Lecture 1: BASICS OF COSMIC RAYS TRANSPORT

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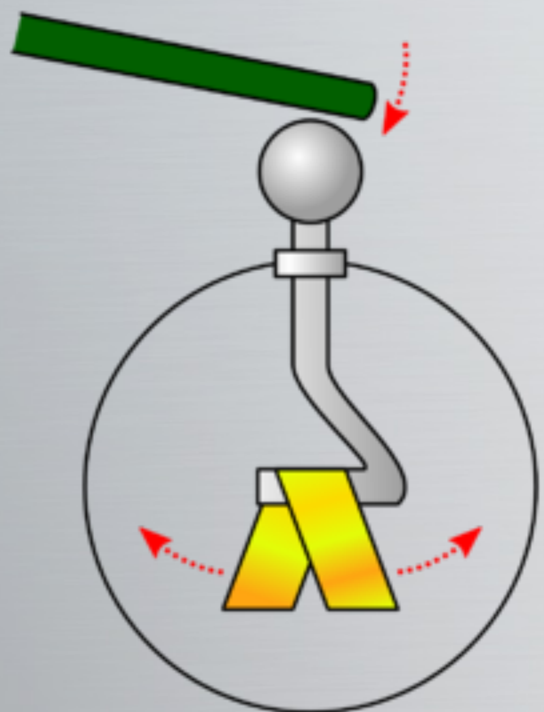
-INTRODUCTION TO CR PHENOMENOLOGY
-PROPAGATION OF A CHARGED PARTICLE IN A MAGNETIC FIELD
-QUASI LINEAR THEORY OF PARTICLE TRANSPORT

Fermi Summer School, Lewes, DE, May 29, 2012

THE COSMOS IN AN ELECTROSCOPE



IT WAS KNOWN SINCE THE END OF 1800 THAT AN ELECTROSCOPE WOULD NOT STAY CHARGED INDEFINITELY

HOW WOULD THE CHARGE FLOW AWAY?

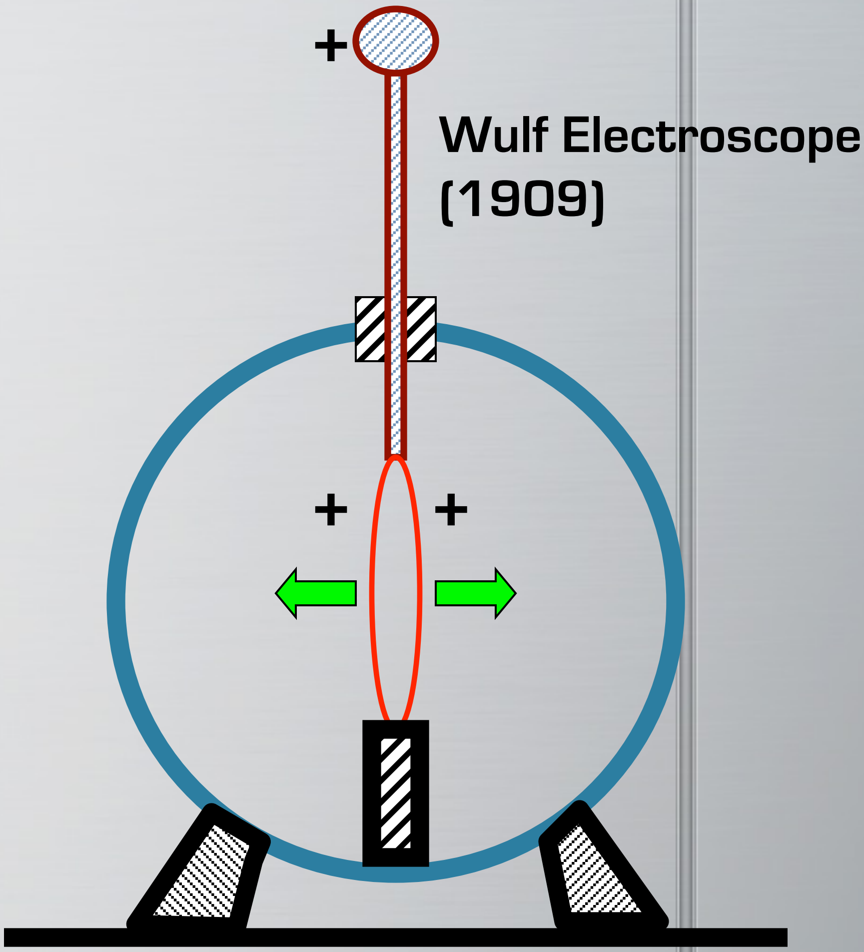
THE AIR MUST HAVE A RESIDUAL IONIZATION, SO THAT THE CHARGE WOULD BE ABLE TO LEAVE THE ELECTROSCOPE...

BUT THEN WHAT IS THE SOURCE OF SUCH A RESIDUAL IONIZATION?

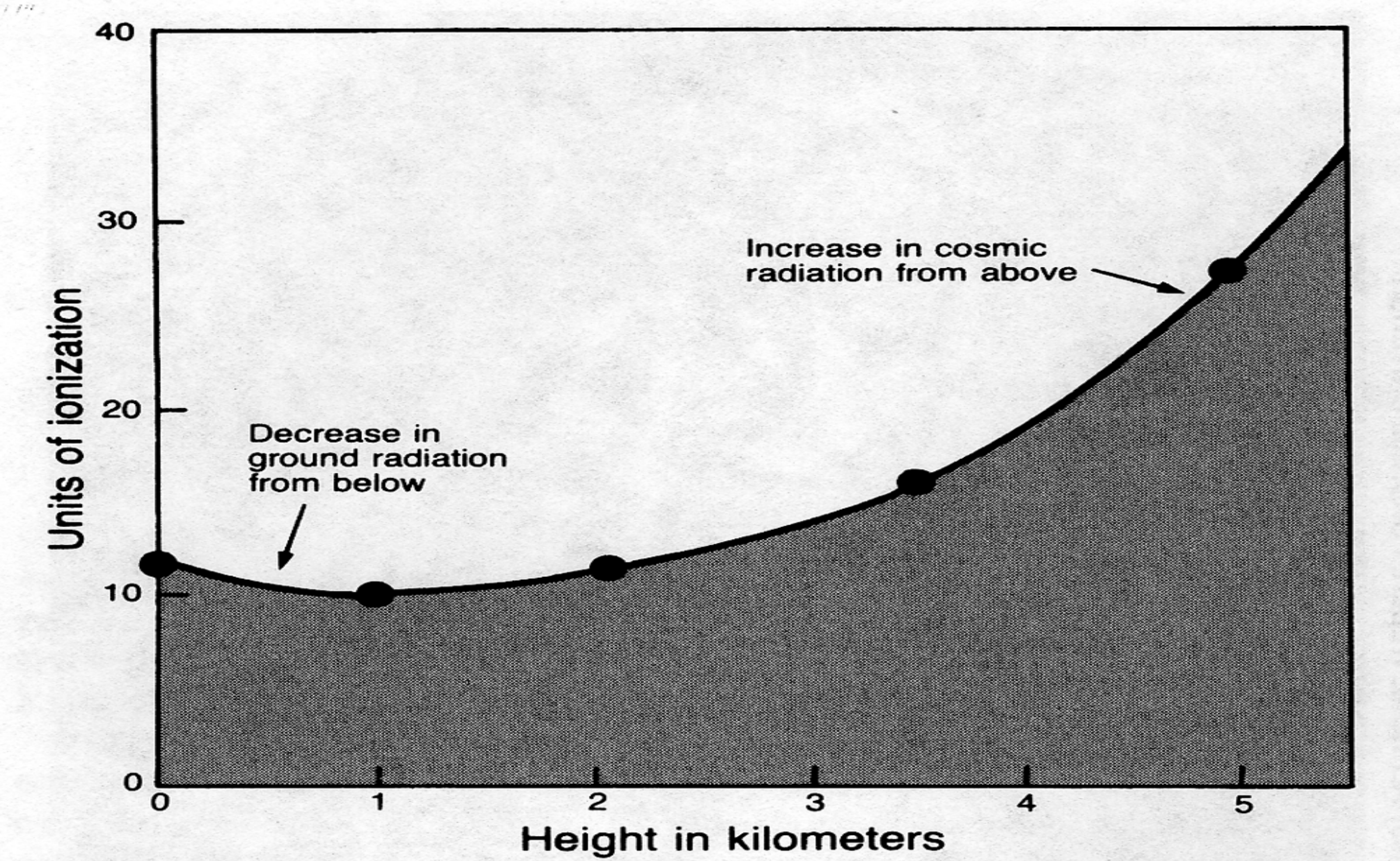
1895: X-RAYS (ROENGTEN)
1896: RADIOACTIVITY (BECQUEREL)

BUT IONIZATION REMAINED WHEN THE ELECTROSCOPE WAS IN A LEAD OR WATER CAVITY: **HIGHLY PENETRATING RADIATION**

Victor F. Hess: the 1912 flight



COSMIC Rays



40
30
20
10
0
Units of ionization
Increase in cosmic radiation from above
Decrease in ground radiation from below
0
1
2
3
4
5
Height in kilometers

MILLIKAN's THEORY

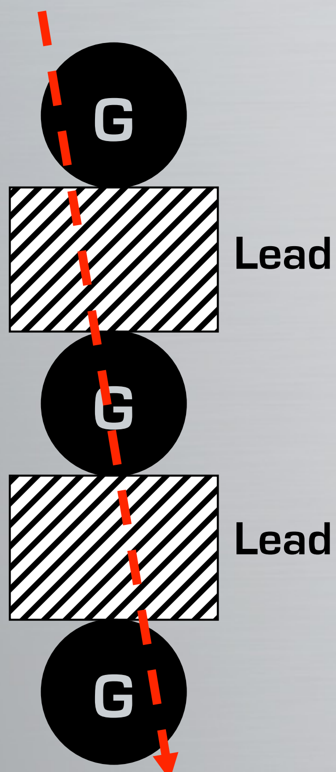
Cosmic Rays (as Millikan called them) are gamma rays as the birth cry of elements heavier than hydrogen

Millikan found that the absorption curve of CR was not compatible with one absorption length, but rather could be fit with a combination of three absorption lengths: 300, 1250 and 2500 g/cm^2, corresponding, according to Compton Theory to gamma ray energies of 26, 110 and 220 MeV

4 p → He	ΔM=27 MeV	**OK**
14 p → N	ΔM=108 MeV	**OK**
12 p → C	**ΔM=85 MeV**	**?**
16 p → O	ΔM=129 MeV	**OK**
28 p → Si	ΔM=239 MeV	**OK**

But gamma rays do not go through lead!

Bruno Rossi had performed several experiments with his coincidence Geiger counters and found that CR could penetrate even 1m of lead



COSMIC RAYS
ARE NO GAMMA RAYS
And
THEIR ENERGY IS > GeV

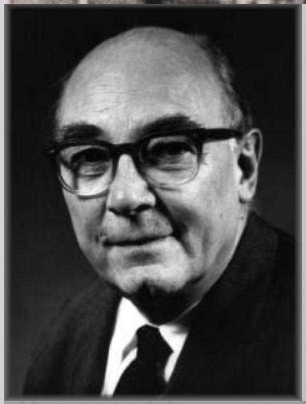
East-West Effect

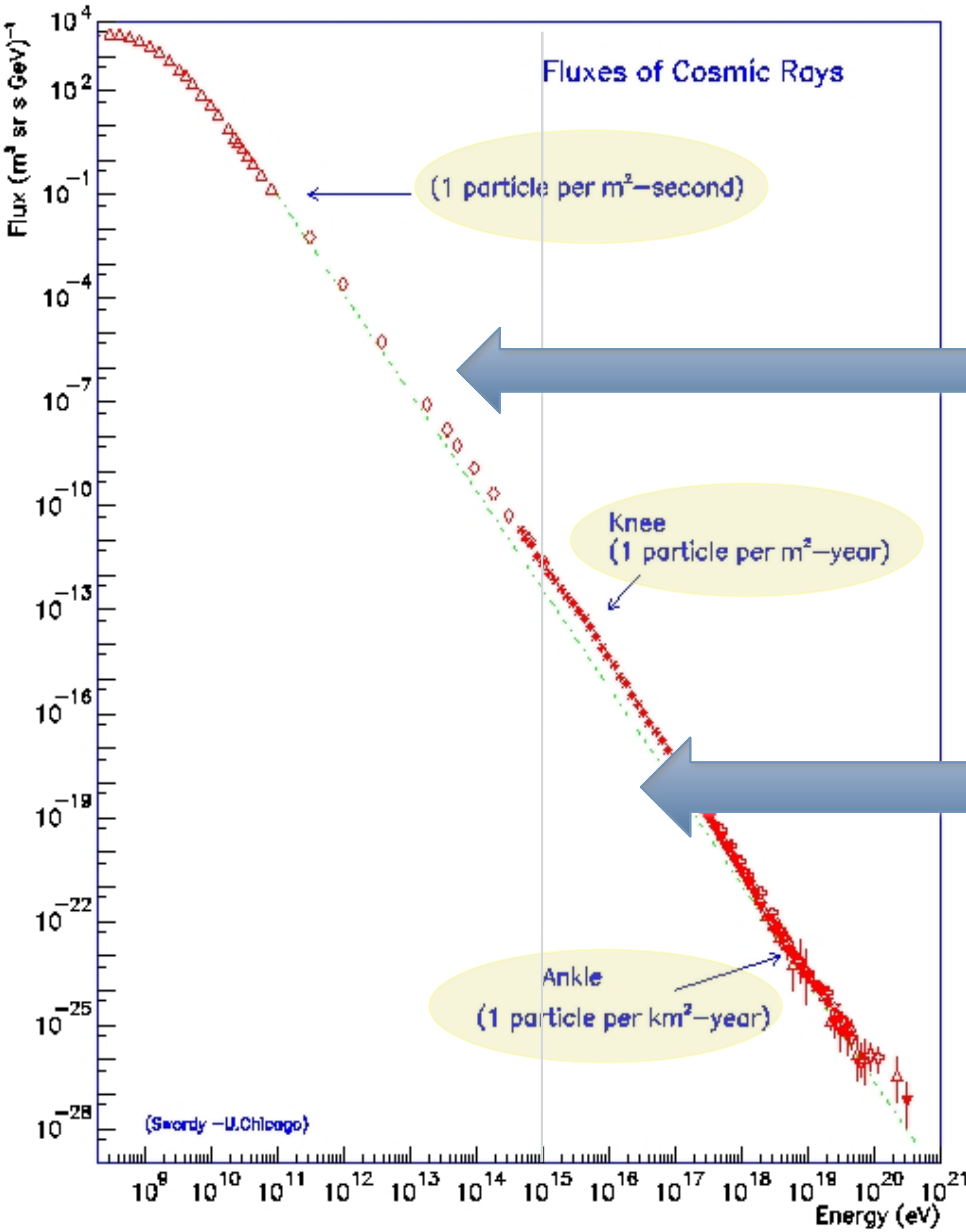
In the 30s Bruno Rossi had predicted that if CR were charged then one should see a latitude dependence of the flux, depending upon the sign of the charge.

THE EFFECT WAS DISCOVERED BY COMPTON IN 1930 AND CONFIRMED BY ROSSI IN 1931.

MOST COSMIC RAYS WERE POSITIVELY CHARGED PARTICLES

- **1930:** B. Rossi in Arcetri predicts the East-West effect
- **1932:** Carl Anderson discovers the positron in CR
- **1934:** Bruno Rossi detects coincidences even at large distance from the center...first evidence of extensive showers!
- **1937:** Seth Neddermeyer and Carl Anderson discover the muon
- **1938-39:** Auger detects first extensive air showers with energy up to $10^{13\text{-}14}$ eV
- **1940's:** Boom of particle physics discoveries in CR
- **1962:** UHECRs by Linsley & Scarsi
- **1966:** Penzias and Wilson discover the CMB

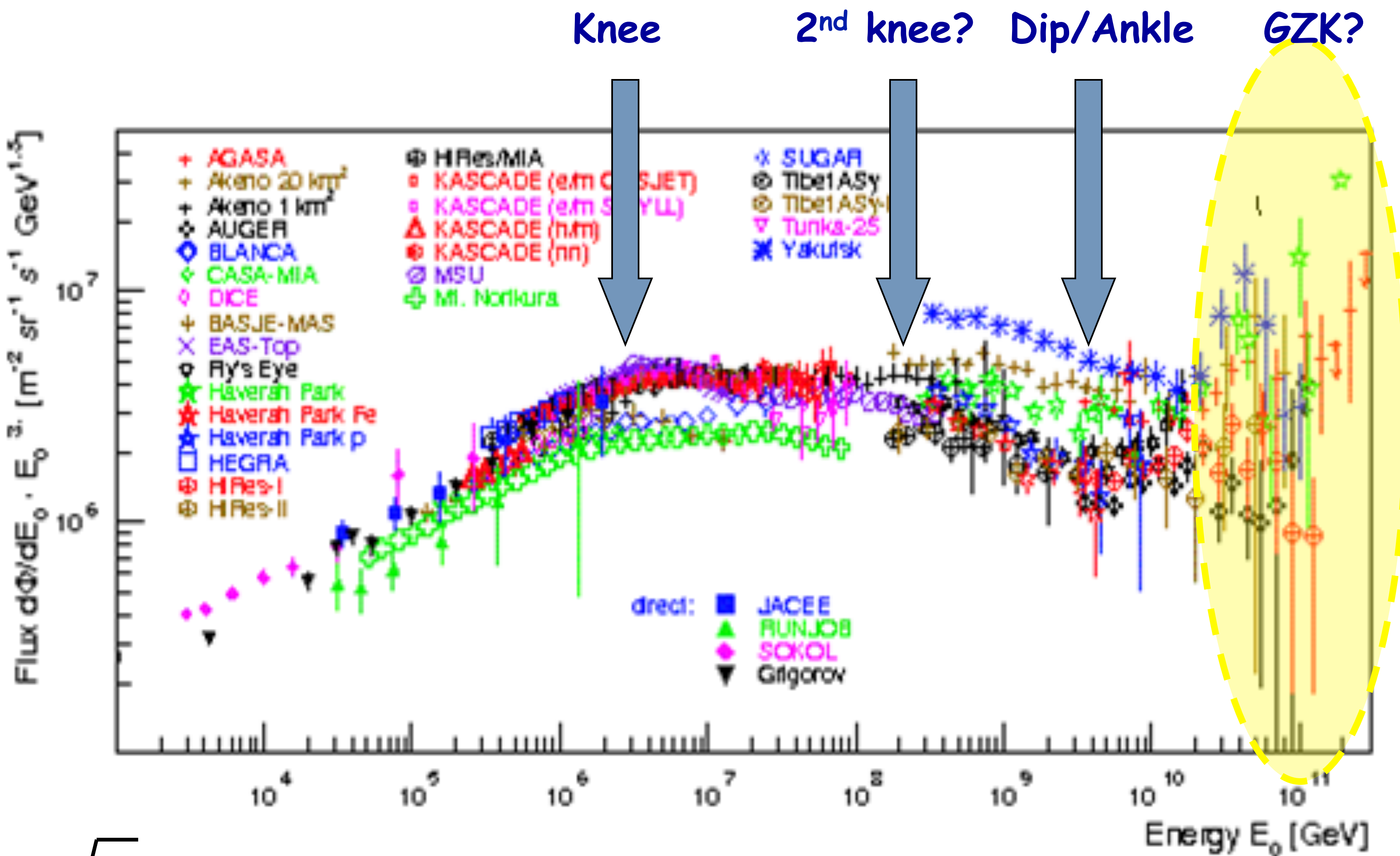




Direct Measurements

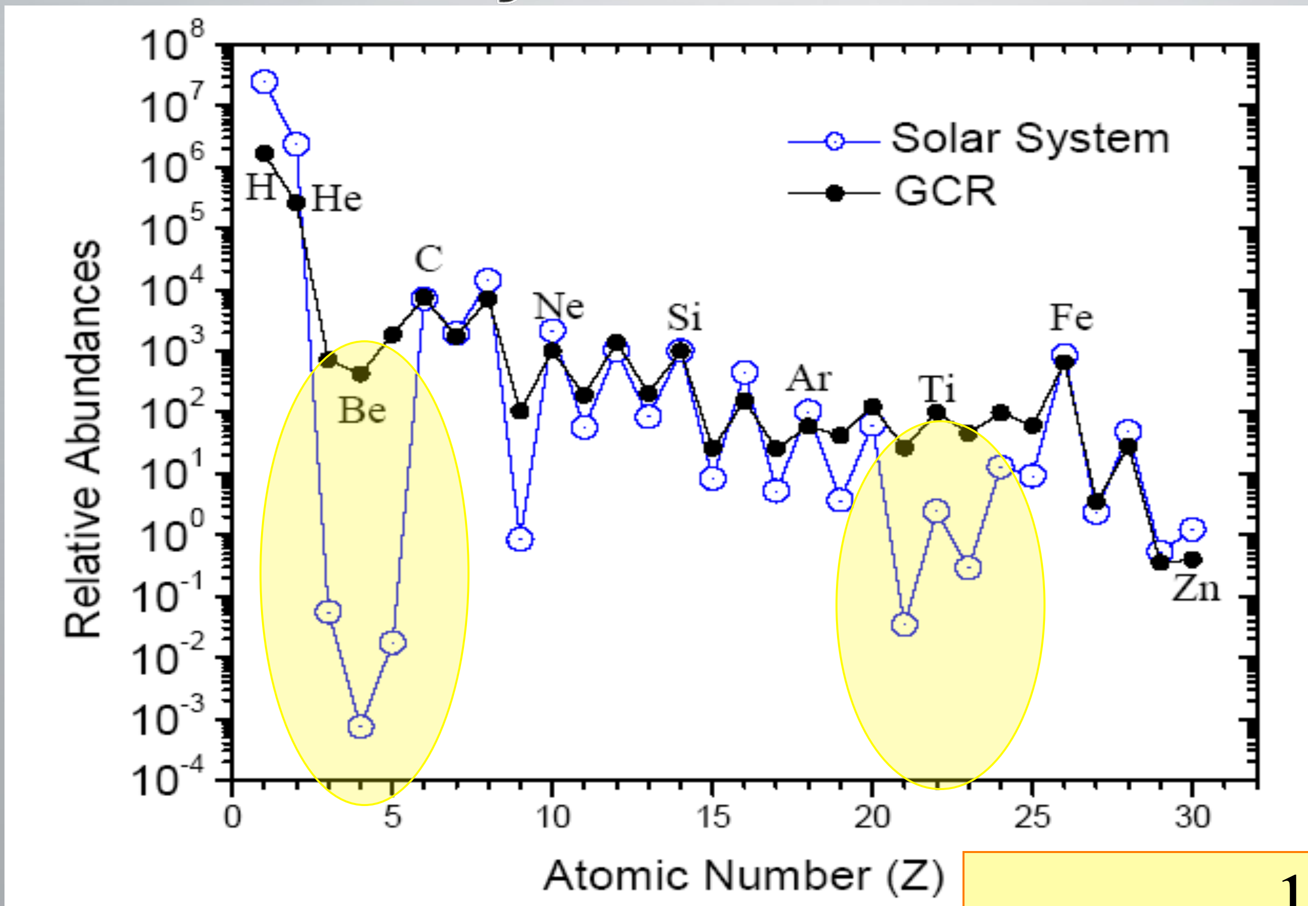
Indirect Measurements

The Spectrum of Cosmic Rays



$\sqrt{s}$ 140 GeV 2.5 TeV 20 TeV 100 TeV 450 TeV

The Chemical Composition of Cosmic Rays



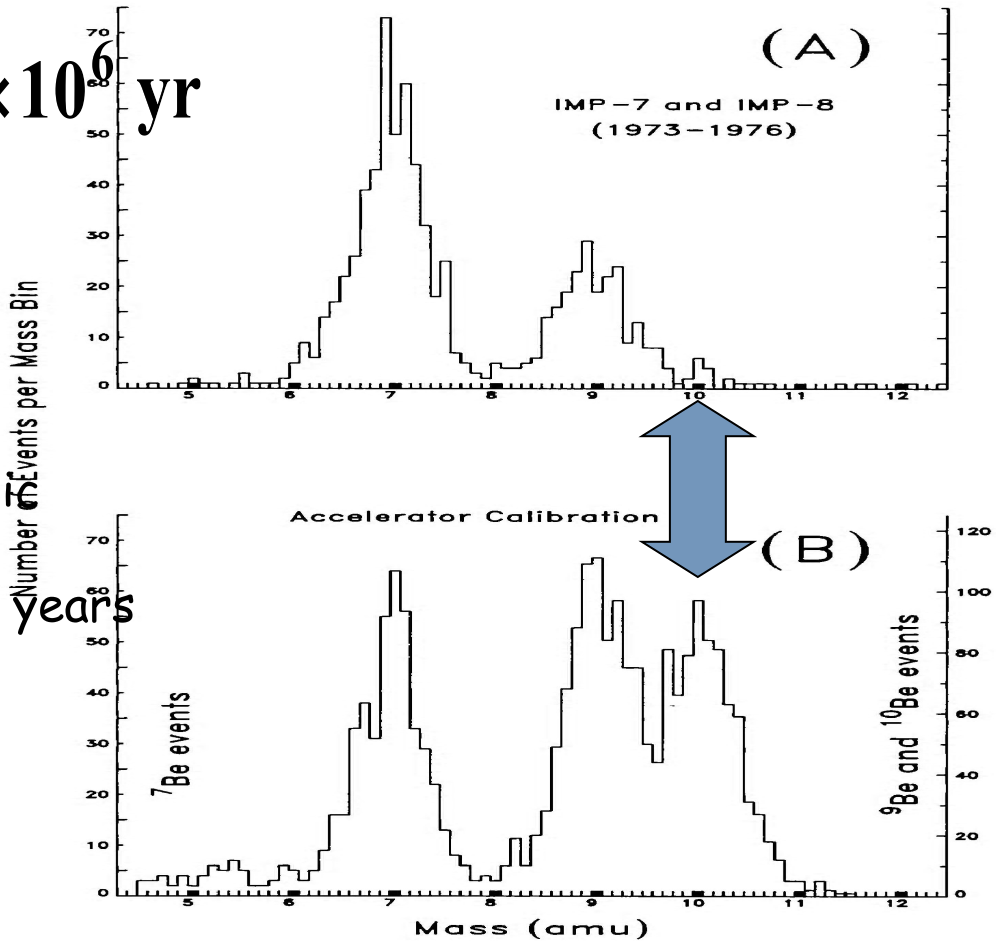
$$\tau_{\rm int} \approx \frac{1}{n_{\rm gas} c\, \sigma_{\rm spall}} \approx \text{few Myr}$$

Unstable Elements

Simpson and Garcia-Munoz 1988

$$\tau_{^{10}Be} = 1.5 \times 10^{6}\ yr$$

Age of Cosmic Rays about 10-15 million years



Balloon flights Cosmic Rays

Laboratory Experiment

PROPAGATION OF COSMIC RAYS

$$\tau_{DISC} = \frac{300\text{ pc}}{(1/3)c} \approx 3000\text{ years}$$

PROPAGATION TIME IN THE DISC

$$\tau_{GAL} = \frac{15\text{ kpc}}{(1/3)c} \approx 150{,}000\text{ years}$$

PROPAGATION TIME ALONG THE ARMS OF THE GALAXY

$$\tau_{HALO} = \frac{3\text{ kpc}}{(1/3)c} \approx 30{,}000\text{ years}$$

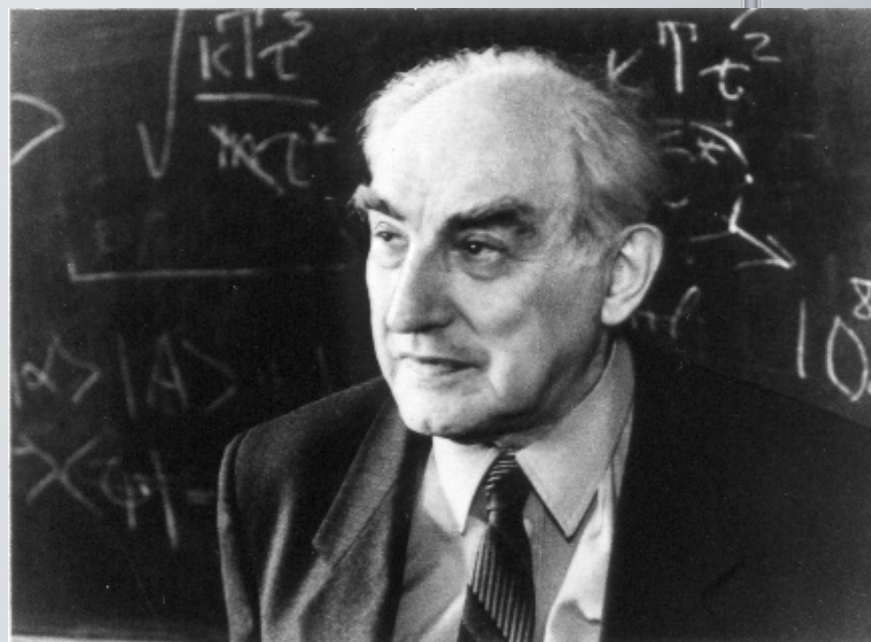
PROPAGATION TIME IN THE HALO

ALL THESE TIME SCALES ARE EXCEEDINGLY SHORT TO BE MADE COMPATIBLE WITH THE ABUNDANCE OF LIGHT ELEMENTS

$\Rightarrow$ DIFFUSIVE PROPAGATION

WHERE DO MOST CRs COME FROM?

Zwicky & Baade were the first to postulate that SNR could be plausible sources of CRs (1934)



Vitali Lazarevich Ginzburg made the argument for SNRs as sources of galactic CR in the 60's in a more quantitative form.

BASICS of the SNR PARADIGM

Zwicky (mid 30s) and Ginzburg & Sirovatsky (mid 60s) realized that On energetic grounds SNR are the only viable source of CR in the Galaxy

TOTAL CR LUMINOSITY OF THE GALAXY ~ $3x10^{40}$ erg/s

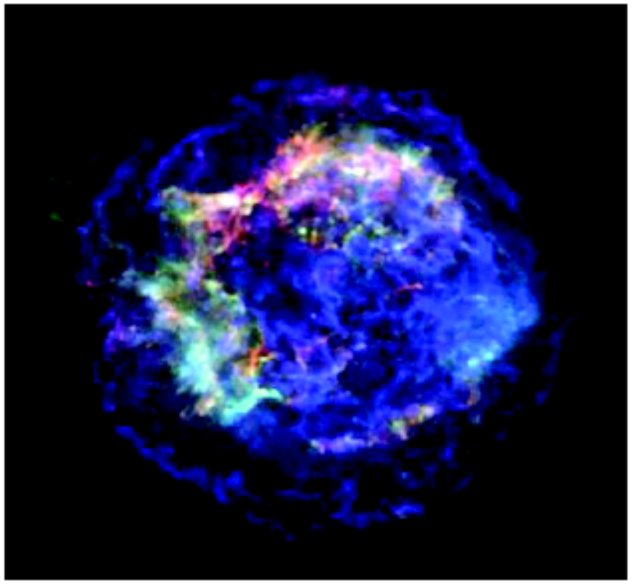
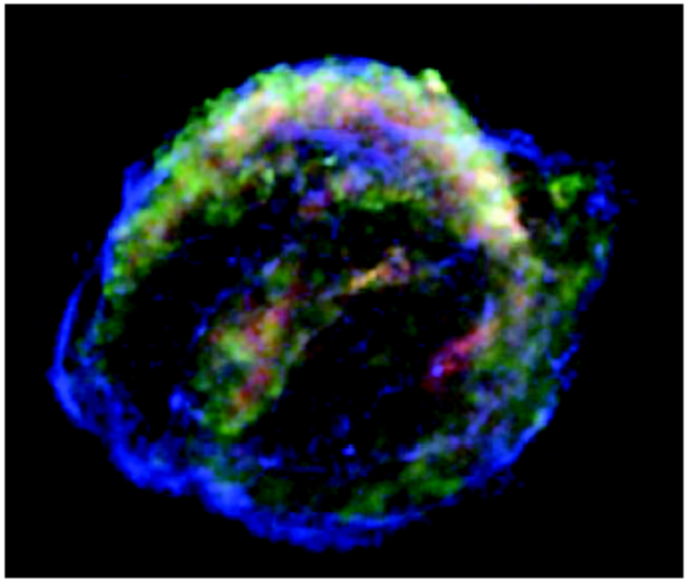
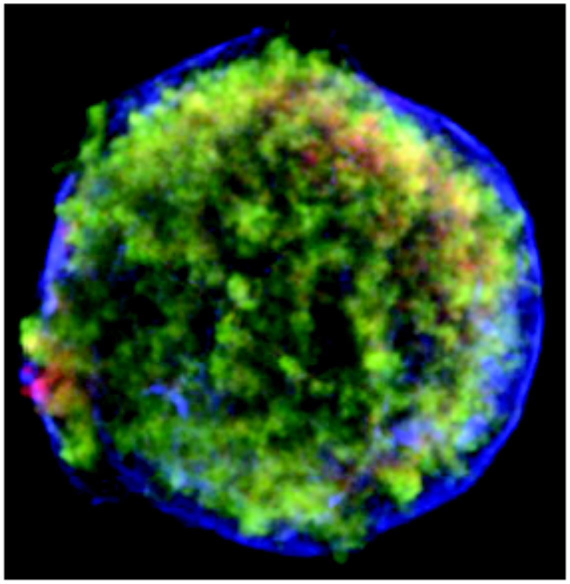
$$L_{SN} = R_{SN}\, E_{kin} \approx 3 \times 10^{41}\ \mathrm{erg\,s^{-1}}$$

IN PRINCIPLE AN EFFICIENCY OF ORDER 10% PER SN IS SUFFICIENT TO ACCOMMODATE THE COSMIC RAY ENERGETICS

THE BIG QUESTION IS HOW DOES NATURE PERFORM THE ENERGY CONVERSION?

ACCELERATION MECHANISM

SUPERNOVA BLAST WAVE

FREE EXPANSION VELOCITY: $V_s = \sqrt{\dfrac{2E_{ej}}{M_{ej}}} = 10^9\ E_{51}^{1/2}\ M_{ej,\Theta}^{-1/2}\ \mathrm{cm/s}$

THE EXPANSION SPEED DROPS DOWN DURING THE SEDOV-TAYLOR PHASE, BUT THE MACH NUMBER IS ~100

A STRONG SHOCK WAVE DEVELOPS

PURPOSE OF THE FIRST THREE LECTURES

1. DESCRIBE THE BASES OF THE THEORY OF TRANSPORT OF CHARGED PARTICLES IN A MAGNETIC FIELD

2. DERIVE THE TRANSPORT EQUATION THAT DESCRIBES THE MOTION OF THESE PARTICLE

3. ACHIEVE A PHYSICAL UNDERSTANDING OF HOW DIFFUSIVE TRANSPORT LEADS TO SECOND ORDER AND FIRST ORDER ENERGY GAIN (ACCELERATION)

4. DISCUSS THE BASIC FACTS OF PARTICLE TRANSPORT IN THE GALAXY

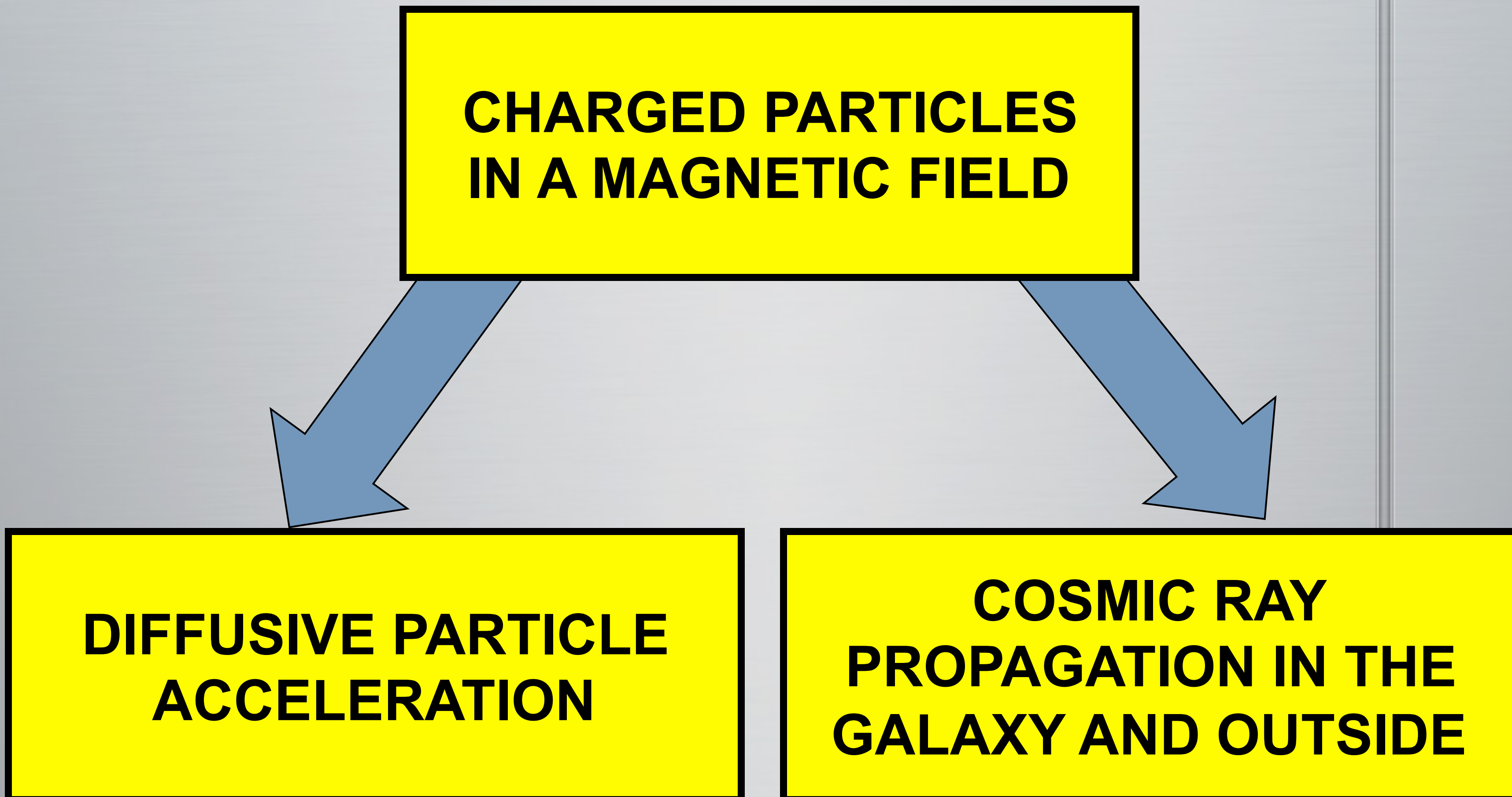
MOST OF THE CONCEPTS DISCUSSED HERE WILL BE APPLICABLE NOT ONLY TO COSMIC RAYS AS OBSERVED AT EARTH, BUT MORE IN GENERAL TO PARTICLES ACCELERATED ANYWHERE AND POSSIBLY RADIATING IN SITU

Purpose of the 3rd and 4th Lectures

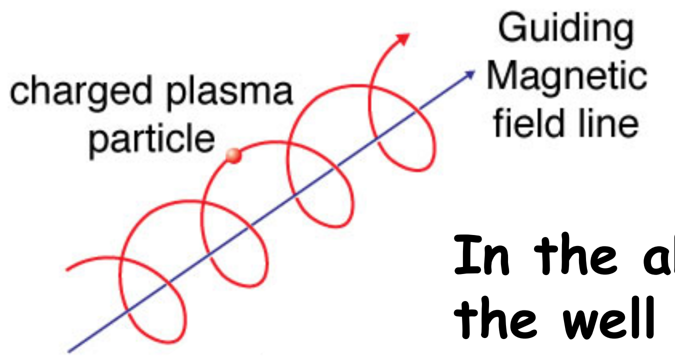
IN THE THIRD LECTURE I WILL COVER A BIT OF THE PHYSICS INVOLVED IN THE PRODUCTION AND PROPAGATION OF ELECTRONS AND POSITRONS IN PULSARS AND PULSAR WIND NEBULAE AND THEIR POSSIBLE CONNECTION WITH THE SO-CALLED POSITRON EXCESS SEEN BY PAMELA AND CONFIRMED BY THE FERMI-LAT

IN THE LAST LECTURE I WILL DISCUSS ACCELERATION PROCESSES IN SITUATIONS OF INTEREST FOR EXTRAGALACTIC SOURCES AND THE PROPAGATION OF COSMIC RAYS ON COSMOLOGICAL DISTANCES.

COSMIC RAY TRANSPORT



Charged Particles in a regular B-field



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

LARMOR FREQUENCY

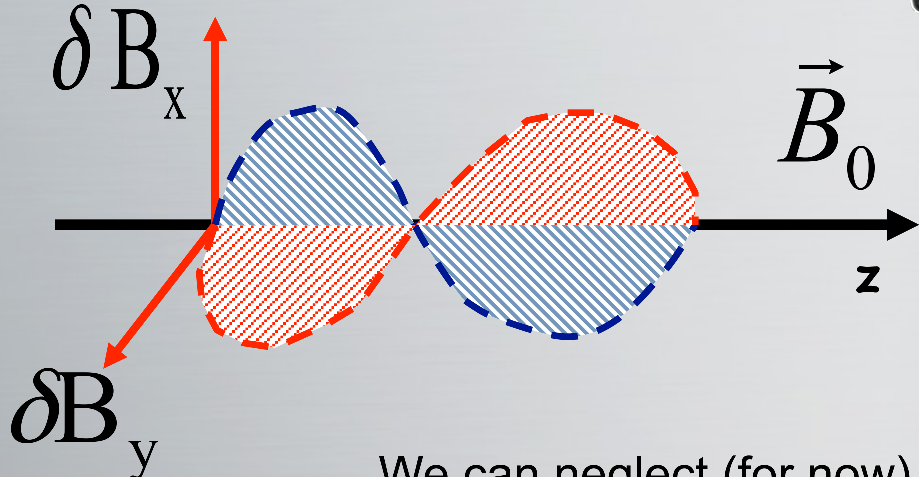
$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

A FEW NOTES...

- THE MAGNETIC FIELD DOES NOT CHANGE PARTICLE ENERGY -> NO ACCELERATION BY B FIELDS

- A RELATIVISTIC PARTICLE MOVES IN THE Z DIRECTION ON AVERAGE AT C/3

Motion of a charged particle in a random magnetic field



Let us consider an Alfven wave propagating in the z direction:

$$\delta B << B_0 \qquad \delta\vec{B} \perp \vec{B}_0$$

We can neglect (for now) the electric field associated with the wave, or in other words we can sit in the reference frame of the wave:

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_Z = P\mu$

Remember that the wave typically moves with the Alfven speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \ cm/s$$

Alfven waves have frequencies << ion gyration frequency $\Omega_p = qB/m_p c$

It is therefore clear that for a relativistic particle these waves, in first approximation, look like static waves.

The equation of motion can be written as:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v} \times (\vec{B}_0 + \vec{\delta}B)$$

If to split the momentum in parallel and perpendicular, the perpendicular component cannot change in modulus, while the parallel momentum is described by

$$\frac{dp \parallel}{dt} = \frac{q}{c}|\vec{v}_\perp \times \delta\vec{B}| \qquad p_\parallel = p \ \mu$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos(\Omega t - kx + \psi)$$

Wave form of the magnetic field with a random phase and frequency

$\Omega = qB_0/mc\gamma$ Larmor frequency

In the frame in which the wave is at rest we can write $x = v\mu t$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos\left[(\Omega - kv\mu)t + \psi\right]$$

It is clear that the mean value of the pitch angle variation over a long enough time vanishes

$$\langle \Delta\mu \rangle_t = 0$$

We want to see now what happens to $\langle \Delta\mu\Delta\mu \rangle$

Let us first average upon the random phase of the waves:

$$\langle \Delta\mu(t')\Delta\mu(t")\rangle_\psi = \frac{q^2v^2(1-\mu^2)\delta B^2}{2c^2p^2} cos\left[(\Omega - kv\mu)(t'-t")\right]$$

And integrating over time:

$$\langle \Delta\mu\Delta\mu\rangle_t = \frac{q^2v^2(1-\mu^2)\delta B^2}{2c^2p^2}\int dt' \int dt"\, cos\left[(\Omega - kv\mu)(t'-t")\right]$$

$$= \frac{q^2v(1-\mu^2)\delta B^2}{c^2p^2\mu}\delta(k-\Omega/v\mu)\Delta t$$

RESONANCE

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \rangle = \frac{q^2(1-\mu^2)\pi}{m^2c^2\gamma^2}\frac{1}{v\mu}4\pi\int dk \frac{\delta B(k)^2}{4\pi}\delta(k-\Omega/v\mu)$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)k_{res}F(k_{res})$$

$$k_{res} = \frac{\Omega}{v\mu}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $=G(k_{res})$

THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

PATHLENGTH FOR DIFFUSION ~ VT

$$\tau \approx \frac{1}{\Omega\, G(k_{res})} \Rightarrow \left\langle \frac{\Delta z \Delta z}{\Delta t} \right\rangle \approx v^2\, \tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.

PARTICLE SCATTERING

- EACH TIME THAT A RESONANCE OCCURS THE PARTICLE CHANGES PITCH ANGLE BY $\Delta\theta \sim \delta B/B$ WITH A RANDOM SIGN
- THE RESONANCE OCCURS ONLY FOR RIGHT HAND POLARIZED WAVES IF THE PARTICLES MOVES TO THE RIGHT (AND VICEVERSA)
- THE RESONANCE CONDITION TELLS US THAT 1) IF $k \ll 1/r_L$ PARTICLES SURF ADIABATICALLY AND 2) IF $k \gg 1/r_L$ PARTICLES HARDLY FEEL THE WAVES

What Equations for Diffusion?

BASIC FORMALISM

$$f(\vec{p}, \vec{x}, t)$$

DISTRIBUTION FUNCTION OF PARTICLES WITH MOMENTUM P AT THE POSITION X AT TIME T

$$\Psi(\vec{p}, \Delta\vec{p})$$

PROBABILITY THAT A PARTICLE WITH MOMENTUM P CHANGES ITS MOMENTUM BY DELTA P

$$\int d\Delta\vec{p}\;\Psi(\vec{p}, \Delta\vec{p}) = 1$$

In general we can write:

$$f(\vec{p}, \vec{x} + \vec{v}\Delta t, t + \Delta t) = \int d\Delta\vec{p}\ f(\vec{p} - \Delta\vec{p}, \vec{x}, t)\Psi(\vec{p} - \Delta\vec{p}, \Delta\vec{p})$$

In the limit of small momentum changes we can Taylor – expand:

$$f(\vec{p}, \vec{x} + \vec{v}\Delta t, t + \Delta t) = f(\vec{p}, \vec{x}, t) + \left(\vec{v}\frac{\partial f}{\partial x} + \frac{\partial f}{\partial t}\right)\Delta t$$

$$f(\vec{p} - \Delta\vec{p}, \vec{x}, t) = f(\vec{p}, \vec{x}, t) - \frac{\partial f}{\partial \vec{p}}\Delta\vec{p} + \frac{1}{2}\Delta\vec{p}\Delta\vec{p}\frac{\partial^2 f}{\partial \vec{p}^2}$$

$$\Psi(\vec{p} - \Delta\vec{p}, \Delta\vec{p}) = \Psi(\vec{p}, \Delta\vec{p}) - \frac{\partial \Psi}{\partial \vec{p}}\Delta\vec{p} + \frac{1}{2}\Delta\vec{p}\Delta\vec{p}\frac{\partial^2 \Psi}{\partial \vec{p}^2}$$

Substituting in the first Equation:

$$f + \Delta t \left(\vec{v}\frac{\partial f}{\partial \vec{x}} + \frac{\partial f}{\partial t} \right) = \int d\Delta\vec{p} \left(f(\vec{p},\vec{x},t) - \frac{\partial f}{\partial \vec{p}}\Delta\vec{p} + \frac{1}{2}\Delta\vec{p}\Delta\vec{p}\frac{\partial^2 f}{\partial \vec{p}^2} \right) \left(\Psi(\vec{p},\vec{x},t) - \frac{\partial \Psi}{\partial \vec{p}}\Delta\vec{p} + \frac{1}{2}\Delta\vec{p}\Delta\vec{p}\frac{\partial^2 \Psi}{\partial \vec{p}^2} \right)$$

Recall that $\int d\Delta\vec{p}\ \Psi(\vec{p},\Delta\vec{p}) = 1$

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} = -\frac{\partial}{\partial \vec{p}}\left[f \langle \frac{\Delta\vec{p}}{\Delta t} \rangle \right] + \frac{1}{2}\frac{\partial}{\partial \vec{p}}\left[\frac{\partial}{\partial \vec{p}} \left(\langle \frac{\Delta\vec{p}\Delta\vec{p}}{\Delta t} \rangle f \right) \right]$$

$$\langle \frac{\Delta\vec{p}}{\Delta t} \rangle = \frac{1}{\Delta t} \int d\Delta\vec{p}\ \Delta\vec{p}\ \Psi(\vec{p},\Delta\vec{p})$$

$$\langle \frac{\Delta\vec{p}\Delta\vec{p}}{\Delta t} \rangle = \frac{1}{\Delta t} \int d\Delta\vec{p}\ \Delta\vec{p}\ \Delta\vec{p}\ \Psi(\vec{p},\Delta\vec{p})$$

We can now use a sort of Principle of Detailed Balance:

$$\Psi(\vec{p}, -\Delta\vec{p}) = \Psi(\vec{p} - \Delta\vec{p}, \Delta\vec{p})$$

and expanding the RHS:

$$\Psi(\vec{p}, -\Delta\vec{p}) = \Psi(\vec{p}, \Delta\vec{p}) - \Delta\vec{p}\frac{\partial\Psi}{\partial\vec{p}} + \frac{1}{2}\Delta\vec{p}\Delta\vec{p}\frac{\partial^2\Psi}{\partial\vec{p}^2}$$

And integrating in Delta p:

$$1 = 1 - \frac{\partial}{\partial\vec{p}}\langle\frac{\Delta\vec{p}}{\Delta t}\rangle + \frac{1}{2}\frac{\partial}{\partial\vec{p}}\frac{\partial}{\partial\vec{p}}\langle\frac{\Delta\vec{p}\Delta\vec{p}}{\Delta t}\rangle$$

$$\Downarrow$$

$$\langle\frac{\Delta\vec{p}}{\Delta t}\rangle - \frac{1}{2}\frac{\partial}{\partial\vec{p}}\langle\frac{\Delta\vec{p}\Delta\vec{p}}{\Delta t}\rangle = \text{Constant}$$

We shall see later that the terms in this Eq. vanish for p→0, therefore the Constant must be zero and we have:

$$\langle \frac{\Delta \vec{p}}{\Delta t} \rangle = \frac{1}{2} \frac{\partial}{\partial \vec{p}} \langle \frac{\Delta \vec{p} \Delta \vec{p}}{\Delta t} \rangle$$

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{x}} = - \frac{\partial}{\partial \vec{p}} \left[f \langle \frac{\Delta \vec{p}}{\Delta t} \rangle \right] + \frac{1}{2} \frac{\partial}{\partial \vec{p}} \left[\frac{\partial}{\partial \vec{p}} \left(\langle \frac{\Delta \vec{p} \Delta \vec{p}}{\Delta t} \rangle f \right) \right]$$

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{x}} = \frac{\partial}{\partial \vec{p}} \left[D_{pp} \frac{\partial f}{\partial \vec{p}} \right] \qquad D_{pp} = \frac{1}{2} \langle \frac{\Delta \vec{p} \Delta \vec{p}}{\Delta t} \rangle$$

BOLTZMANN EQUATION

COLLISION TERM

IN ONE SPATIAL DIMENSION ONE EASILY OBTAINS:

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

WHERE

$$D_{\mu\mu} = \frac{1}{2}\langle\frac{\Delta\mu\Delta\mu}{\delta t}\rangle$$

IS THE PITCH ANGLE DIFFUSION COEFFICIENT.

THE PREVIOUS EQUATION CAN BE VIEWED AS THE BOLTZMANN EQUATION WITH A SCATTERING TERM DEFINED BY DIFFUSION.

From pitch to Spatial Diffusion

IT IS INTUITIVELY CLEAR HOW A PARTICLE THAT IS DIFFUSING IN ITS PITCH ANGLE MUST BE ALSO DIFFUSING IN SPACE. LET US SEE HOW THE TWO ARE RELATED TO EACH OTHER BY INTEGRATING THE BOLTZMANN EQUATION IN PITCH ANGLE:

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

$$f_0(p,t,z) = \frac{1}{2}\int_{-1}^{1} d\mu f(p,t,\mu,z)$$

ISOTROPIC PART OF THE PARTICLE DISTRIBUTION FUNCTION. FOR MOST PROBLEMS THIS IS ALSO VERY CLOSE TO THE ACTUAL DISTRIBUTION FUNCTION

$$\frac{\partial f_0}{\partial t} + \frac{1}{2}v\int_{-1}^{1} d\mu\mu\frac{\partial f}{\partial z} \equiv 0$$

ONE CAN SEE THAT THE QUANTITY

$$J = \frac{1}{2} v \int_{-1}^{1} d\mu \mu f$$

BEHAVES AS A PARTICLE CURRENT, AND THE BOLTMANN EQUATION BECOMES:

$$\frac{\partial f_0}{\partial t} = -\frac{\partial J}{\partial z}$$

NOTICE THAT YOU CAN ALWAYS WRITE:

$$\mu = -\frac{1}{2} \frac{\partial}{\partial \mu} \left(1 - \mu^2\right)$$

WITH THIS TRICK:

$$J = \frac{1}{2} v \int_{-1}^{1} d\mu \mu f = \frac{v}{4} \int_{-1}^{1} d\mu (1-\mu^2) \frac{\partial f}{\partial \mu}$$

RECONSIDER THE INITIAL EQUATION

$$\boxed{\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[D_{\mu\mu} \frac{\partial f}{\partial \mu}\right]}$$

AND INTEGRATE IT AGAIN FROM -1 TO m:

$$\frac{\partial}{\partial t} \int_{-1}^{\mu} d\mu' f + \int_{-1}^{\mu} d\mu' v \mu' \frac{\partial f}{\partial z} = D_{\mu\mu} \frac{\partial f}{\partial \mu}$$

AND MULTIPLYING BY $(1-\mu^2)/D_{\mu\mu}$

$$(1-\mu^2)\frac{\partial f}{\partial \mu} = \frac{1-\mu^2}{D_{\mu\mu}}\frac{\partial}{\partial t}\int_{-1}^{\mu} d\mu' f + \frac{1-\mu^2}{D_{\mu\mu}}\int_{-1}^{\mu} d\mu' v\mu' \frac{\partial f}{\partial z}$$

NOW RECALL THAT THE DISTRIBUTION FUNCTION TENDS TO ISOTROPY, SO THAT AT THE LOWEST ORDER IN THE ANISOTROPY ONE HAS:

$$(1-\mu^2)\frac{\partial f}{\partial \mu} = \frac{1-\mu^2}{D_{\mu\mu}}\frac{\partial f_0}{\partial t}(1+\mu) + \frac{1-\mu^2}{D_{\mu\mu}}\frac{1}{2}v(\mu^2-1)\frac{\partial f_0}{\partial z}$$

AND RECALLING THE DEFINITION OF CURRENT:

$$J = \frac{v}{4}\frac{\partial f_0}{\partial t}\int_{-1}^{1} d\mu \frac{1-\mu^2}{D_{\mu\mu}}(1+\mu) - \frac{v^2}{8}\frac{\partial f_0}{\partial z}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}} = \kappa_t \frac{\partial f_0}{\partial t} - \kappa_z \frac{\partial f_0}{\partial z}$$

USING THE TRANSPORT EQ IN TERMS OF CURRENT:

$$J = -\kappa_t \frac{\partial J}{\partial z} - \kappa_z \frac{\partial f_0}{\partial z}$$

NOW WE RECALL THE TRANSPORT EQUATION IN CONSERVATIVE FORM:

$$\frac{\partial f_0}{\partial t} = -\frac{\partial J}{\partial z}$$

AND PUTTING THINGS TOGETHER:

$$\frac{\partial f_0}{\partial t} = -\frac{\partial}{\partial z}\left[-\kappa_t \frac{\partial J}{\partial z} - \kappa_z \frac{\partial f_0}{\partial z}\right]$$

BUT IT IS EASY TO SHOW THAT THE FIRST TERM MUST BE NEGLIGIBLE:

$$J = \frac{v}{2}\int_{-1}^{1} d\mu \mu f_0 (1+\delta\mu) = \frac{1}{3} v \delta f_0 \ll v f_0 \qquad \delta \ll 1$$

IT FOLLOWS THAT THE ISOTROPIC PART OF THE DISTRIBUTION FUNCTION MUST SATISFY THE DIFFUSION EQUATION:

$$\boxed{\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial z}\left[\kappa_z \frac{\partial f_0}{\partial z}\right]}$$

DIFFUSION EQUATION

$$\kappa_z = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}} = \frac{1}{3} v \lambda_{\parallel}$$

SPATIAL DIFFUSION COEFFICIENT

WRAPPING UP

I OUTLINED THE VERY BASICS OF THE THEORY OF DIFFUSIVE TRANSPORT OF CHARGED PARTICLES IN A MAGNETIC FIELD

THESE TOOLS WILL BE APPLIED IN THE NEXT TWO LECTURES TO THE DESCRIPTION OF

1. PARTICLE ACCELERATION

2. TRANSPORT OF PARTICLES IN THE GALAXY

THE BASIC CONCEPT IS THAT A CHARGED PARTICLE IN A MAGNETIC FIELD WITH A SMALL TURBULENT COMPONENT PERFORMS A RANDOM DIFFUSIVE MOTION PARALLEL TO THE ORDERED MAGNETIC FIELD

THERE IS A SMALL DIFFUSIVE MOTION PERPENDICULAR TO THE ORDERED FIELD. THE TWO BECOME COMPARABLE WHEN THE TURBULENT COMPONENT IS LARGE