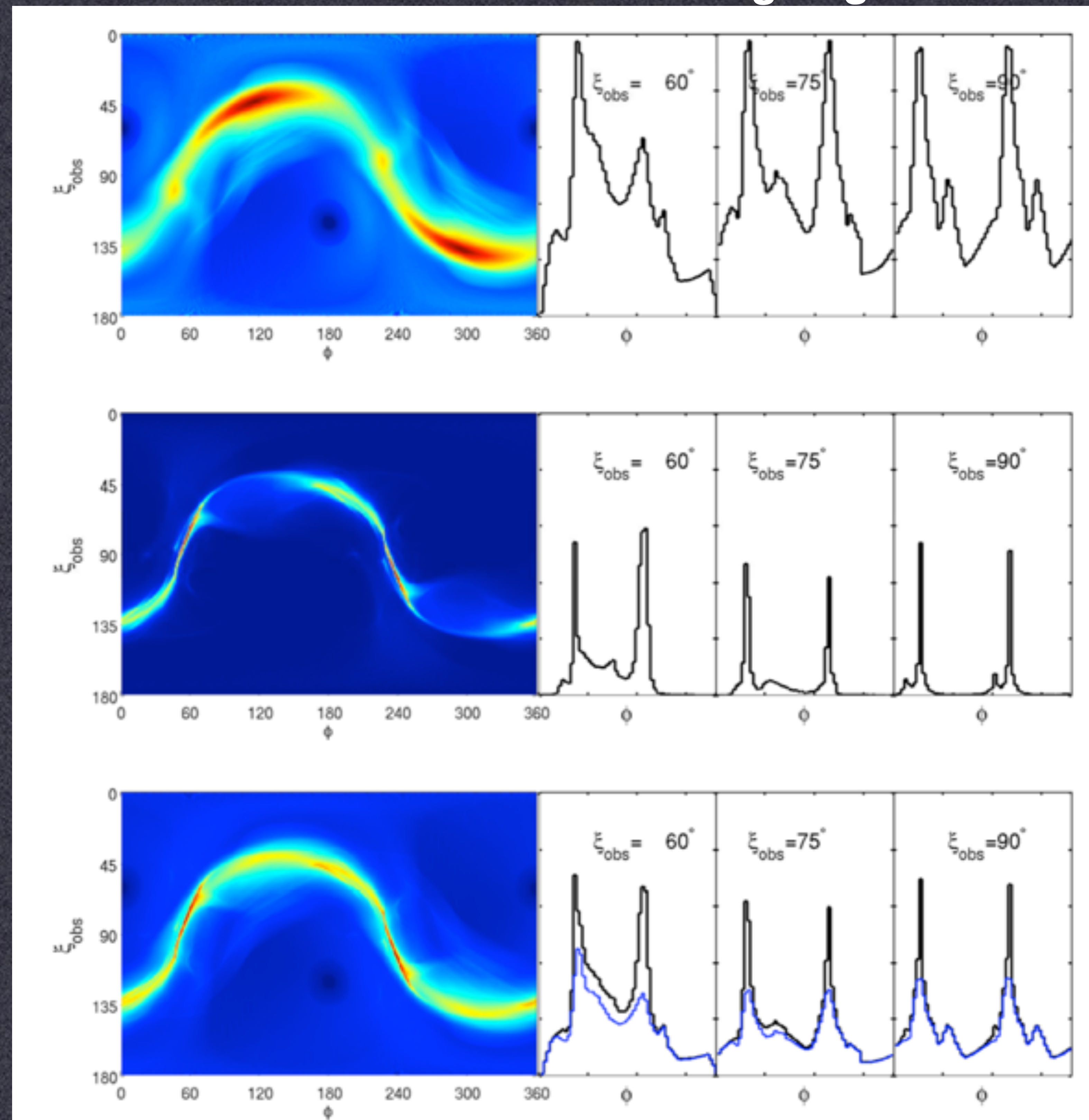
Beaming: along interpolated B field into the sheet. Results in radial beaming. Other beaming does not work!

Combine emission from current layer (<Rlc) for bridge emission with current sheet (>Rlc) for peaks



Viewing angle

Inclination angle



Li, AS, Tchekhovskoy 2014

Weak pulsars

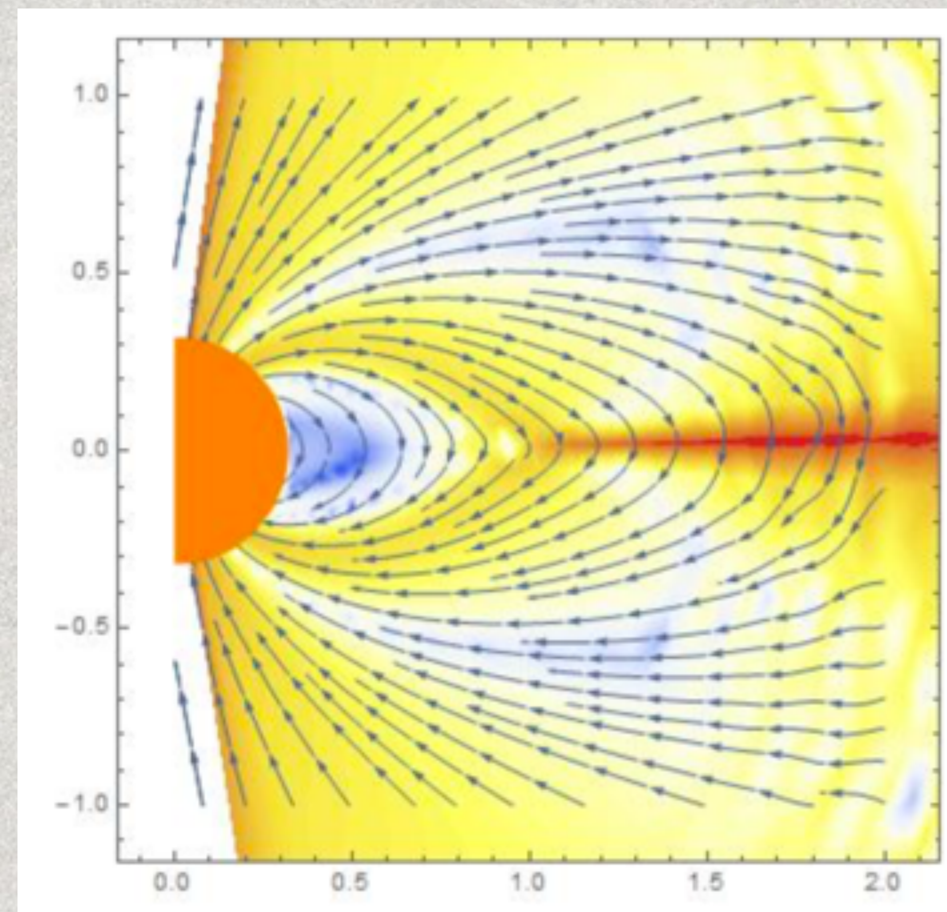
- Force-free disconnects current and charge density (j can be larger or smaller than rho c)
- Weak pulsar solutions connect charge and current: Contopoulos (16), Gruzinov (11+), Beskin (1980s+). Current is tied to GJ density*v. v can be <c, but hard to guess which lines are <c.
- Charge density determines corotation. Resistive solutions break corotation. Weak pulsar solutions allow E>B, but try to keep corotation.

$$\vec{j} = \frac{\rho c \vec{E} \times \vec{B}}{B^2 + E_0^2} + \frac{\left(-\beta_{||}\rho c + \sqrt{\frac{B^2+E_0^2}{B_0^2+E_0^2}(1-\beta_{||}^2)}\,\sigma E_0\right)(B_0\vec{B} + E_0\vec{E})}{B^2 + E_0^2}$$

Li et al 11

$$\mathbf{J}_{\mathrm{AE}} = \frac{\rho_e c \mathbf{E} \times \mathbf{B} + |\rho_e| c (E_0 \mathbf{E} + B_0 \mathbf{B})}{B^2 + E_0^2}$$

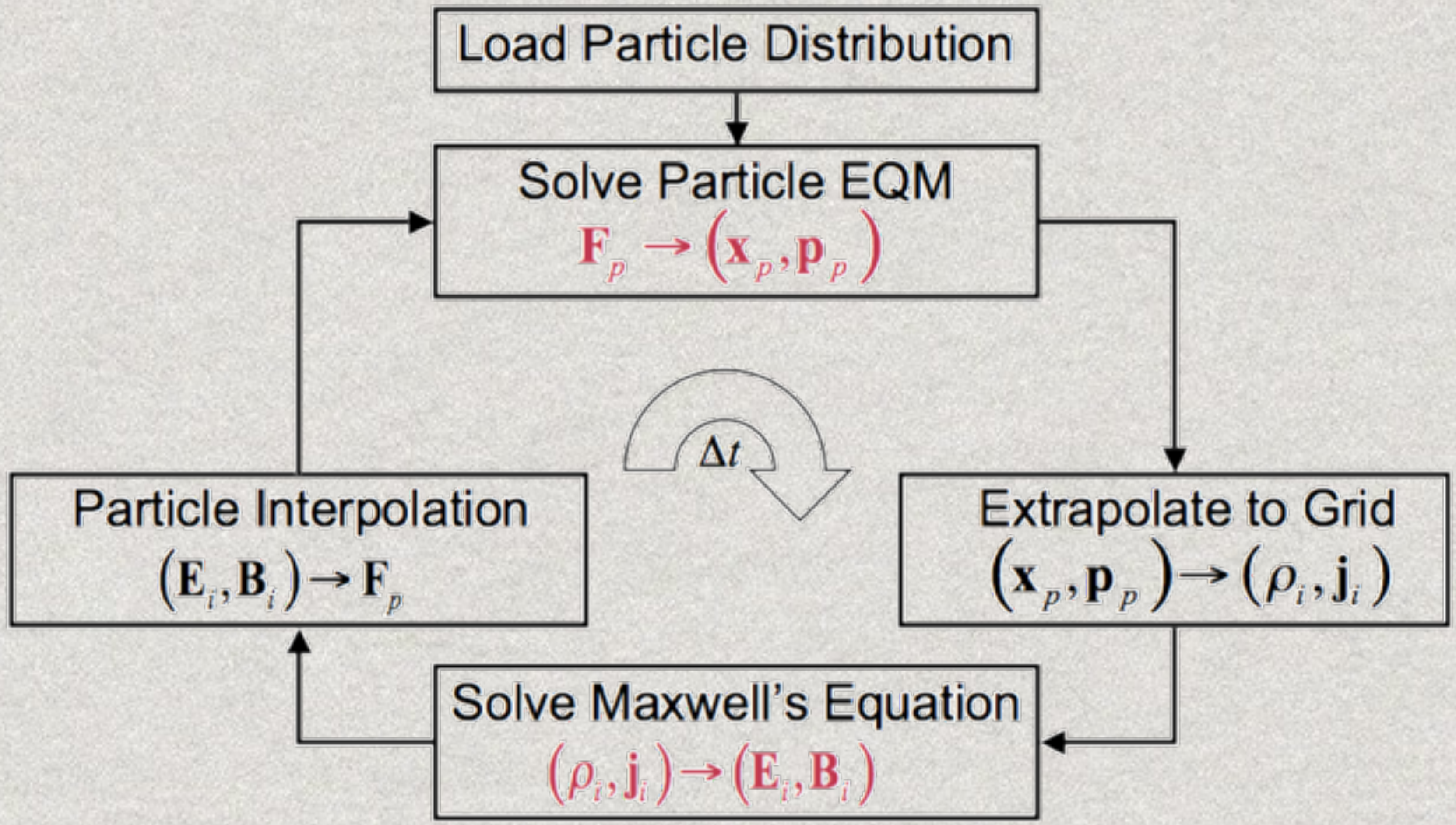
Contopoulos 16



Kinetic method: particle-in-cell (PIC) simulations

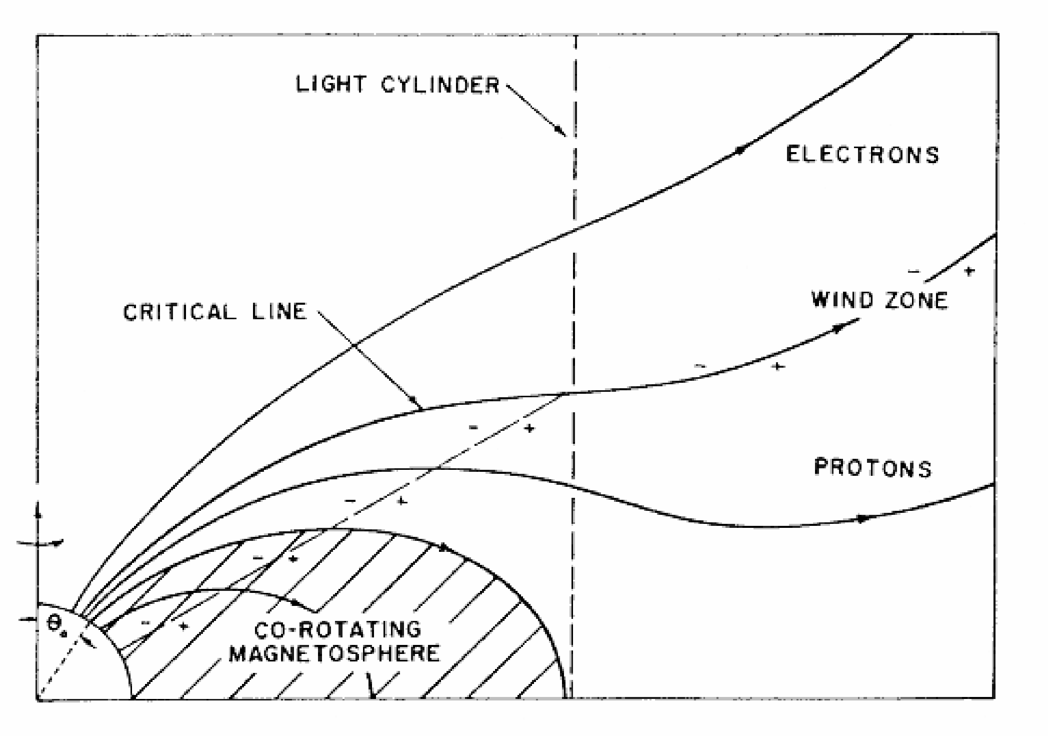
$$\partial E/\partial t = c(\nabla \times B) - 4\pi J\,, \qquad \nabla \cdot E = 4\pi\varrho\,, \quad \nabla \cdot B = 0$$

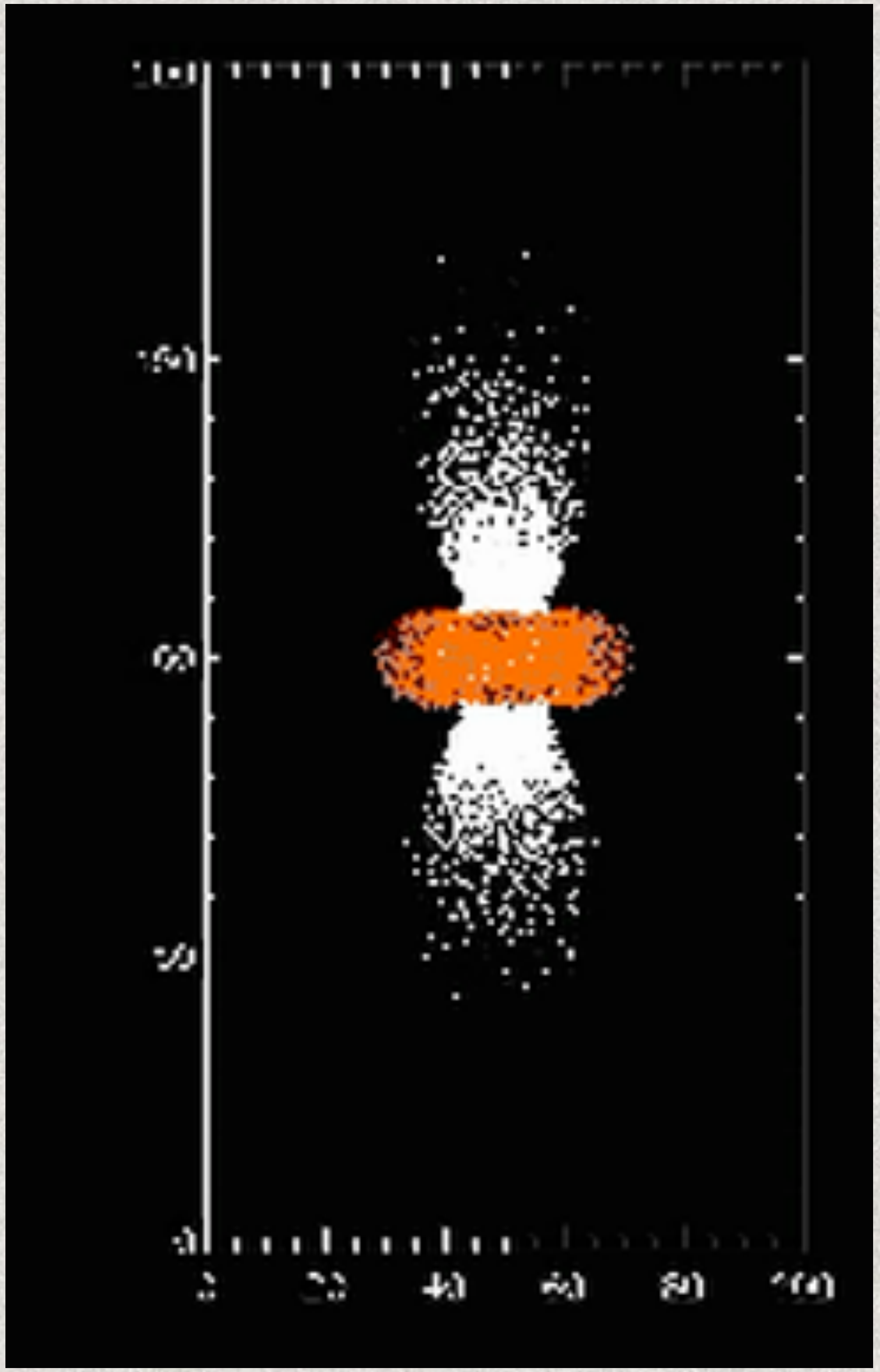
$$\partial B/\partial t = -c(\nabla \times E)\,, \qquad \frac{d}{dt}\gamma m\mathbf{v} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$$



Charge-separated models

AS & Arons 02;
Michel et al 84, 01





Free escape from the surface, plasma density ~ GJ.

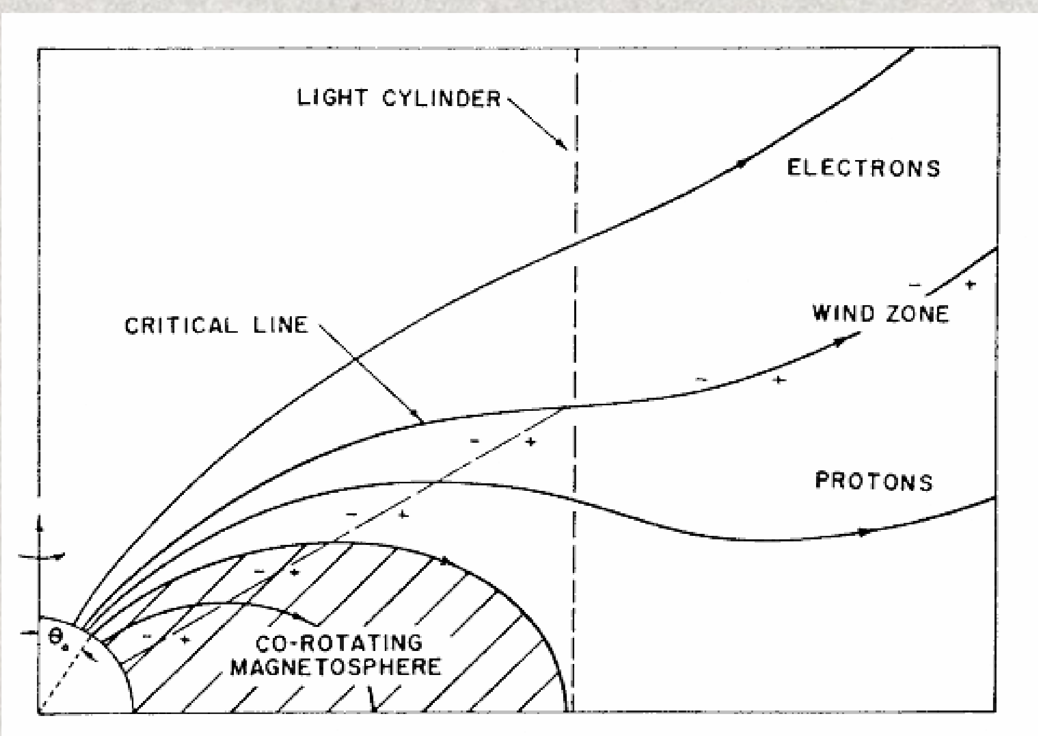
Use particle-in-cell simulations

Disk+dome electrospheres

No spin-down

Are these the dead pulsars after pair production ends?

Charge-separated models



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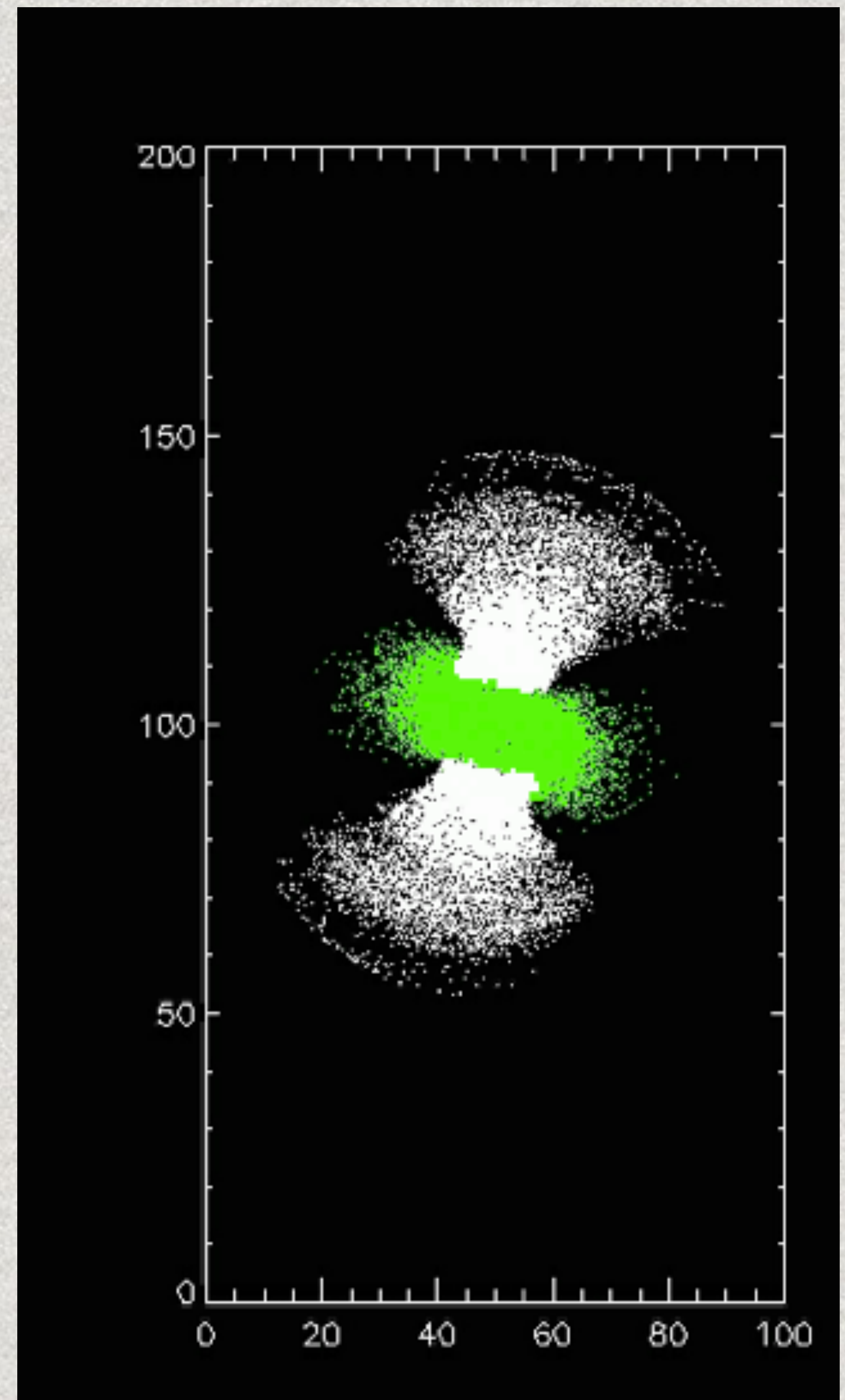


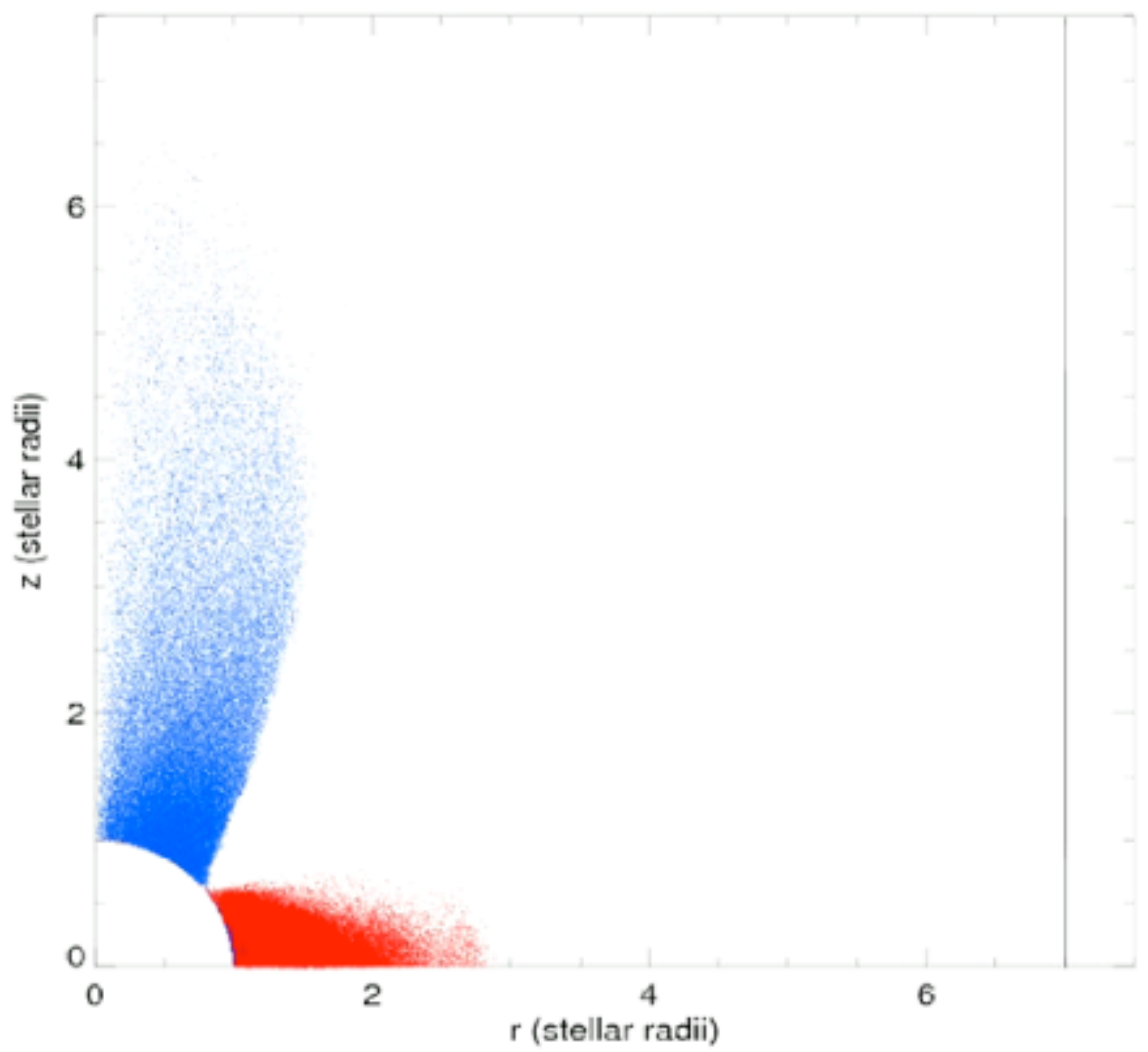
AS & Arons 02;
Michel et al 84, 01

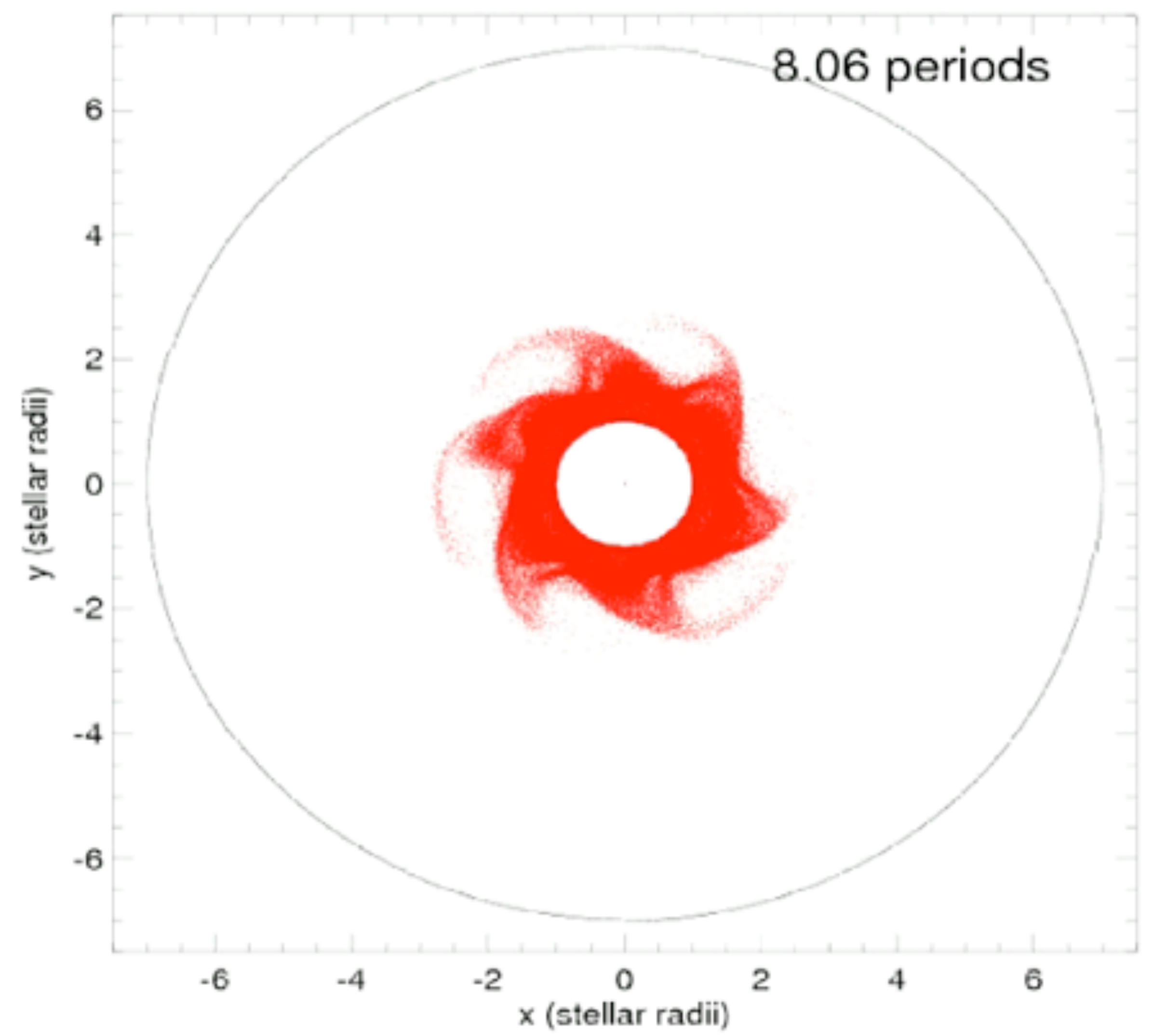
Disk+dome electrospheres

No spin-down

Are these the dead pulsars after pair production ends?

Non-axisymmetric instabilities





Belyaev & AS (unpub)

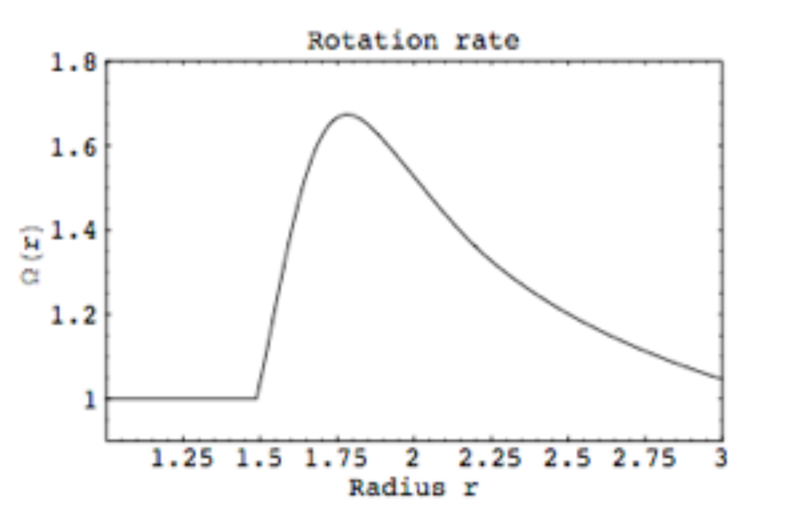
Disk-Torus Electrosphere

Michel et al `84-01

Diocotron instability

AS & Arons 02;

Petri et al 02-



Possibility of radial current

Electrospheres are a curiosity

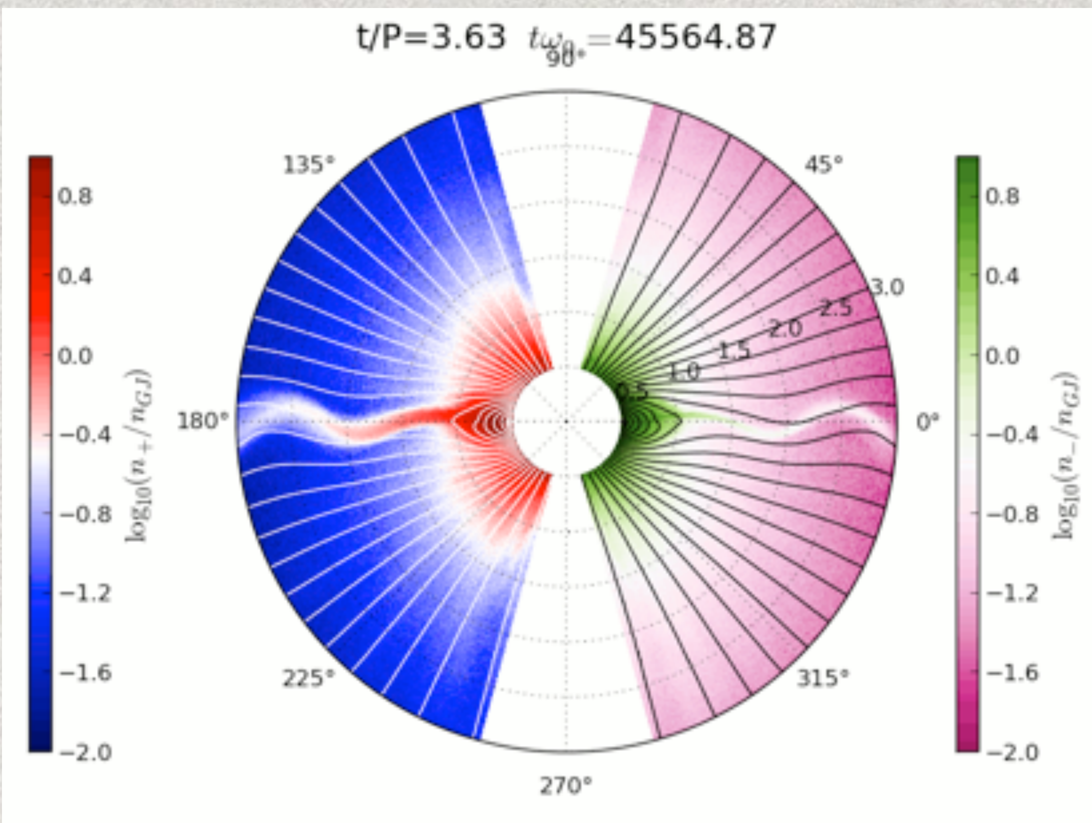
Add pairs?

Petri et al 02

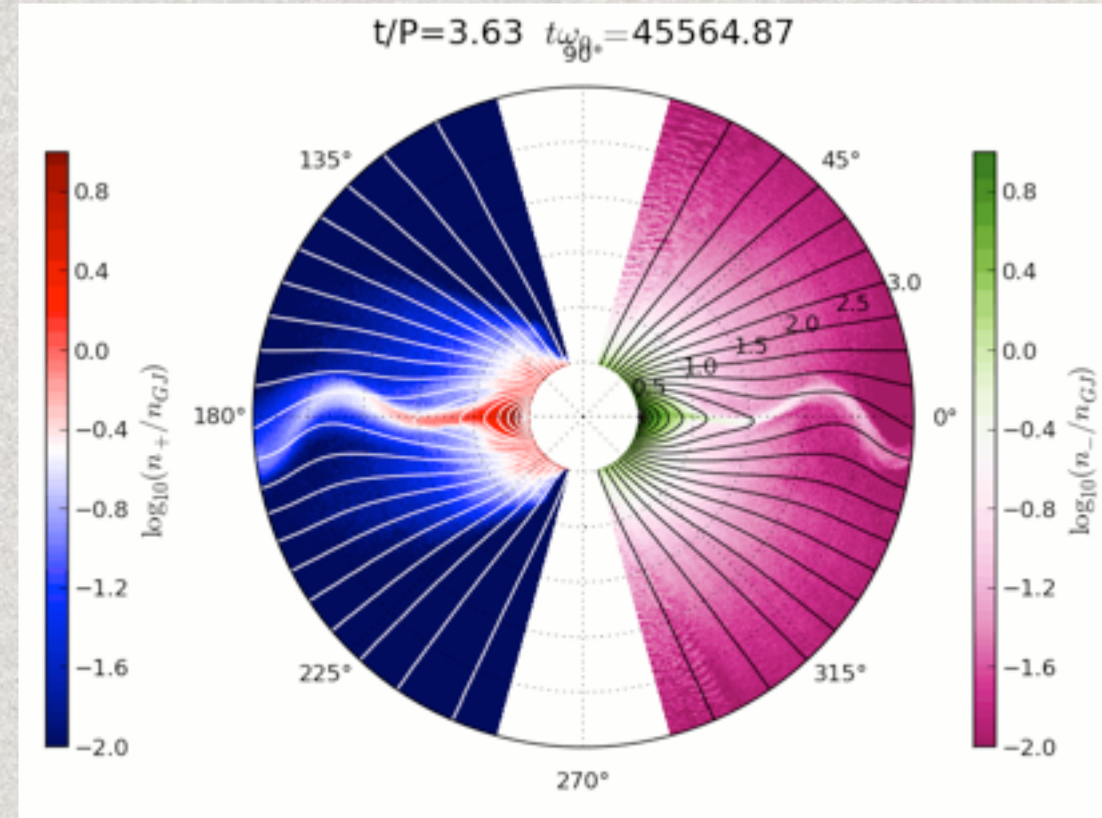
Continuum of intermediate states?

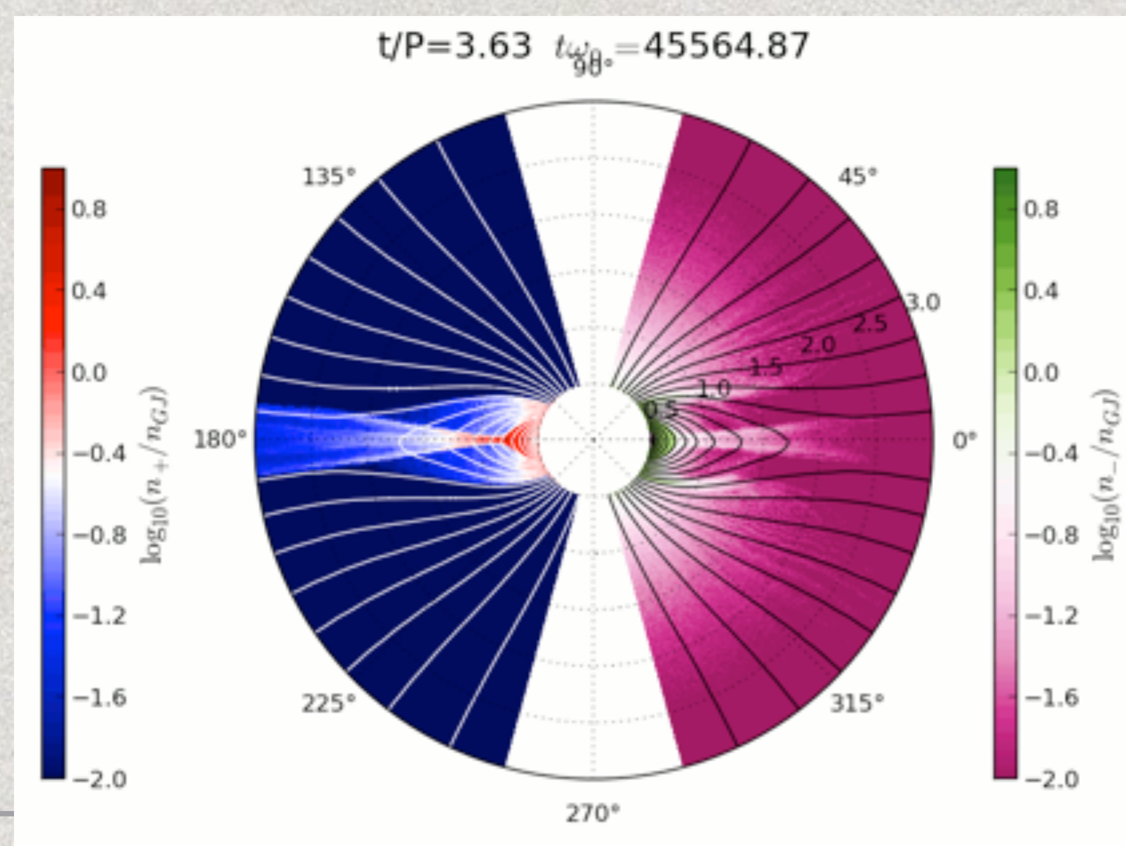
Injection of pairs from surface v=0.5c

$n/n_{GJ}=5$



$n/n_{GJ}=2$





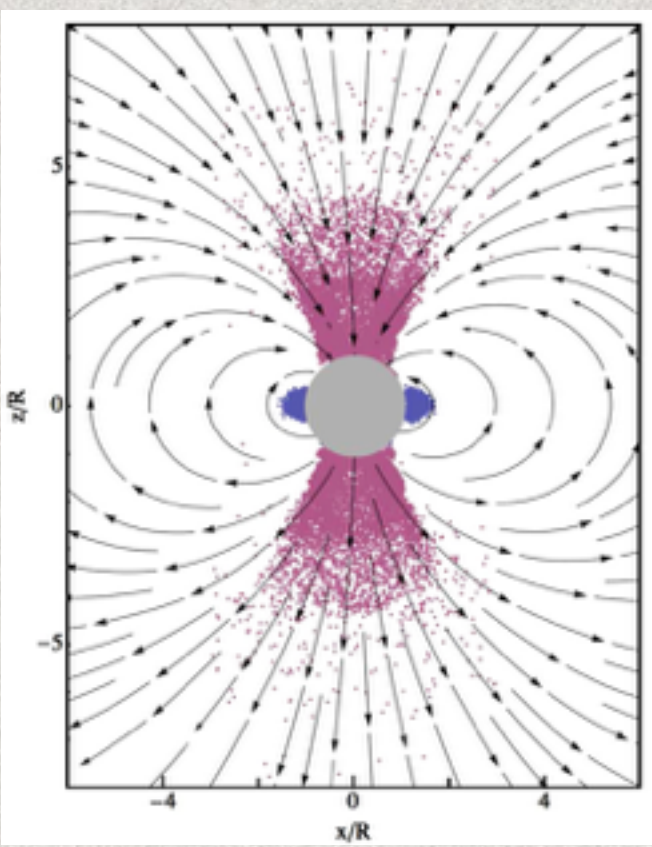
$n/n_{GJ}=1$

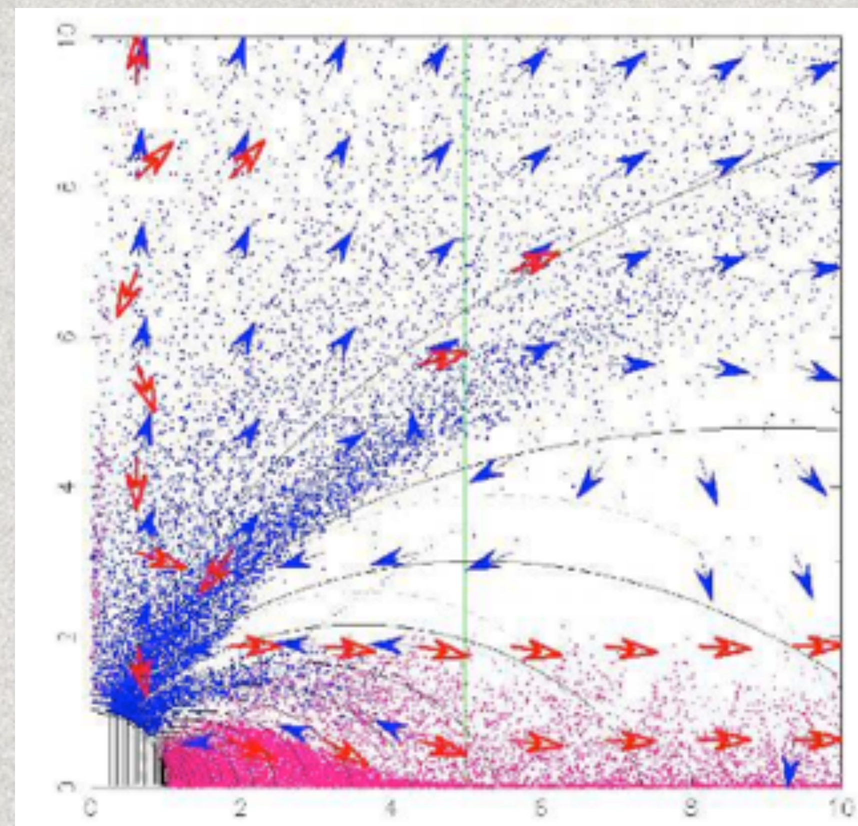
Cerutti et al 2014

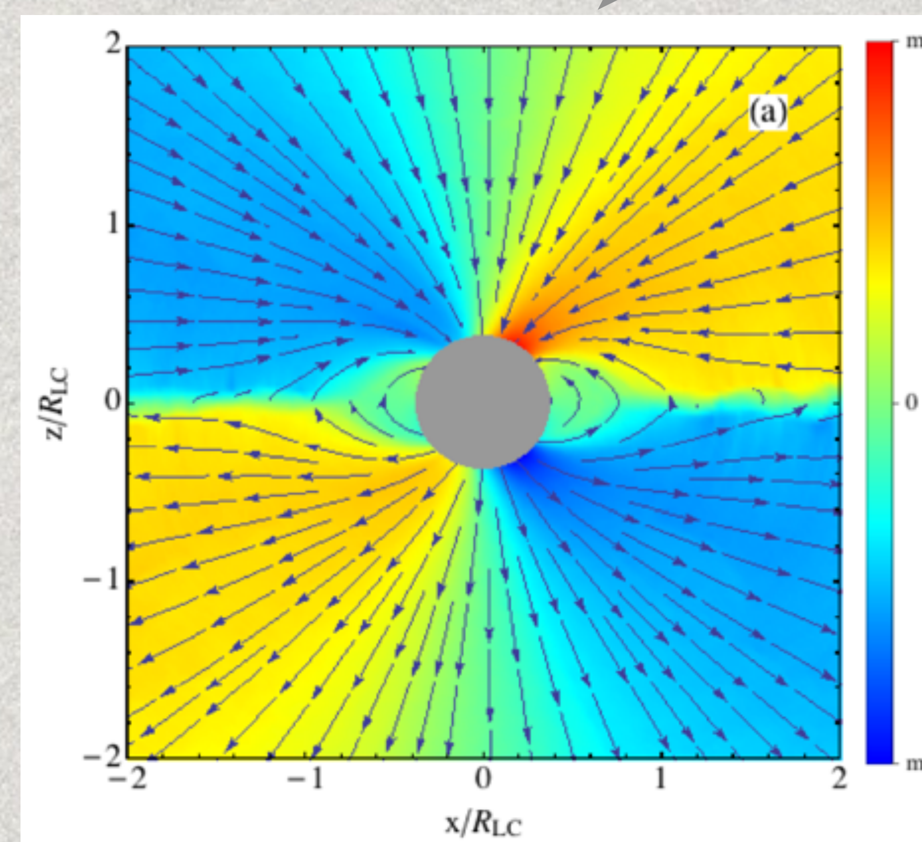
Ab-initio pulsars

- There may be other solutions depending on plasma supply; experimenting with pair formation prescriptions — see Sasha's talk

Plasma supply







There is a class of solutions with E>B and accelerated particles (e.g. Gruzinov; Yuki+Shibata). They must be low-multiplicity states, that may not produce abundant pulsar wind as needed by observations.

Weak pulsars?

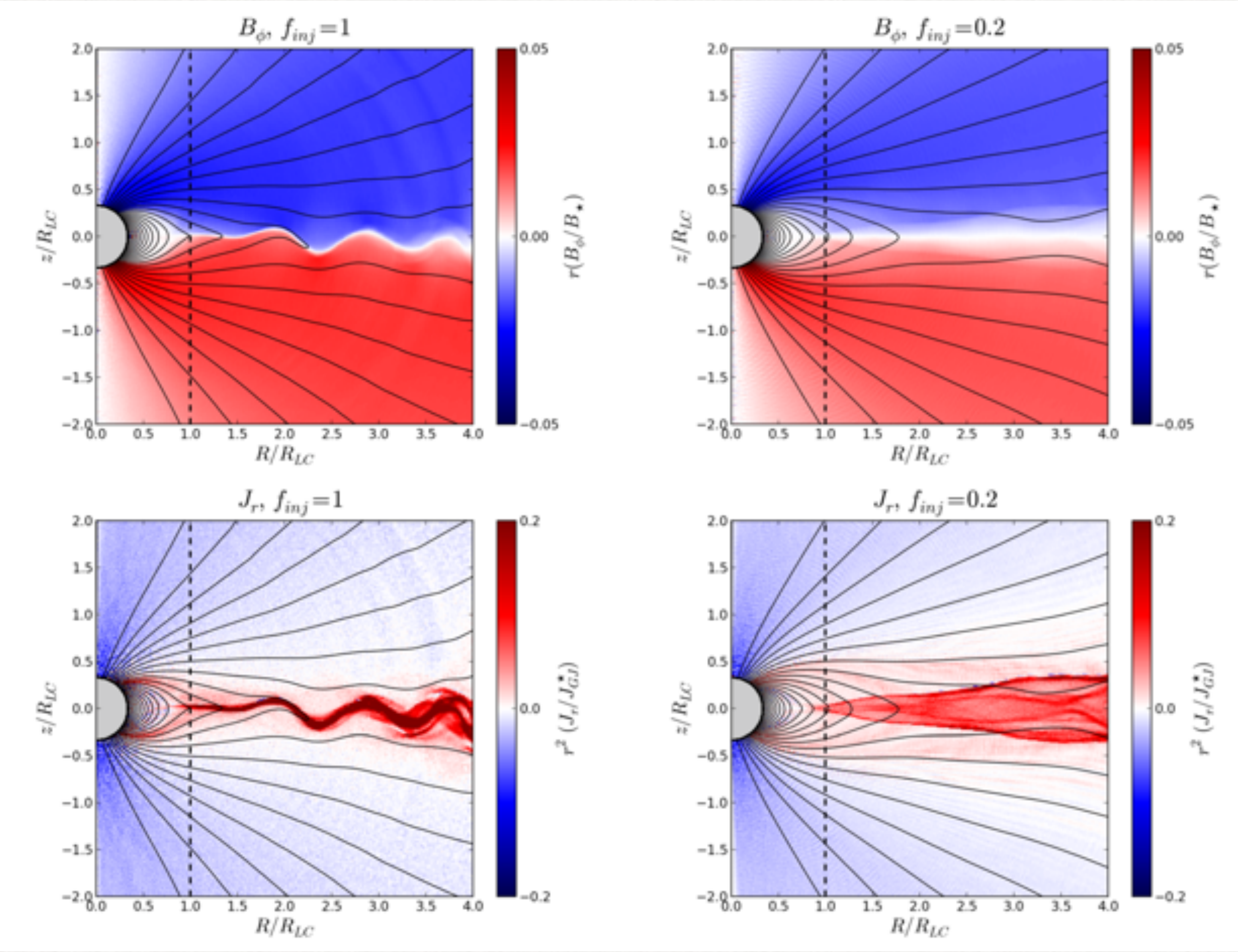
- Existence of pair formation at and beyond the LC is necessary for spin-down.

Cerutti et al 2014

- Weak pulsars only have pairs from near the star. Do they work?

- When pairs are continually injected — reach E>B solutions

- Self-consistent pair production — collapses to disk-dome (see next talk)



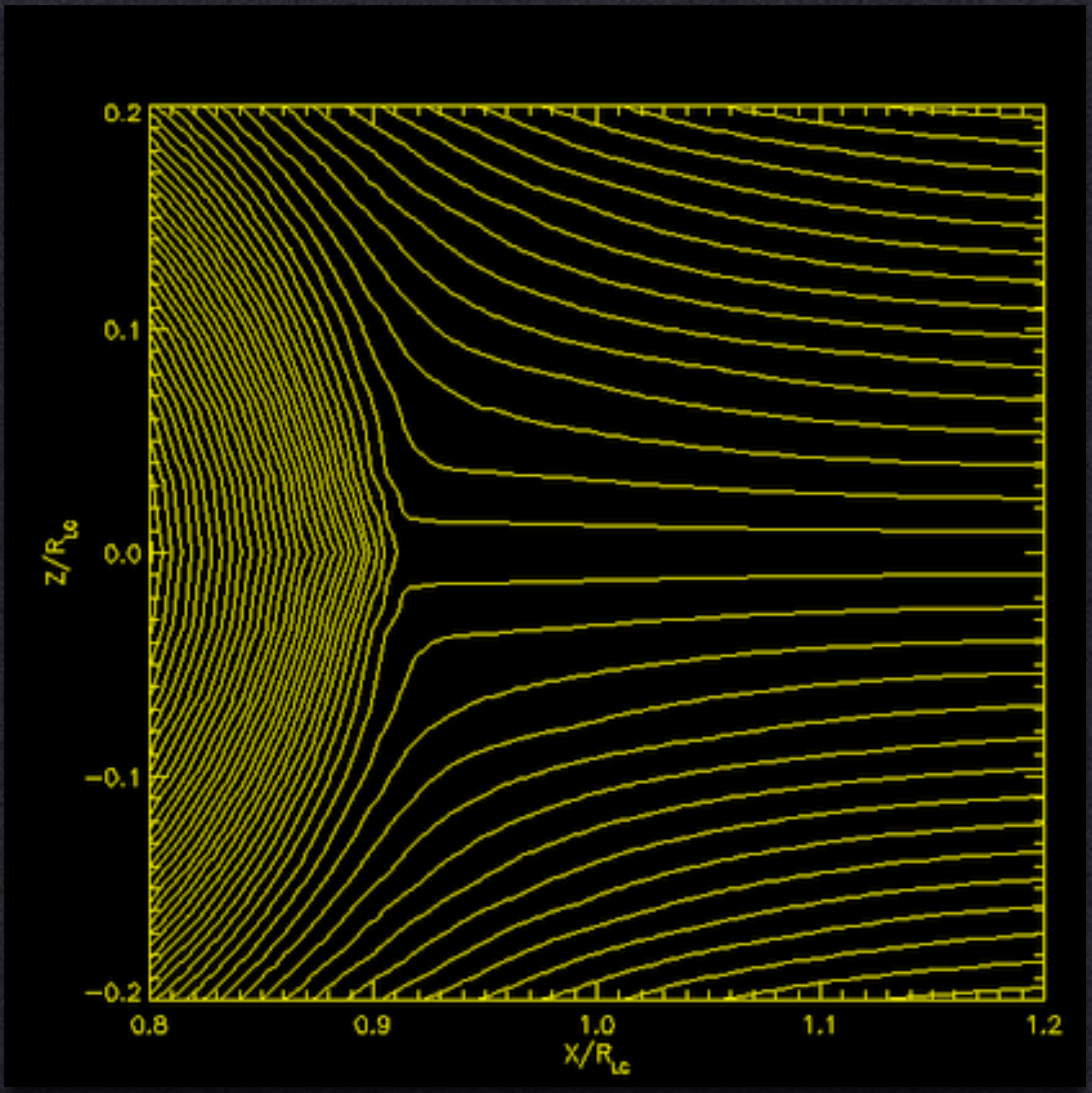
Source of emission

- Emission is geometrically associated with the current sheet
- What is the acceleration and radiation mechanism in current sheet?

  Most likely culprit -- relativistic reconnection. This is different from conventional picture of accelerating gaps starved of plasma and curvature emission

- Boosted synchrotron from heated plasma can work

Reconnection controls magnetospheric shape!

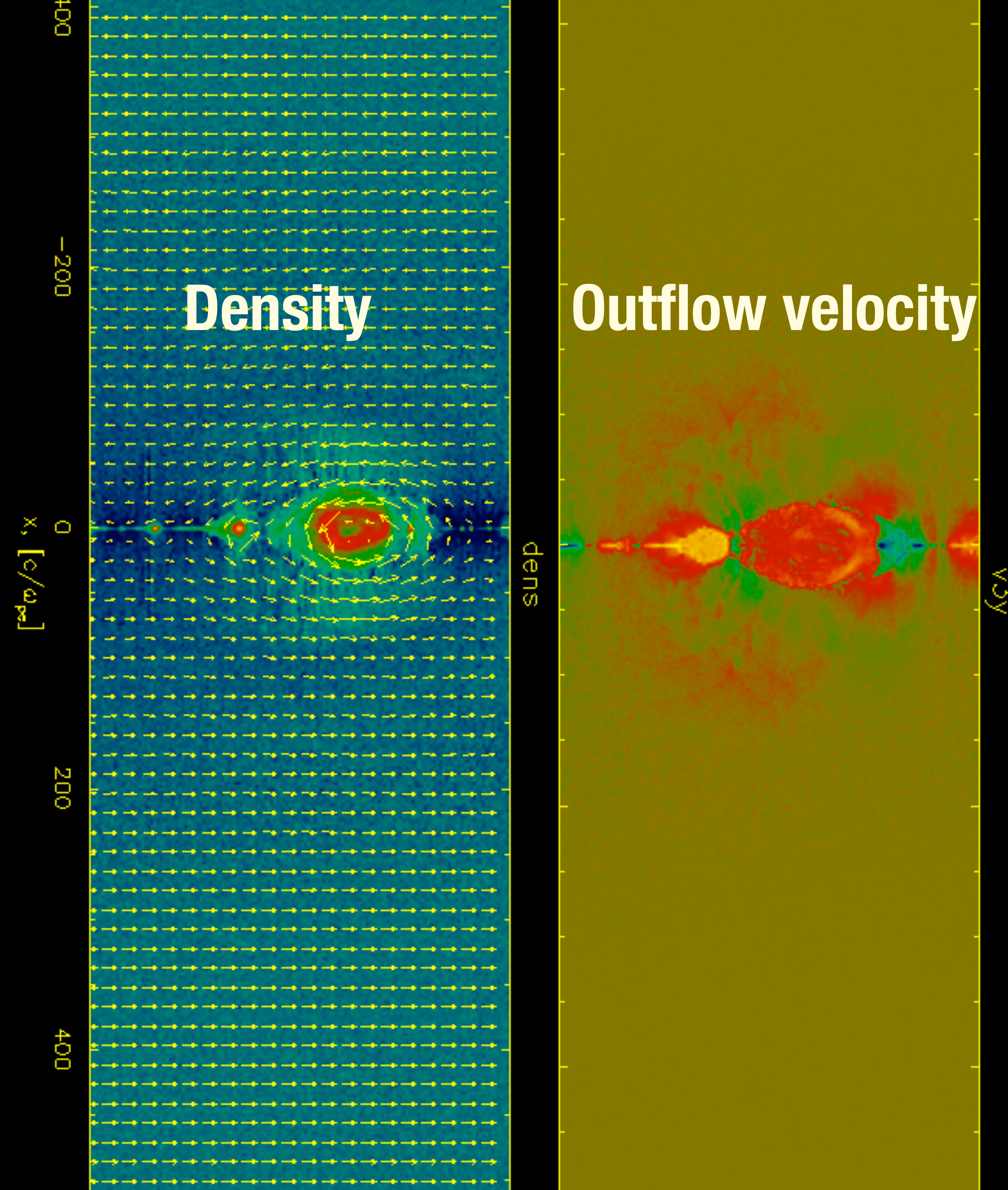


Better ideas of flow direction in the current sheet needed.

In PIC simulations get outflows near sqrt(sigma).

Minijets?

Since beaming along extrapolated B field in the current sheet makes double peaks, it's a contender



Why reconnection makes sense

- Conditions in the sheet can be obtained from:

Pressure balance

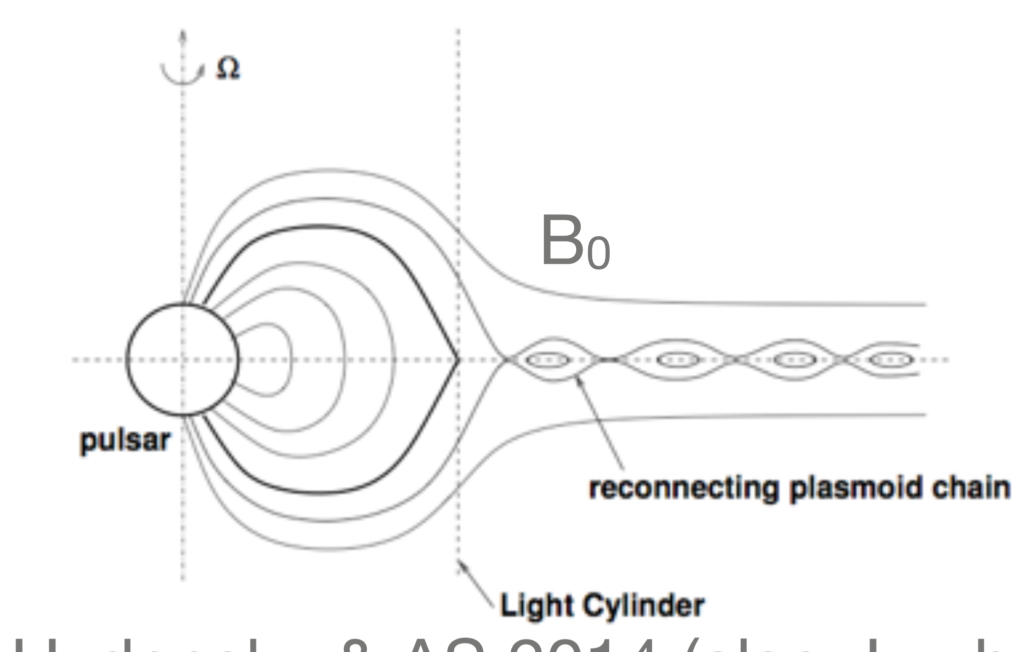
$B_0^2/8\pi = 2\, n\, T$

Strong synchr. cooling:

$S_{in} = (c/4\pi)\, E_z\, B_0 \sim Q_{rad} \sim \delta\, (2n)\, P_{sync}(T)$

Ampere's law:

$j_z = 2ne\, v_{dr} = 2\, nec\, \beta_{dr} \sim (c/4\pi)\, B_0/\delta$



Uzdensky & AS 2014 (also, Lyubarsky 96, Petri 12, Arca 12)

Temperature, density and thickness depend on B at LC. $\gamma_T = T/m_e c^2 \sim [\beta_{dr}\, \beta_{rec}\, 8\pi e/\sigma\, B_0]^{1/2}$

$\sim (\beta_{dr}\, \beta_{rec})^{1/2}\, 4 \times 10^4$

Temperature at 10GeV comoving --> 160MeV synch radiation --> GeV pulsed emission in the lab boosted by bulk gamma of ~10. IC gives VHE.

Conclusions

- Magnetospheric shape is now known and confirmed in the limit of abundant plasma in 3D.
- Geometrically these models are being contrasted with gamma-ray observations (Separatrix Layer vs Gaps).
- More realistic models with 3D RMHD, cascade physics and full PIC are advancing
- Reconnection may play an important and under-appreciated role in both emission and determining the magnetospheric shape.