Lecture 2:
PARTICLE ACCELERATION

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-PARTICLE ACCELERATION IN RANDOM SCATTERING WITH MOVING WAVES
-FIRST ORDER DIFFUSIVE SHOCK ACCELERATION / TEST PARTICLE THEORY
-FIRST ORDER DIFFUSIVE SHOCK ACCELERATION / NON-LINEAR THEORY
-APPLICATION TO MULTI-NU OBSERVATIONS OF SUPERNOVA REMNANTS
-ADVANCED ASPECTS (VELOCITY OF SCATTERING CENTERS, NEUTRALS)

ACCELERATION OF NONTHERMAL PARTICLES

The presence of non-thermal particles is ubiquitous in the Universe (solar wind, Active galaxies, supernova remnants, gamma ray bursts, Pulsars, micro-quasars)

WHEREVER THERE ARE MAGNETIZED PLASMAS THERE ARE NON-THERMAL PARTICLES

PARTICLE ACCELERATION

BUT THERMAL PARTICLES ARE USUALLY DOMINANT, SO WHAT DETERMINES THE DISCRIMINATION BETWEEN THERMAL AND ACCELERATED PARTICLES?

INJECTION

ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

MAGNETIC FIELD CANNOT MAKE WORK ON CHARGED PARTICLES THEREFORE ELECTRIC FIELDS ARE NEEDED FOR ACCELERATION TO OCCUR

REGULAR ACCELERATION

THE ELECTRIC FIELD IS LARGE SCALE:

$$\langle \vec{E} \rangle \neq 0$$

STOCHASTIC ACCELERATION

THE ELECTRIC FIELD IS SMALL SCALE:

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

REGULAR ACCELERATION

$$\langle \vec{E} \rangle \neq 0$$

Very special conditions are necessary in Astrophysical environments in order to achieve this condition, because of the high electrical conductivity of astrophysical plasmas.
Few exceptions:

UNIPOLAR INDUCTOR: this occurs in the case of rotating magnetic fields, such as in pulsars, rotating black holes. An electric potential is established between the surface of the rotating object (neutrons star, BH) and infinity. The potential difference is usable only in places (gaps) where the condition $\vec{E} \cdot \vec{B} = 0$ is violated. MHD is broken in the gaps.

RECONNECTION: Locally, regions with opposite orientation of magnetic field merge, giving rise to a net local electric field E~LB, where L is the size of the reconnection region. It occurs in the sun and solar wind, but probably also in the magnetosphere of rotating neutron stars and BHs.

STOCHASTIC ACCELERATION

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

Most acceleration mechanisms that are operational in astrophysical environments are of this type. We have seen that the action of random magnetic fluctuations is that of scattering particles when the resonance is achieved. In other words, the particle distribution is isotropized in the reference frame of the wave.

Although in the reference frame of the waves the momentum is conserved (B does not make work) in the lab frame the particle momentum changes by

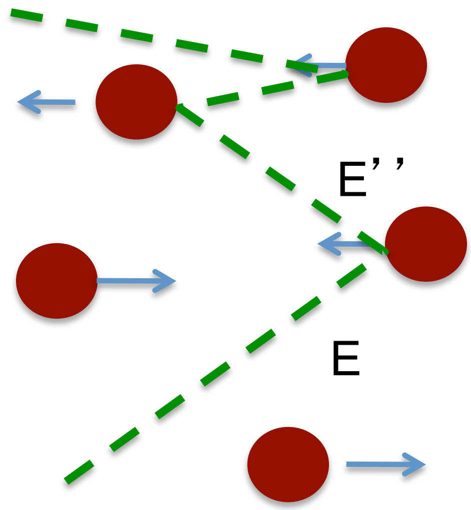
$$\Delta p \sim p \frac{v_A}{c}$$

In a time T which is the diffusion time as found in the last lecture. It follows that

$$D_{pp} = \langle \frac{\Delta p \Delta p}{\Delta t} \rangle \sim p^2 \frac{1}{T} \left(\frac{v_A}{c} \right)^2 \rightarrow \tau_{pp} \sim \frac{p^2}{D_{pp}} \sim T \left(\frac{v_A}{c} \right)^{-2} \gg T$$

THE MOMENTUM CHANGE IS A SECOND ORDER PHENOMENON !!!

2^{nd} Order Fermi Acceleration



We inject a particle with energy E. In the reference frame of a cloud moving with speed b **the particle energy is:**

$$E' = \gamma E + \beta\gamma p\mu$$

and the momentum along x is:

$$p'_x = \beta\gamma E + \gamma p\mu$$

Assuming that the cloud is very massive compared with the particle, we can assume that the cloud is unaffected by the scattering, therefore the particle energy in the cloud frame does not change and the momentum along x is simply inverted, so that after ‘scattering’ $p'_x \rightarrow -p'_x$. The final energy in the Lab frame is therefore:

$$E'' = \gamma E' + \beta\gamma p'_x =$$

$$\gamma^2 E\left(1 + \beta^2 + 2\beta\mu\frac{p}{E}\right)$$

$$\frac{p}{E} = \frac{mv\gamma}{m\gamma} = v$$

Where v is now the dimensionless Particle velocity

It follows that:

$$E" = \gamma^2 E \left(1 + \beta^2 + 2\beta\mu v\right)$$

and:

$$\frac{E" - E}{E} = \gamma^2 \left(1 + 2\beta v\mu + \beta^2\right) - 1$$

and finally, taking the limit of non-relativistic clouds g→1:

$$\frac{E" - E}{E} \approx 2\beta^2 + 2\beta v\mu$$

We can see that the fractional energy change can be both positive or negative, which means that particles can either gain or lose energy, depending on whether the particle-cloud scattering is head-on or tail-on.

We need to calculate the probability that a scattering occurs head-on or Tail-on. The scattering probability along direction m **is proportional to the Relative velocity in that direction:**

$$P(\mu) = Av_{rel} = A\frac{\beta\mu + v}{1 + v\beta\mu} \rightarrow_{v\rightarrow 1} \approx A(1+\beta\mu)$$

The condition of normalization to unity:

$$\int_{-1}^{1} P(\mu) d\mu = 1$$

leads to A=1/2. It follows that the mean fractional energy change is:

$$\langle \frac{\Delta E}{E} \rangle = \int_{-1}^{1} d\mu P(\mu) \left(2\beta^2 + 2\beta\mu\right) = \frac{8}{3}\beta^2$$

NOTE THAT IF WE DID NOT ASSUME RIGID REFLECTION AT EACH CLOUD BUT RATHER ISOTROPIZATION OF THE PITCH ANGLE IN EACH CLOUD, THEN WE WOULD HAVE OBTAINED (4/3) b^2 **INSTEAD OF (8/3)** b^2

THE FRACTIONAL CHANGE IS A SECOND ORDER QUANTITY IN b<<1. This is the reason for the name *SECOND ORDER FERMI ACCELERATION*

The acceleration process can in fact be shown to become more Important in the relativistic regime where b→1

THE PHYSICAL ESSENCE CONTAINED IN THIS SECOND ORDER DEPENDENCE IS THAT IN EACH PARTICLE-CLOUD SCATTERING THE ENERGY OF THE PARTICLE CAN EITHER INCREASE OR DECREASE → WE ARE LOOKING AT A PROCESS OF DIFFUSION IN MOMENTUM SPACE

THE REASON WHY ON AVERAGE THE MEAN ENERGY INCREASES IS THAT HEAD-ON COLLISIONS ARE MORE PROBABLE THAN TAIL-ON COLLISIONS

WHAT IS DOING THE WORK?

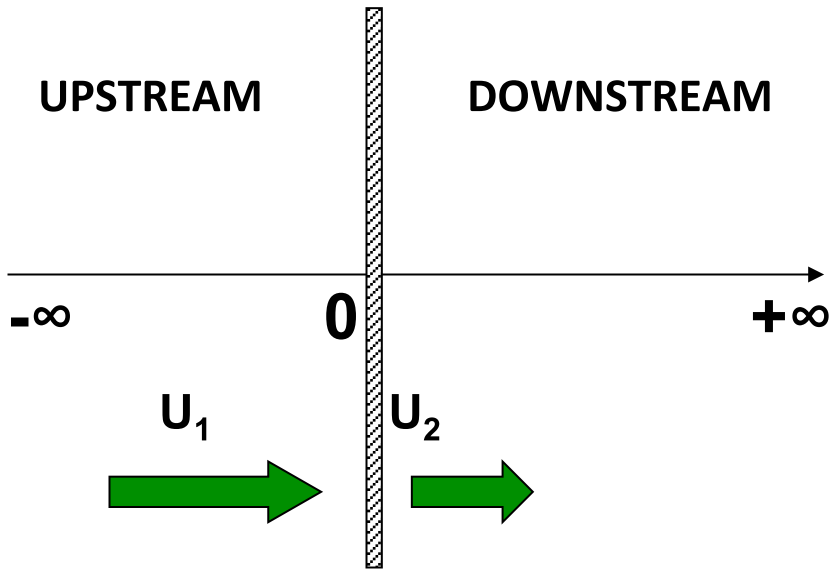
We just found that particles propagating in a magnetic field can change their momentum (in modulus and direction)… BUT WHAT IS THE SOURCE OF THE ELECTRIC FIELDS??? ***Moving Magnetic Fields***

The induced electric field is responsible for this first instance of particle acceleration

The scattering leads to momentum transfer, but to WHAT?

Recall that particles isotropize in the reference frame of the waves…

Shock Solutions



Let us sit in the reference frame in which the shock is at rest and look for stationary solutions

$$\frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial x}\left(\rho u^2 + P\right) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP\right) = 0$$

It is easy to show that aside from the trivial solution in which all quantities remain spatially constant, there is a discontinuous solution:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

M_1 is the upstream Fluid Mach number

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - \gamma(\gamma-1)][(\gamma-1)M_1^2+2]}{(\gamma+1)^2 M_1^2}$$

Strong Shocks $M_1^2 >> 1$

In the limit of strong shock fronts these expressions get substantially simpler and one has:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2, \quad T_2 = 2\frac{\gamma-1}{(\gamma+1)^2} m u_1^2$$

ONE CAN SEE THAT SHOCKS BEHAVE AS VERY EFFICENT HEATING MACHINES IN THAT A LARGE FRACTION OF THE INCOMING RAM PRESSURE IS CONVERTED TO INTERNAL ENERGY OF THE GAS BEHIND THE SHOCK FRONT…

Collisionless shocks

While shocks in the terrestrial environment are mediated by particle-particle collisions, astrophysical shocks are almost always of a different nature. The pathlength for ionized plasmas is of the order of:

$$\lambda \simeq \frac{1}{n\sigma} = 3.2 Mpc\; n_1^{-1} \left(\frac{\sigma}{10^{-25} cm^2}\right)^{-1}$$

Absurdly large compared with any reasonable length scale. It follows that astrophysical shocks can hardly form because of particle-particle scattering But REQUIRE the mediation of magnetic fields. In the downstream gas the Larmor radius of particles is:

$$r_{L,th} \approx 10^{10} B_\mu T_8^{1/2}\ \mathrm{cm}$$

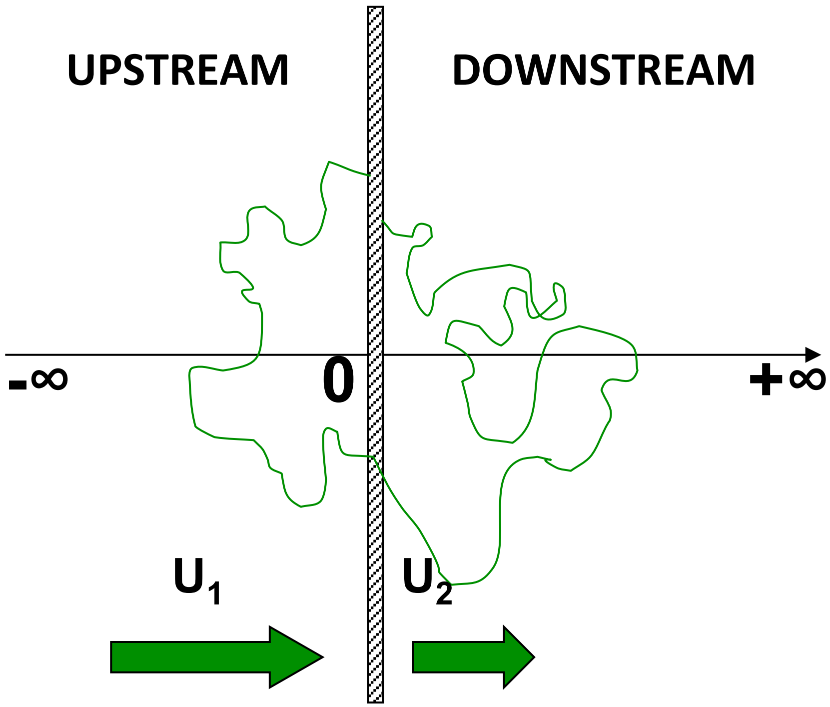
The slowing down of the incoming flow and its isotropization (thermalization) Is due to the action of magnetic fields in the shock region (COLLISIONLESS SHOCKS)

DIFFUSIVE SHOCK ACCELERATION OF CHARGED PARTICLES

or

FIRST ORDER FERMI ACCELERATION

Bouncing between approaching magnetic mirrors



Let us take a relativistic particle with energy E~p upstream of the shock. In the downstream frame:

$$E_d = \gamma E(1+\beta\mu) \quad 0 \le \mu \le 1$$

where b **=** u_1**-**u_2**>0. In the downstream frame the direction of motion of the particle is isotropized and reapproaches the shock with the same energy but pitch angle** m'

$$E_u = \gamma E_d - \beta E_d \gamma \mu' = \gamma^2 E(1+\beta\mu)(1-\beta\mu')$$

$$-1 \le \mu' \le 0$$

In the non-relativistic case the particle distribution is, at zeroth order, isotropic Therefore:

TOTAL FLUX

$$J = \int_0^1 d\Omega \frac{N}{4\pi} v\mu = \frac{Nv}{4} \Rightarrow P(\mu)d\mu = \frac{ANv\mu}{\frac{Nv}{4}} d\mu = 2\mu d\mu$$

The mean value of the energy change is therefore:

$$\langle \frac{E_u - E}{E} \rangle = -\int_0^1 d\mu 2\mu \int_{-1}^0 d\mu' 2\mu' \left[\gamma^2(1+\beta\mu)(1-\beta\mu') - 1\right] \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

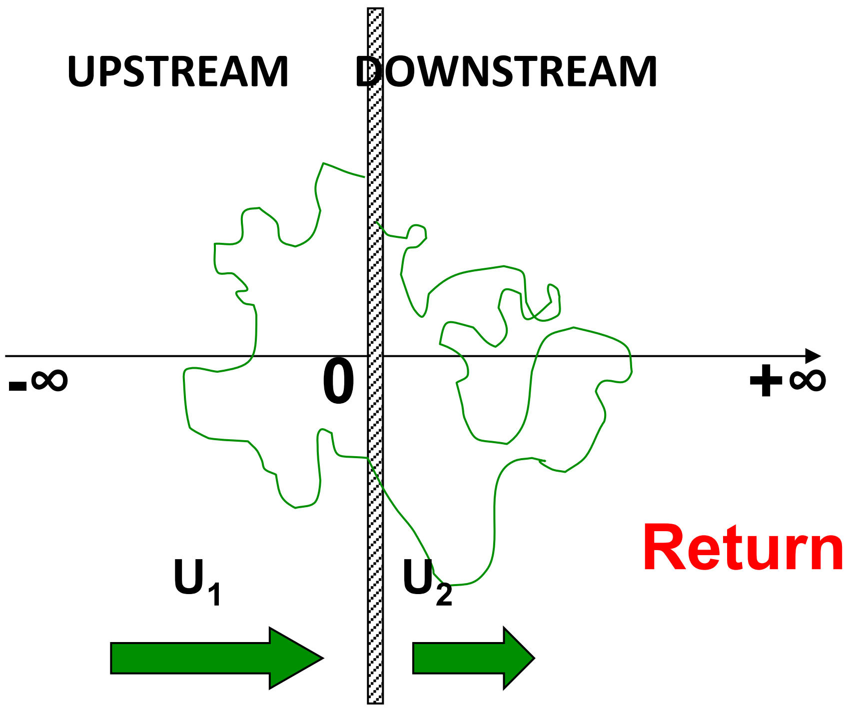
A FEW IMPORTANT POINTS:

I. **There are no configurations that lead to losses**

II. **The mean energy gain is now first order in b**

III. **The energy gain is basically independent of any detail on how particles scatter back and forth!**

RETURN PROBABILITIES AND SPECTRUM OF ACCELERATED PARTICLES



$$\varphi_{in} = \int_{-u_2}^{1} d\mu f_0(u_2 + \mu) = \frac{1}{2}(1 + u_2)^2$$

$$\varphi_{out} = \int_{-1}^{-u_2} d\mu f_0(u_2 + \mu) = \frac{1}{2}(1 - u_2)^2$$

Return Probability from Downstream

$$P_d = \frac{\varphi_{out}}{\varphi_{in}} = \frac{(1 - u_2)^2}{(1 + u_2)^2} \approx 1 - 4u_2$$

HIGH PROBABILITY OF RETURN FROM DOWNSTREAM

BUT TENDS TO ZERO FOR HIGH U_2

ENERGY GAIN: $$E_{k+1} = \left(1 + \frac{4}{3}V\right)E_k$$

$$\mathbf{E_0 \rightarrow E_1 \rightarrow E_2 \rightarrow --- \rightarrow E_K=[1+(4/3)V]^K\ E0}$$

$$\ln\left(\frac{E_K}{E_0}\right) = K \ln\left(1 + \frac{4}{3}\left(U_1 - U_2\right)\right)$$

$$\mathbf{N_0 \rightarrow N_1=N_0\ P_{ret} \rightarrow --- \rightarrow N_K=N_0\ P_{ret}^{K}}$$

$$\ln\left(\frac{N_K}{N_0}\right) = K \ln\left(1 - 4U_2\right)$$

Putting these two expressions together we get:

$$K = \frac{\ln\left[\dfrac{N_K}{N_0}\right]}{\ln\left[1-4U_2\right]} = \frac{\ln\left[\dfrac{E_K}{E_0}\right]}{\ln\left[1+\dfrac{4}{3}(U_1-U_2)\right]}$$

Therefore, after expanding for U<<1:

$$N(>E_K) = N_0\left(\frac{E_K}{E_0}\right)^{-\gamma} \qquad \gamma = \frac{3}{r-1} \qquad r = \frac{U_1}{U_2}$$

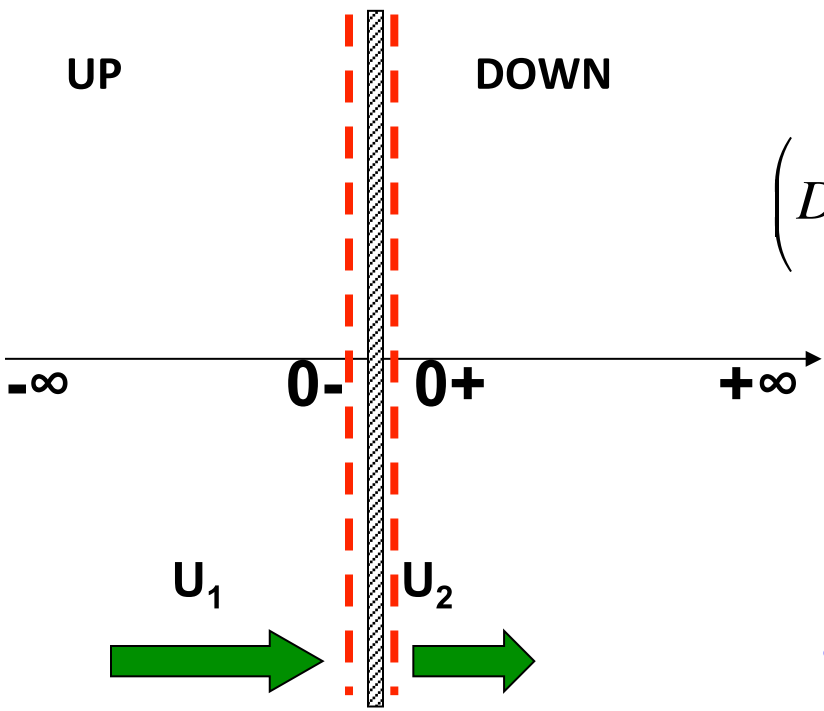
THE SLOPE OF THE DIFFERENTIAL SPECTRUM WILL BE

γ+1=(r+2)/(r-1) → 2 FOR r→4 **(STRONG SHOCK)**

THE TRANSPORT EQUATION APPROACH

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

DIFFUSION ADVECTION COMPRESSION INJECTION



Integrating around the shock:

$$\left(D\frac{\partial f}{\partial x}\right)_2 - \left(D\frac{\partial f}{\partial x}\right)_1 + \frac{1}{3}(u_2-u_1)p\frac{df_0(p)}{dp} + Q_0(p) = 0$$

Integrating from upstr. infinity to 0-:

$$\left(D\frac{\partial f}{\partial x}\right)_1 = u_1 f_0$$

and requiring homogeneity downstream:

$$p\frac{df_0}{dp} = \frac{3}{u_2-u_1}\left(u_1 f_0 - Q_0\right)$$

THE TRANSPORT EQUATION APPROACH

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p) = \frac{3u_1}{u_1 - u_2} \frac{N_{inj}}{4\pi p_{inj}^2} \left(\frac{p}{p_{inj}} \right)^{\frac{-3u_1}{u_1 - u_2}}$$

DEFINE THE COMPRESSION FACTOR $r = u_1/u_2 \rightarrow 4$ (strong shock)

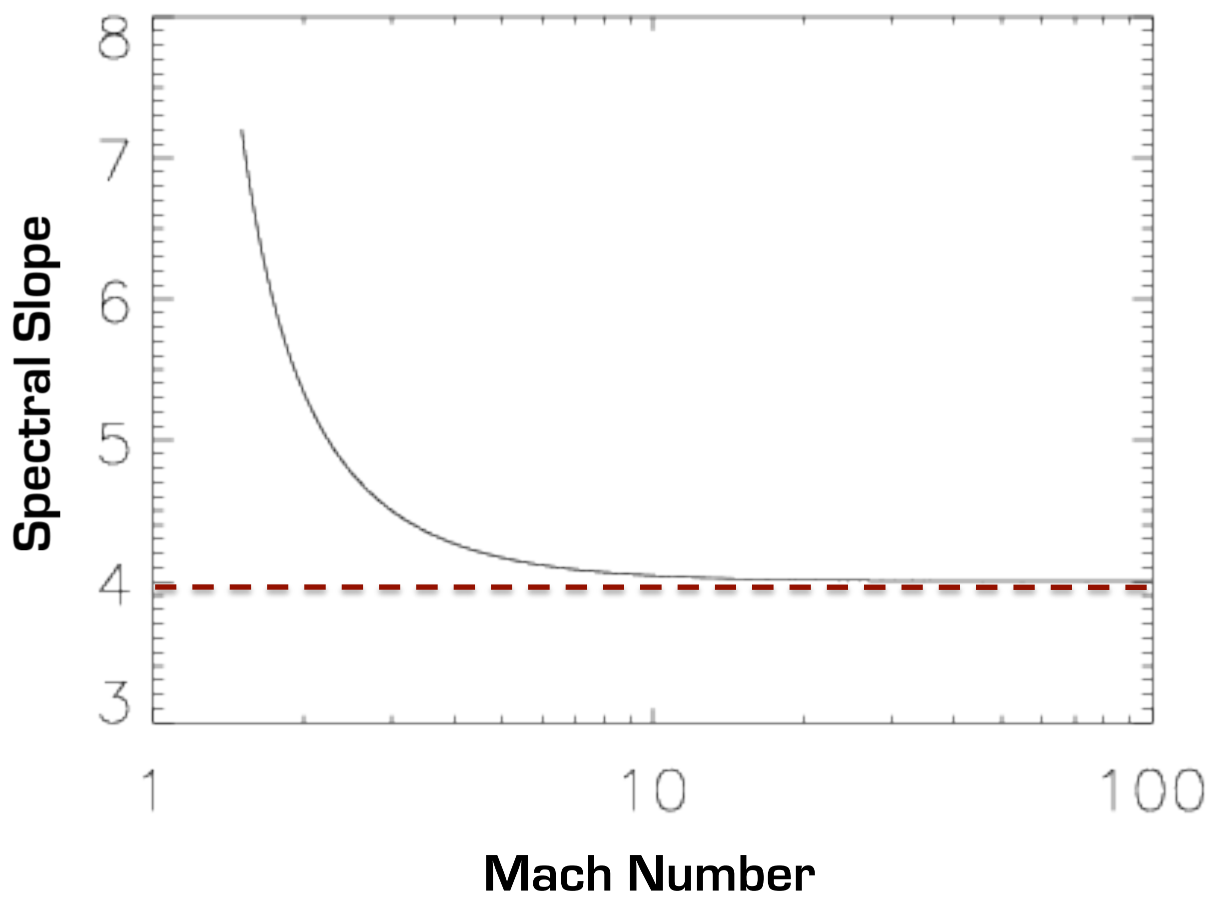
THE SLOPE OF THE SPECTRUM IS

$$\frac{3u_1}{u_1 - u_2} = \frac{3}{1 - 1/r} \rightarrow 4 \quad \text{if r} \rightarrow 4$$

NOTICE THAT: $N(p)dp = 4\pi p^2 f(p) dp \rightarrow N(p) \propto p^{-2}$

1. **THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW EXTENDING TO INFINITE MOMENTA**
2. **THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES**
3. **INJECTION IS TREATED AS A FREE PARAMETER WHICH DETERMINES THE NORMALIZATION**

TEST PARTICLE SPECTRUM



SOME IMPORTANT COMMENTS

- THE STATIONARY PROBLEM DOES NOT ALLOW TO HAVE A MAX MOMENTUM!
- THE NORMALIZATION IS ARBITRARY THEREFORE THERE IS NO CONTROL ON THE AMOUNT OF ENERGY IN CR
- AND YET IT HAS BEEN OBTAINED IN THE TEST PARTICLE APPROXIMATION
- THE SOLUTION DOES NOT DEPEND ON WHAT IS THE MECHANISM THAT CAUSES PARTICLES TO BOUNCE BACK AND FORTH
- FOR STRONG SHOCKS THE SPECTRUM IS UNIVERSAL AND CLOSE TO E^{-2}
- IT HAS BEEN IMPLICITELY ASSUMED THAT WHATEVER SCATTERS THE PARTICLES IS AT REST (OR SLOW) IN THE FLUID FRAME

Maximum Energy

The maximum energy in an accelerator is determined by either the age of the accelerator compared with the acceleration time or the size of the system compared with the diffusion length D(E)/u. The hardest condition is the one that dominates.

Using the diffusion coefficient in the ISM derived from the B/C ratio:

$$D(E) \approx 3 \times 10^{28} E_{GeV}^{1/3} cm^2/s$$

and the velocity of a SNR shock as u=5000 km/s one sees that:

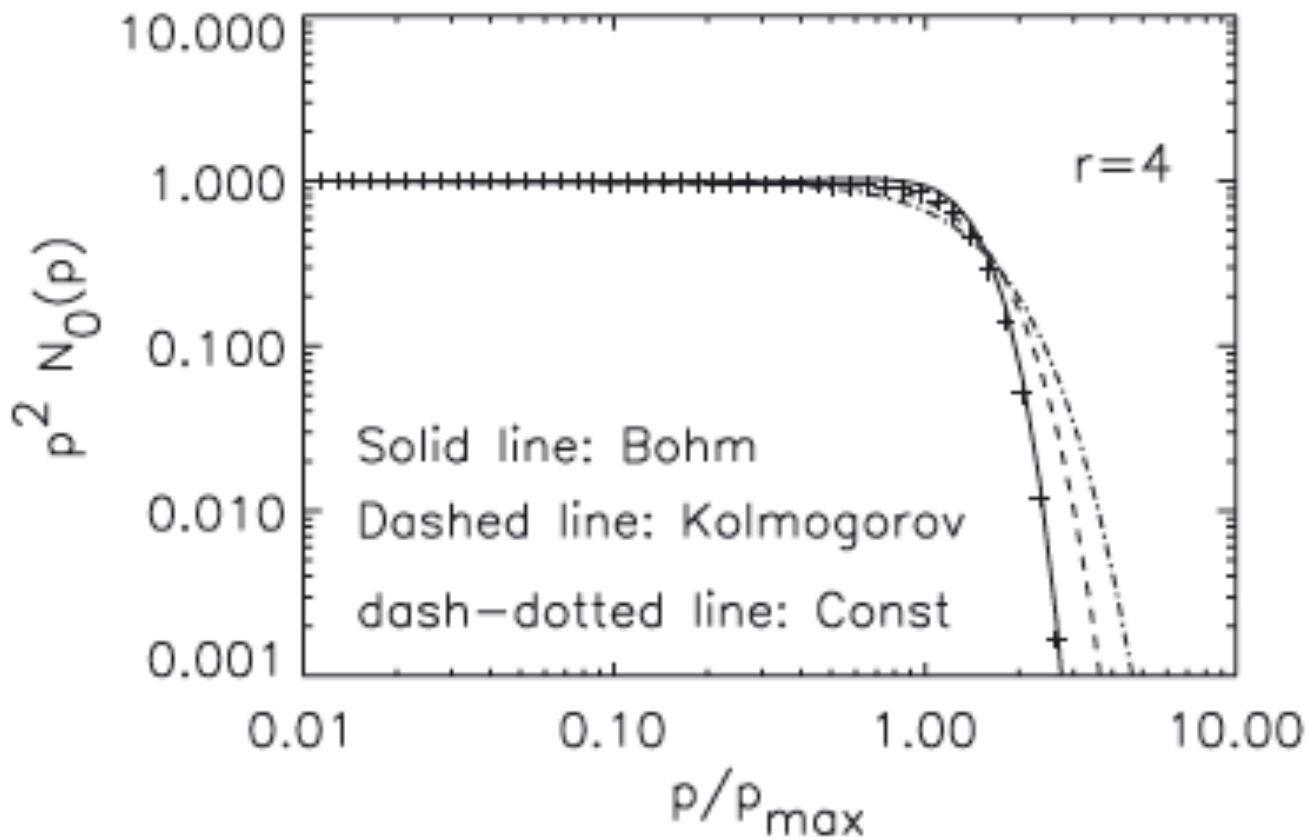
$$t_{acc} \sim D(E)/u^2 \sim 4 \times 10^3 E_{GeV}^{1/3} \; years$$

Too long for any useful acceleration → NEED FOR ADDITIONAL TURBULENCE

NOTE: IN APPENDIX 1 I DERIVE THE ACTUAL EXPRESSION FOR THE ACCELERATION TIME:

$$t_{acc}(p) = \langle t \rangle = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \frac{dp'}{p'} \left[\frac{D_1(p')}{u_1} + \frac{D_2(p')}{u_2} \right]$$

ELECTRONS IN ONE SLIDE



PB 2010

Energy losses and electrons

For electrons, energy losses make acceleration even harder.

The maximum energy of electrons is determined by the condition:

$$t_{acc} \leq Min\left[Age, \tau_{loss}\right]$$

Where the losses are mainly due to synchrotron and inverse Compton Scattering.

WHY DO WE NEED MORE THAN THIS? NON LINEAR THEORY

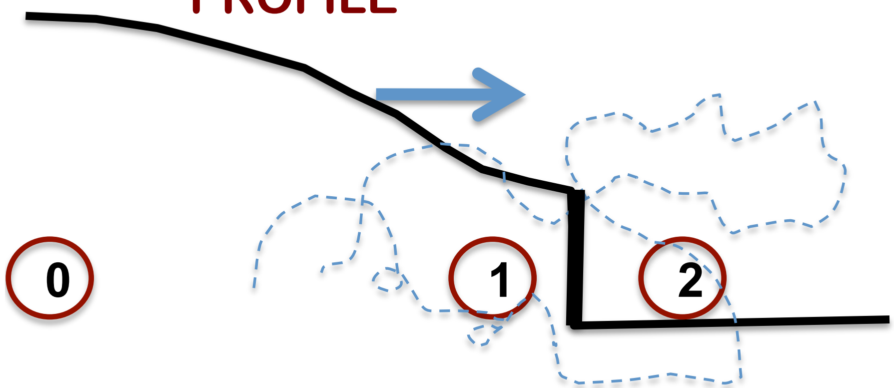
We want a theory of particle acceleration that allows one to describe:

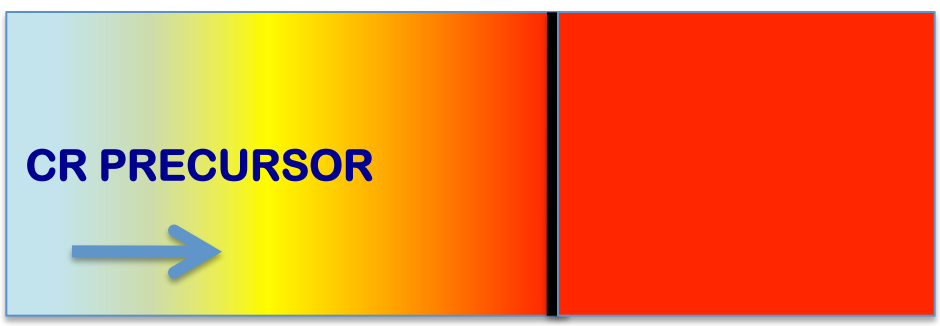
1. ***Dynamical reaction of accelerated particles***
2. ***The interaction between particles and magnetic fields (remember that E_{max} is not large enough!)***
3. ***Dynamical reaction of amplified fields***
4. ***Physical understanding of injection (self-regulation of the system)***
5. ***Escape of particles: how do particles become Cosmic Rays?***

DIFFUSIVE ACCELERATION AT COLLISIONLESS NEWTONIAN SHOCKS

non linear theory

VELOCITY PROFILE





SUBSHOCK

DENSITY OF ACCELERATED PARTICLES

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$$

MASS CONSERVATION

$$\frac{\partial(\rho u)}{\partial t} = -\frac{\partial}{\partial x}\left[\rho u^2 + P_g + P_c + P_W\right]$$

MOMENTUM CONSERVATION

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_g}{\gamma_g - 1}\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1}\right] - u\frac{\partial}{\partial x}\left[P_c + P_W\right] + \Gamma E_W$$

ENERGY CONSERVATION

$$\frac{\partial f(t,x,p)}{\partial t} + \tilde{u}(x)\frac{\partial f(t,x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial f(t,x,p)}{\partial x}\right] + \frac{p}{3}\frac{\partial f(t,x,p)}{\partial p}\frac{d\tilde{u}(x)}{dx}$$

Closing the system with waves and CR

$$u\frac{\partial P_{\rm g}}{\partial x}+\gamma_{\rm g}P_{\rm g}\frac{{\rm d}u}{{\rm d}x}=(\gamma_{\rm g}-1)\Gamma E_{\rm W}$$

GAS PRESSURE AND WAVES

$$\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3+\frac{\gamma_{\rm g}P_{\rm g}u}{\gamma_{\rm g}-1}+\frac{\gamma_{\rm c}P_{\rm c}\tilde{u}}{\gamma_{\rm c}-1}+F_{\rm W}-\bar{D}(x)\frac{\partial E_{\rm c}}{\partial x}\right]=0$$

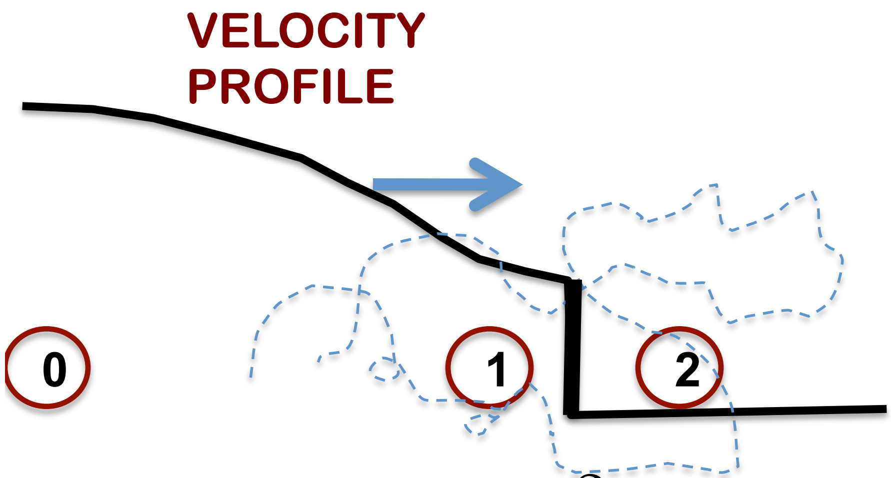
$$\frac{\partial F_{\rm W}}{\partial x}=u\frac{\partial P_{\rm W}}{\partial x}+\sigma E_{\rm W}-\Gamma E_{\rm W}$$

ADVECTION, GROWTH AND DAMPING OF WAVES

$$\sigma E_{\rm W}=v_{\rm A}\frac{\partial P_{\rm c}}{\partial x}$$

ONLY FOR ALFVEN WAVES!!!
AMPLIFICATION OF B-FIELD AS DUE TO CR STREAMING INSTABILITY

Formation of a precursor



$$\frac{\partial}{\partial x}[\rho u] = 0 \rightarrow \rho(x)u(x) = \rho_0 u_0$$

$$\frac{\partial}{\partial x}\left[P_g + \rho u^2 + P_{CR}\right] = 0$$

$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

AND DIVIDING BY THE RAM PRESSURE AT UPSTREAM INFINITY:

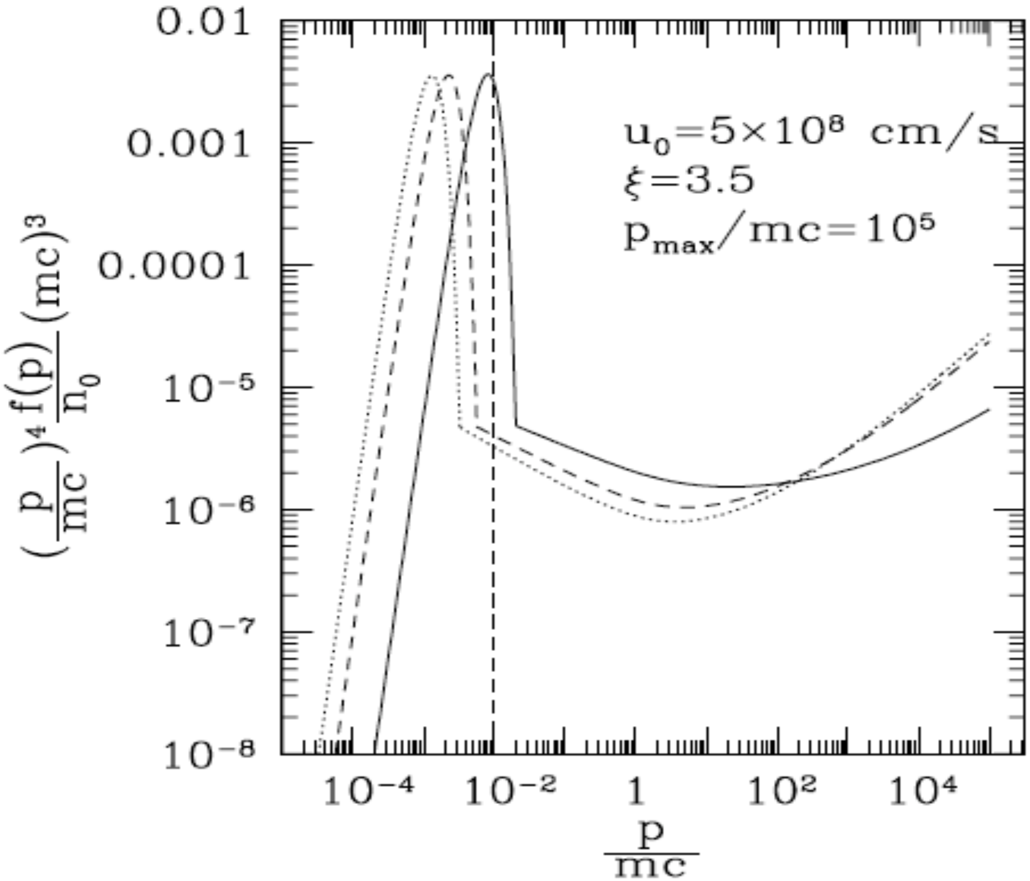
$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1 \Longrightarrow \frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

WHERE WE NEGLECTED TERMS OF ORDER $1/M^2$

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

DIFFUSIVE ACCELERATION AT COLLISIONLESS NEWTONIAN SHOCKS

non linear theory: BASIC PREDICTIONS



COMPRESSION FACTOR BECOMES FUNCTION OF ENERGY

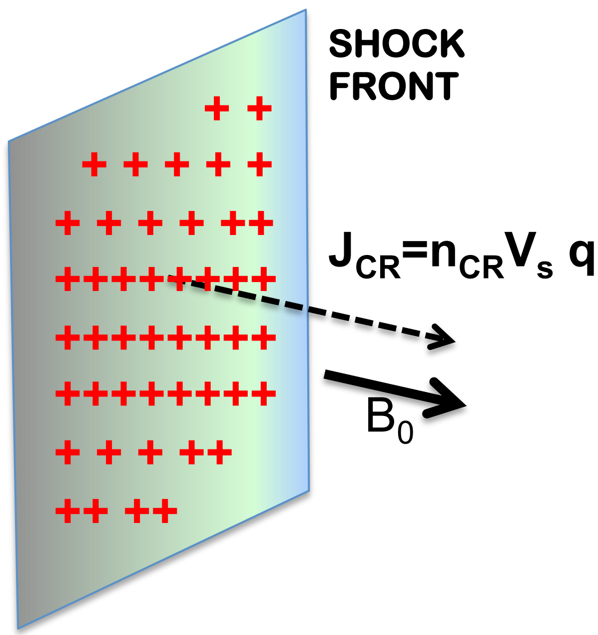
SPECTRA ARE NOT PERFECT POWER LAWS (CONCAVE)

GAS BEHIND THE SHOCK IS COOLER FOR EFFICIENT SHOCK ACCELERATION

SYSTEM SELF REGULATED

EFFICIENT GROWTH OF B-FIELD IF ACCELERATION EFFICIENT

Basics of CR streaming instability



THE UPSTREAM PLASMA REACTS TO THE UPCOMING CR CURRENT BY CREATING A RETURN CURRENT TO COMPENSATE THE POSITIVE CR CHARGE

THE SMALL INDUCED PERTURBATIONS ARE UNSTABLE (ACHTERBERG 1983, ZWEIBEL 1978, BELL 1978, BELL 2004, AMATO & PB 2009)

CR MOVE WITH THE SHOCK SPEED (>> V_A). THIS UNSTABLE SITUATION LEADS THE PLASMA TO REACT IN ORDER TO SLOW DOWN CR TO <V_A BY SCATTERING PARTICLES IN THE PERP DIRECTION (B-FIELD GROWTH)

Magnetic Field Amplification

CR streaming with the shock leads to growth of waves. The general idea is simple to explain:

$$n_{CR} m v_D \rightarrow n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau} \qquad \frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}$$

and assuming equilibrium:

$$\gamma_W = \sqrt{2} \frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

And for parameters typical of SNR shocks:

$$\gamma_W \simeq \sqrt{2}\, \xi_{CR} \left(\frac{V_s}{c}\right)^2 \frac{V_s}{V_A} \Omega_{cyc} \sim \mathcal{O}(10^{-4}\ seconds^{-1})$$

MAGNETIC FIELD AMPLIFICATION

OR THE QUEST OF WHETHER THE CHICKEN OR THE EGG CAME FIRST

SMALL PERTURBATIONS IN THE LOCAL B-FIELD CAN BE AMPLIFIED BY THE SUPER-ALFVENIC STREAMING OF THE ACCELERATED PARTICLES

Particles are accelerated because there is High magnetic field in the acceleration region

High magnetic field is present because particles are accelerated efficiently

Without this non-linear process, no acceleration of CR to High energies (and especially not to the knee!)

BUT…

...MAGNETIC FIELD CAN BE AMPLIFIED BY

1. **RESONANT STREAMING** (Bell 78, Achterberg 83, Zweibel 78)

   Fast generation, fast scattering … saturation?

2. **NON RESONANT STREAMING** (Bell 04, Amato & PB 09)

   Probably more efficient generation rate but inefficient scattering

3. **SHOCK CORRUGATION** (DOWNSTREAM) Giacalone & Jokipii 07
   Not CR induced!
   It happens downstream only, it does not help with particle acceleration unless perpendicular shock

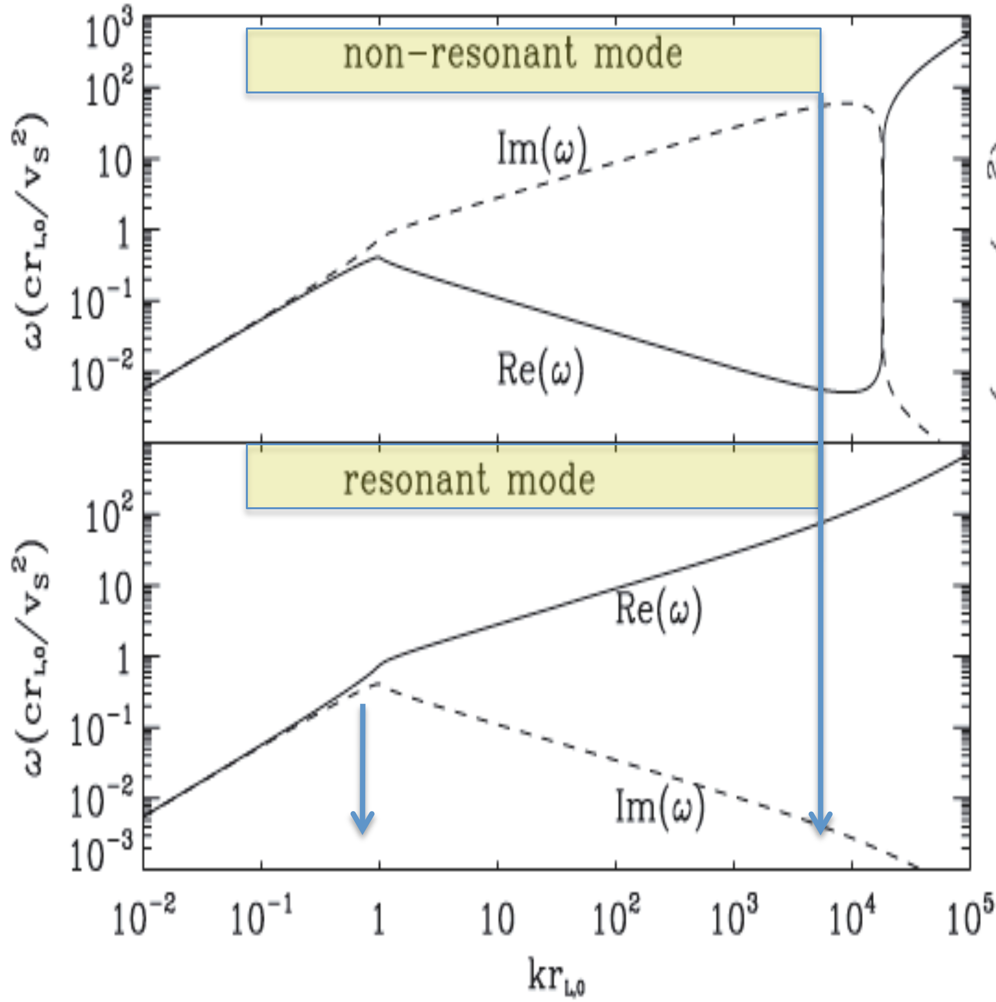
4. **VORTICITY IN THE PRECURSOR** (PB, Matthaeus, et al. 12)

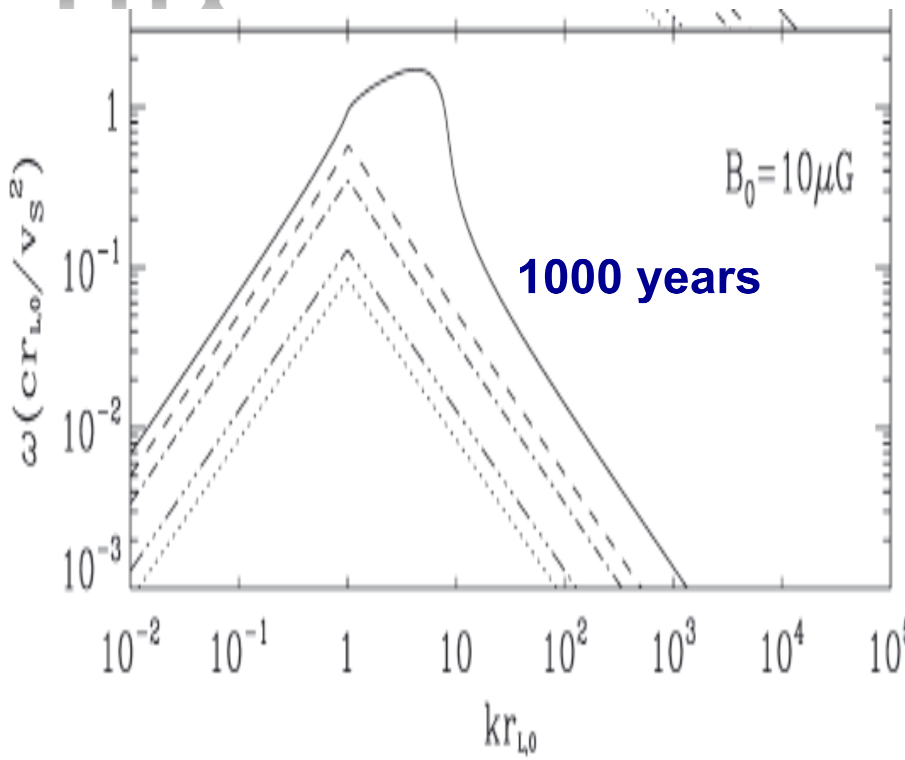
   Potentially very interesting, power on large scales

5. **FIREHOSE INSTABILITY** (Shapiro et al. 98)

   Potentially very interesting, power on large scales

GROWING MODES in CR STREAMING INSTABILITY





NON RESONANT MODES GROW FASTER BUT THEY DO NOT SCATTER PARTICLES EFFECTIVELY UNLESS FAST INVERSE CASCADE

Amato & PB 2009, Bell 2004

SATURATION OF GROWTH

Extremely uncertain. It depends on:

a) Damping (type of waves?)

b) Backreaction of fields on the CR current

c) Coupling between large and small spatial scales

A NAÏVE EXTRAPOLATION OF QLT WOULD LEAD TO:

$$\frac{\delta B^2}{8\pi} = \frac{1}{M_A}\rho V_s^2 \xi_{CR}$$

IN THE RESONANT CASE, UPSTREAM (OR POSSIBLY $\delta B/B \sim 1$ BECAUSE RESONANCE GETS LOST)

$$\frac{\delta B^2}{4\pi} = \frac{1}{2}\rho V_s^2 \xi_{CR} \frac{V_s}{c}$$

ESTIMATED ANALYTICALLY FROM SATURATION CONDITION OF NON RESONANT MODES (BELL 2004)

X-ray rims and B-field amplification

TYPICAL THICKNESS OF FILAMENTS: ~ 10^{-2} pc

The synchrotron limited thickness is:

$$\Delta x \approx \sqrt{D(E_{max})\tau_{loss}(E_{max})} \approx 0.04\ B_{100}^{-3/2}\ \mathrm{pc}$$

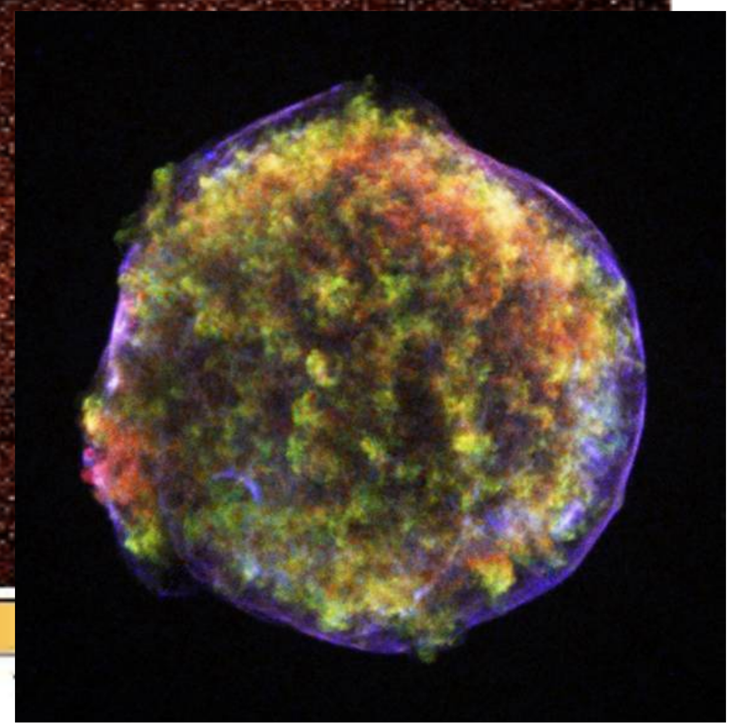
$$B \approx 100\ \mu\mathrm{Gauss}$$

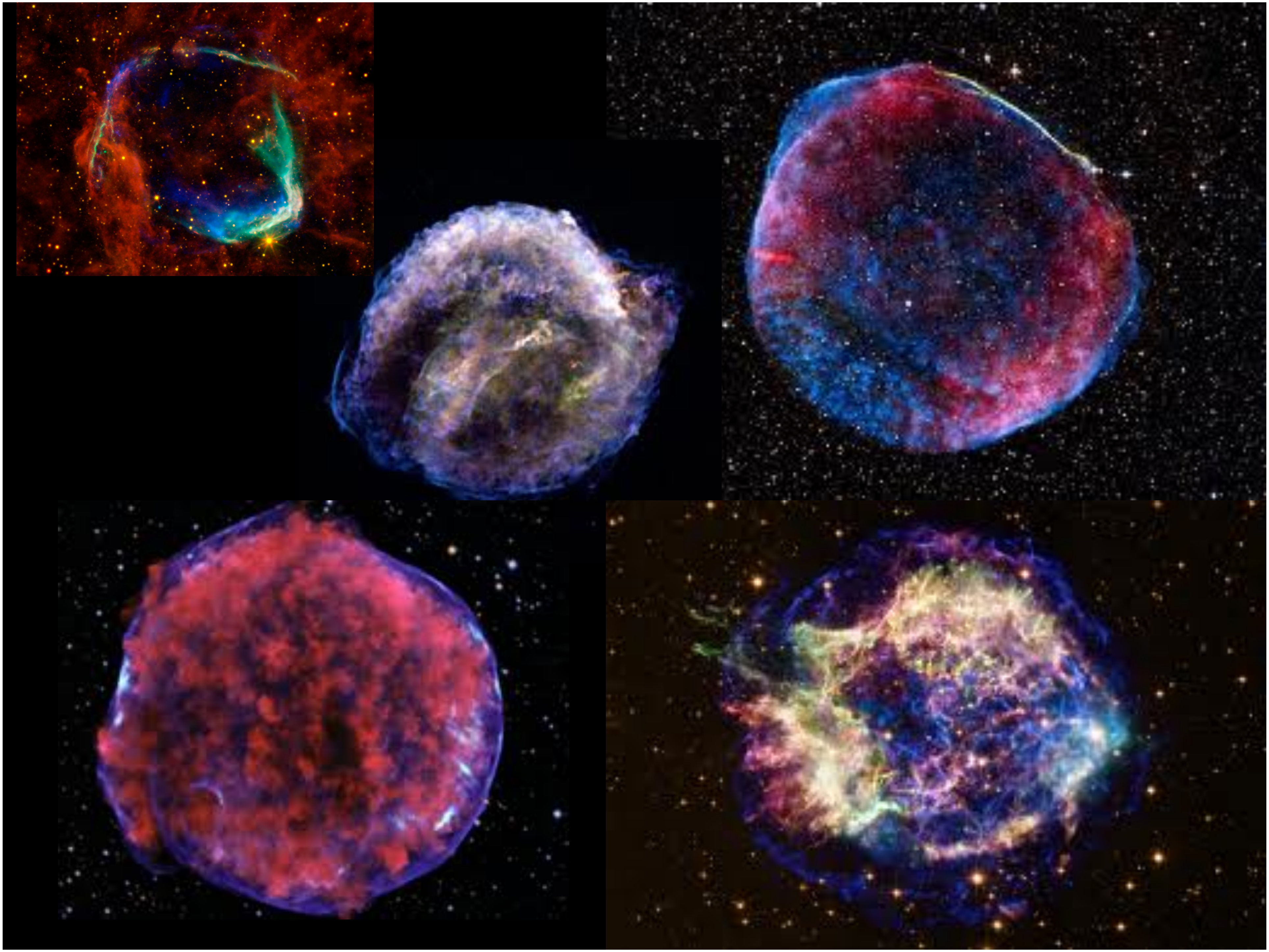
$$E_{max} \approx 10\ B_{100}^{-1/2}\ u_8\ \mathrm{TeV}$$

$$\nu_{max} \approx 0.2\ u_8^2\ \mathrm{keV}$$

In some cases the strong fields are confirmed by time variability of X-rays

Uchiyama & Aharonian, 2007





SPECTRA

THE SPECTRA OF ACCELERATED PARTICLES ARE IN GENERAL CONCAVE AND FLATTER THAN E^{-2} AT HIGH ENERGY

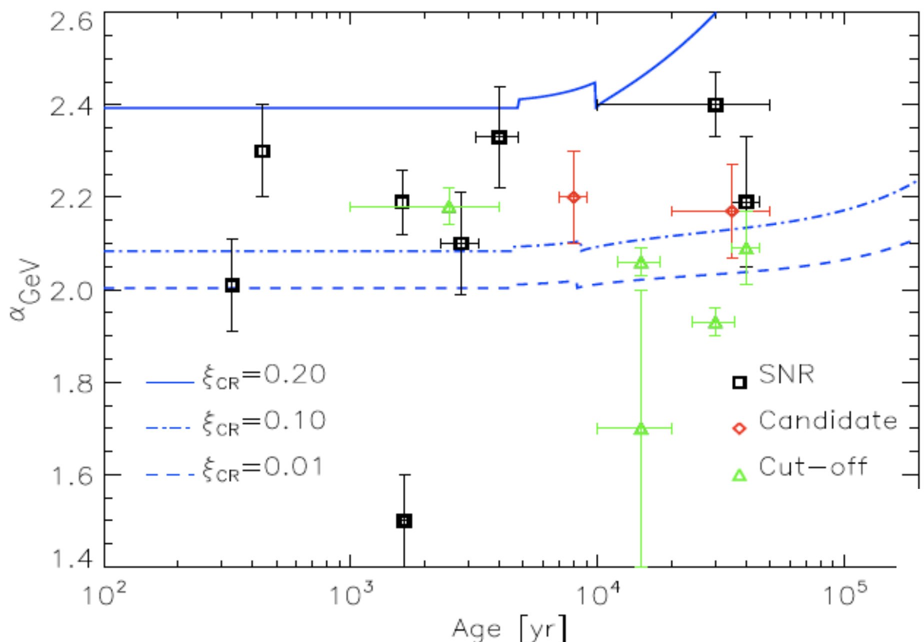
THE MAXIMUM ENERGY WITH B-FIELD AMPLIFICATION REACHS UP TO ~10^{15} eV FOR PROTONS (Z TIMES HIGHER FOR NUCLEI)

THESE SPECTRA SHOULD REFLECT IN THE GAMMA RAY SPECTRA (IF DUE TO PP SCATTERING) AND OF NEUTRINOS

BUT THE OBSERVED SPECTRA OF GAMMAS ARE TYPICALLY ~ $E^{-2.3}$

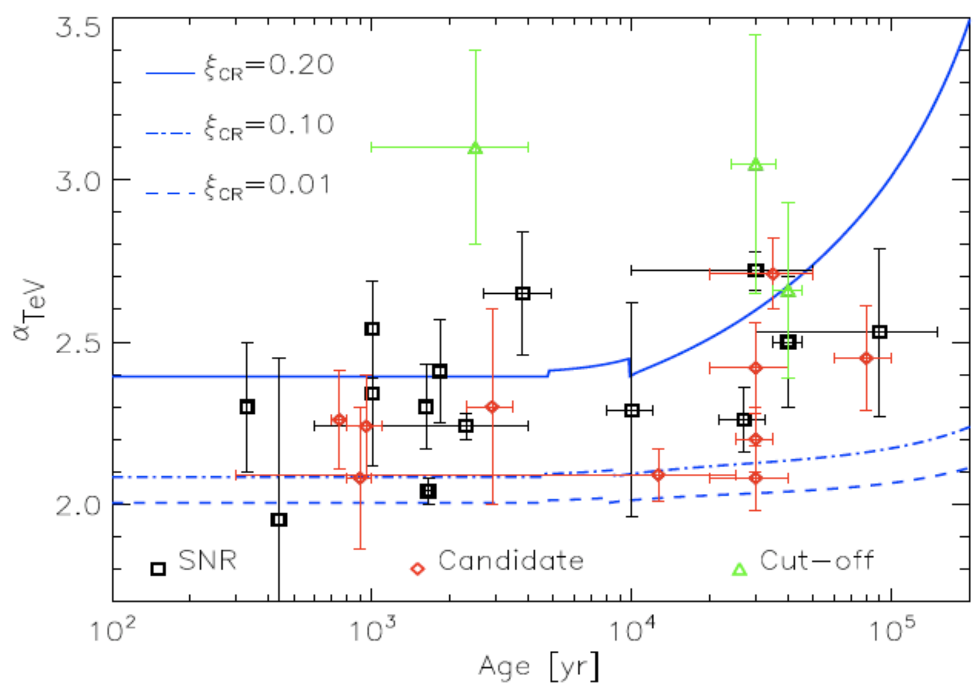
CLEARLY INCOMPATIBLE WITH LEPTONIC MODELS! BUT ALSO NOT COMPATIBLE WITH THE SIMPLEST PREDICTION OF NLDSA

TROUBLE WITH SLOPES ?



Caprioli 2011

VERY SURPRISING TO SEE THAT THE REQUIRED ACCELERATION EFFIC. ARE HIGH BUT THE SPECTRA ARE STEEP



THE EFFECT OF THE VELOCITY OF WAVES

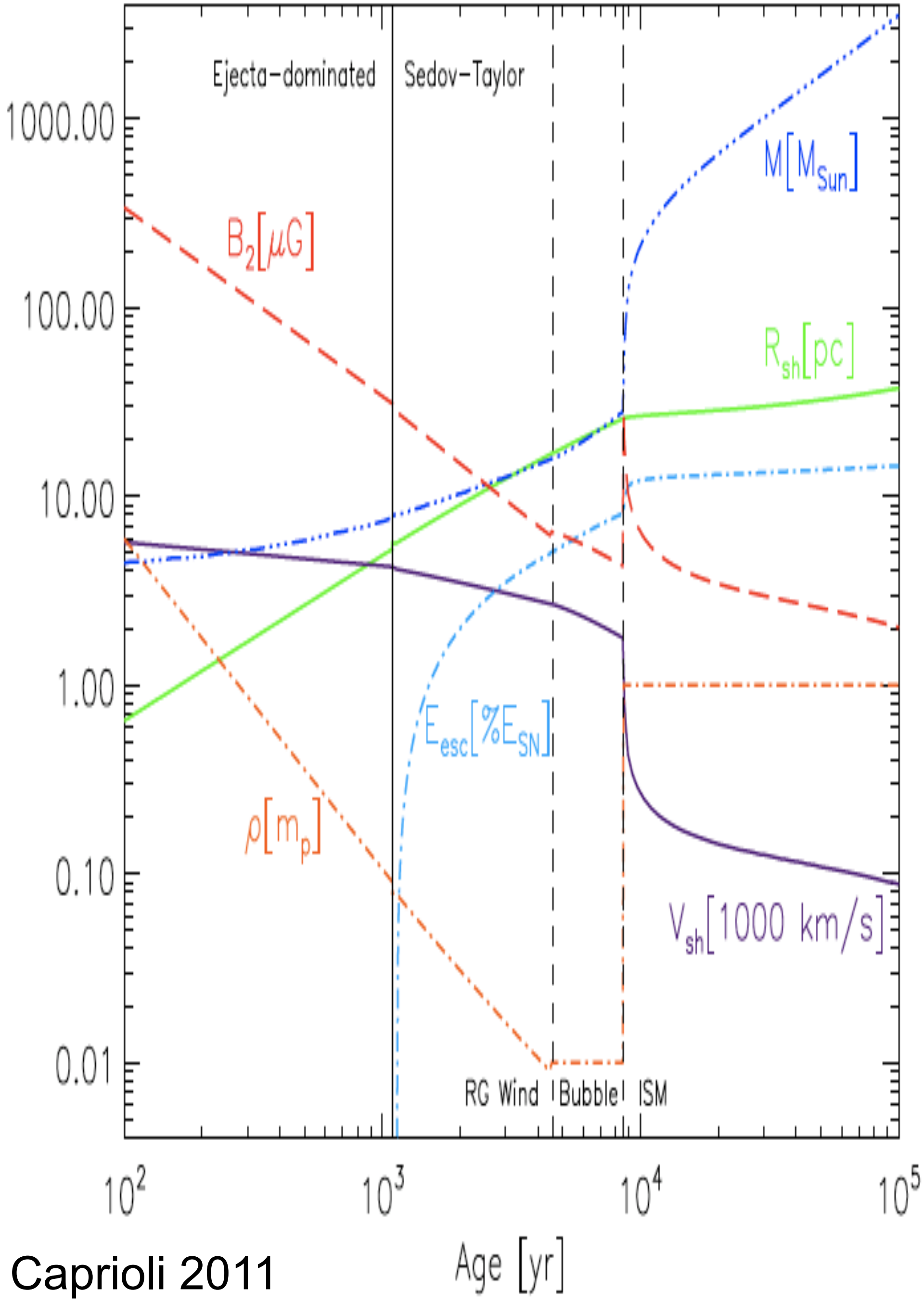
One should remember that the compression factor that counts in shock acceleration is not that of fluid velocity, but that of the scattering centers velocity

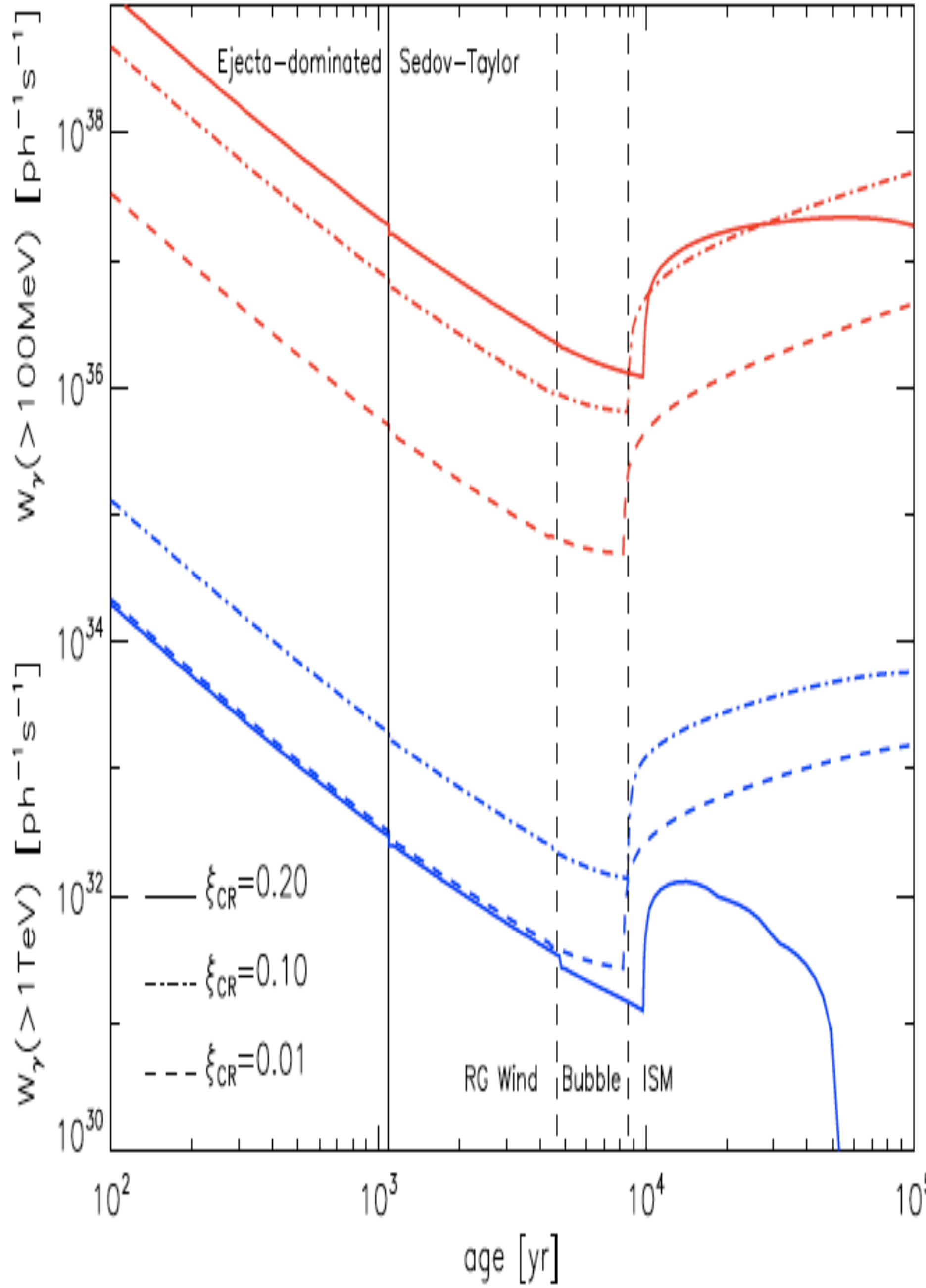
$$r = \frac{u_1}{u_2} \rightarrow \tilde{r} = \frac{u_1 + v_{A,1}}{u_2 + v_{A,2}}$$

When the magnetic field is amplified the Alfven speed is not well defined and one may argue that it should be calculated in the amplified field (it depends on helicity!):

$$\tilde{r} = r\left(1 - \frac{1}{M_{A,1}}\right) = \frac{\gamma_{\rm eff} + 1}{\gamma_{\rm eff} - 1 + 2/M_s^2}\left[1 - \frac{\xi_{cr}(2 - \xi_{cr})}{2(1 - \xi_{cr})^{5/2}}\right] \qquad \gamma_{\rm eff} = \frac{1}{3}\frac{5 + 3\xi_{cr}}{1 + \xi_{cr}}$$

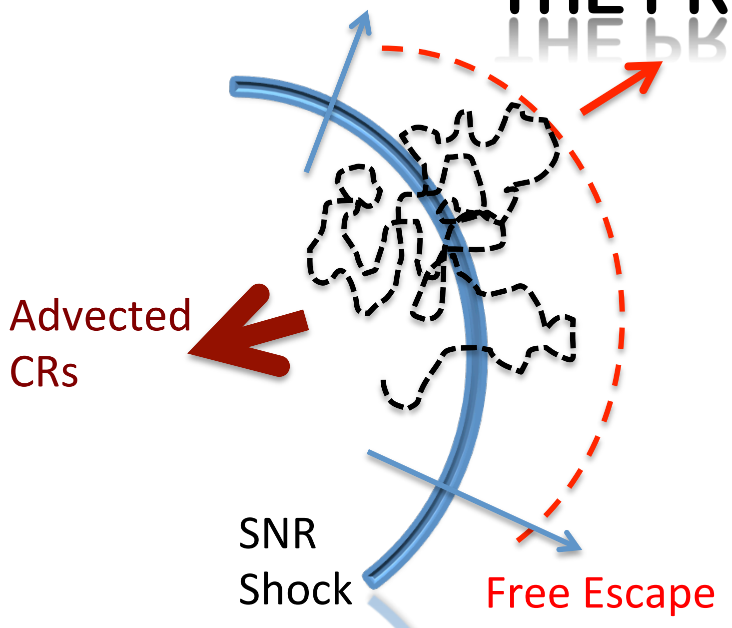
THIS EFFECT LEADS TO STEEPER SPECTRA WHEN ACCELERATION IS EFFICIENT (BUT VERY MODEL DEPENDENT)





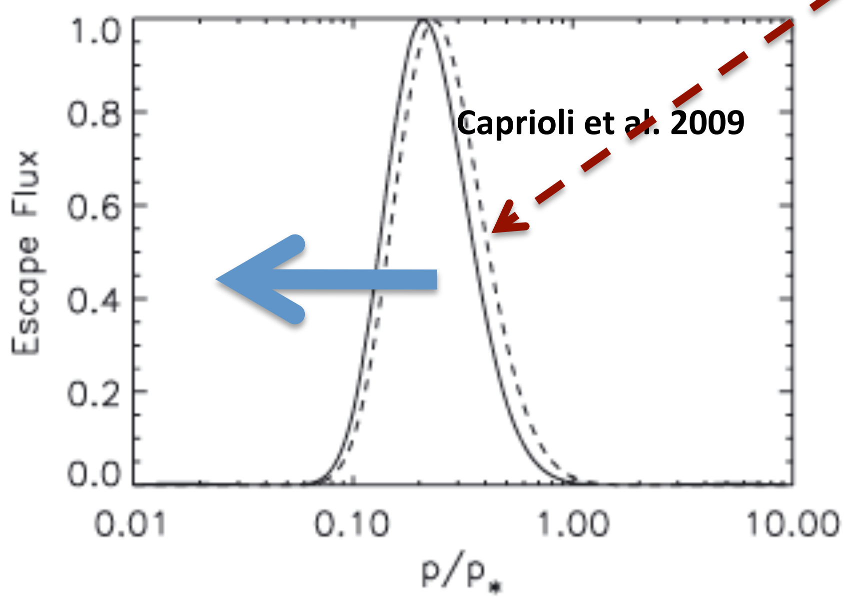
Caprioli 2011

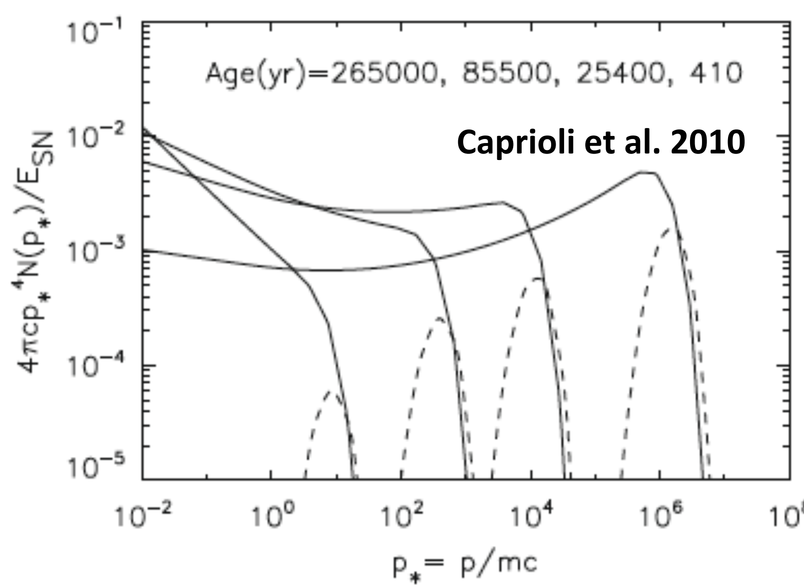
HOW DO ACCELERATED PARTICLES BECOME CRs? THE PROBLEM OF ESCAPE



The escape flux can be calculated using the transport equation IF one assumes a free escape boundary surface **(DURING ST PHASE)**

$$\Phi_{esc}(E,x) = D(E)\left(\frac{\partial f(E,x)}{\partial x}\right)_{x=x_{fe}}$$





CR ESCAPE AND CLOUDS

TWO SCENARIOS:

SNR SHOCK ENTERS THE MC

Collisionless shock only involves the small fraction of Ions (low density)

Ion-neutral density kills waves → low E_{max}

MC IS ILLUMINATED BY CR FROM SNR

The mc only acts as a target for pp

Gamma ray flux depends on

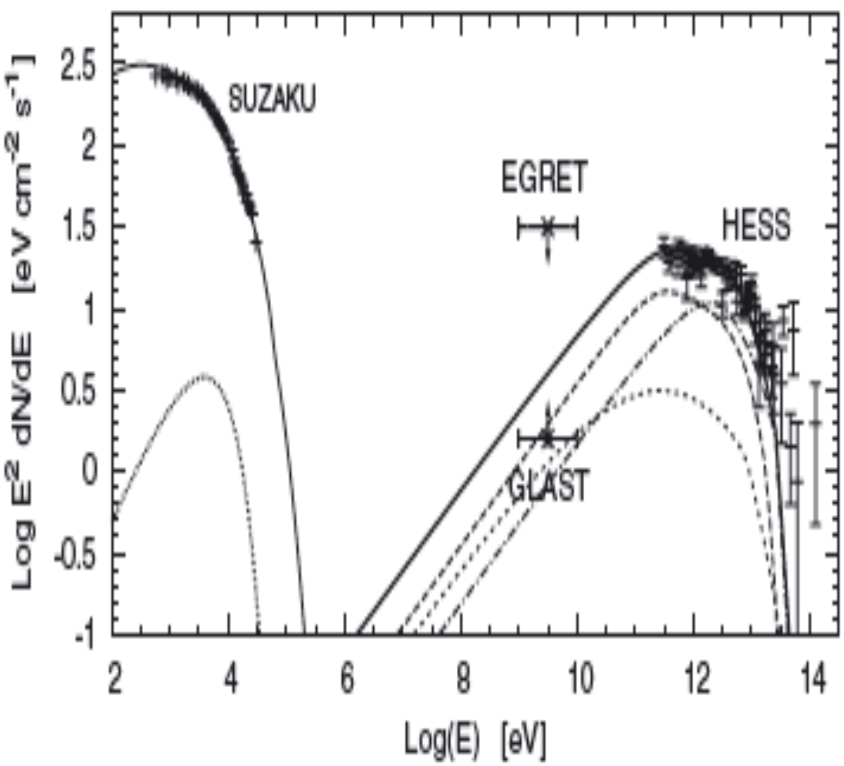
-Age of SNR

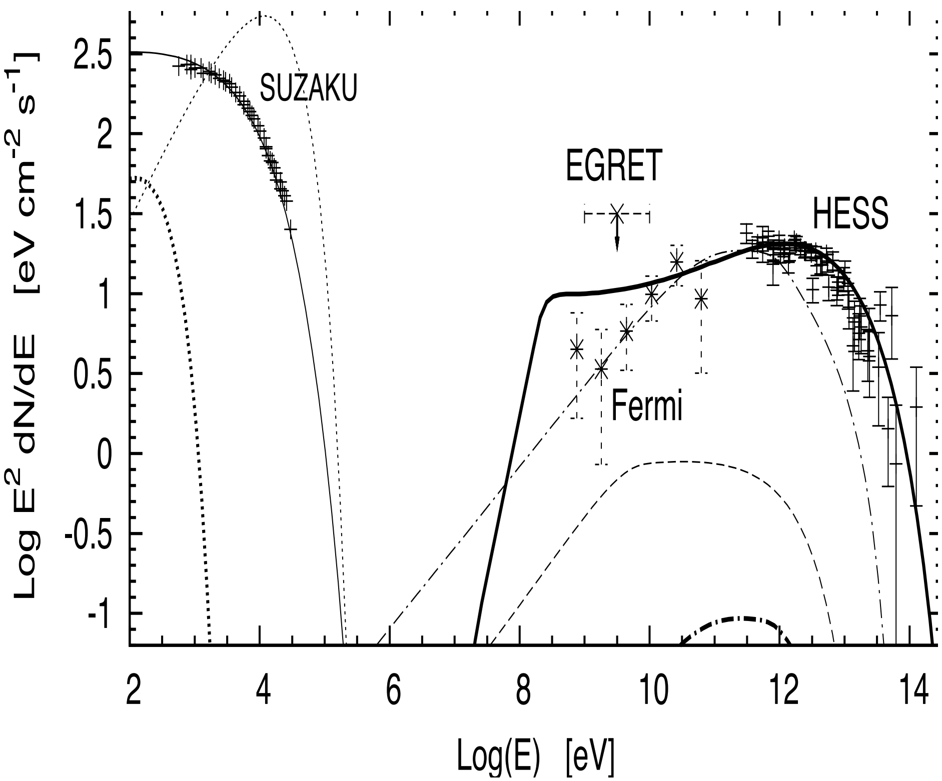
-Diffusion coefficient around the SNR

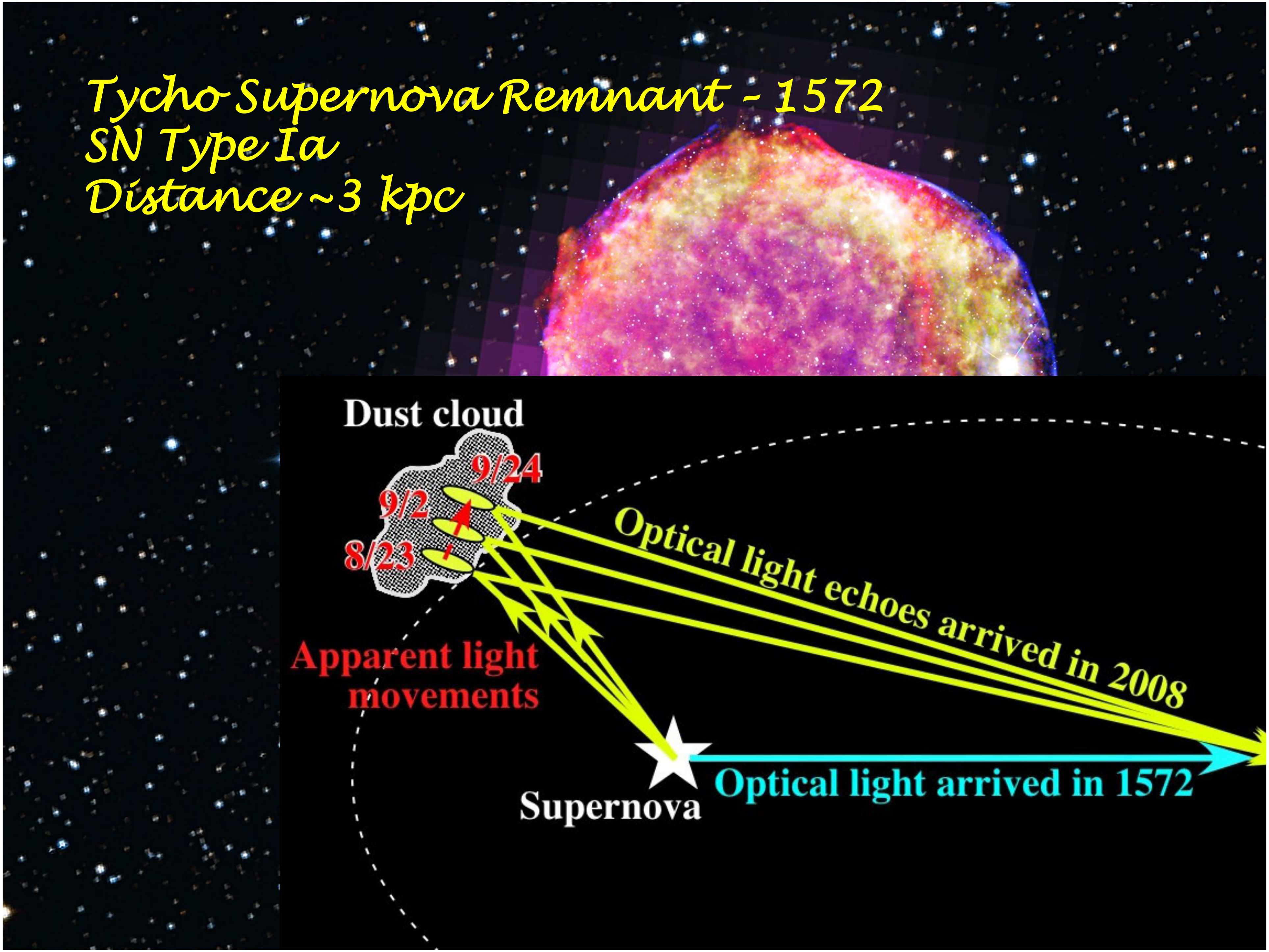
-Escape physics

The case of RX J1713

Morlino et al. 2009

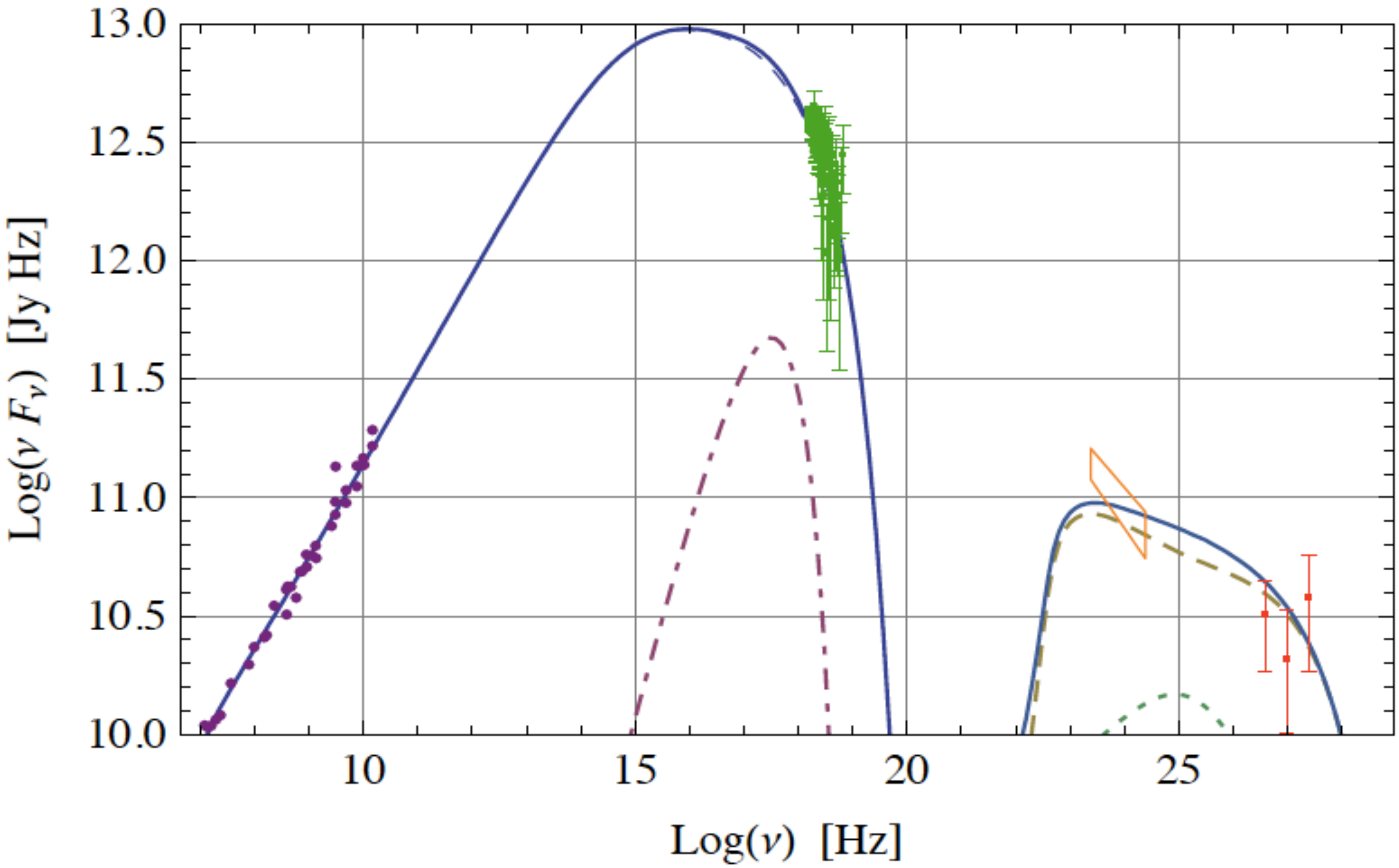





Tycho Supernova Remnant - 1572
SN Type Ia
Distance ~3 kpc
Dust cloud
9/24
9/2
8/23
Optical light echoes arrived in 2008
Apparent light movements
Supernova
Optical light arrived in 1572

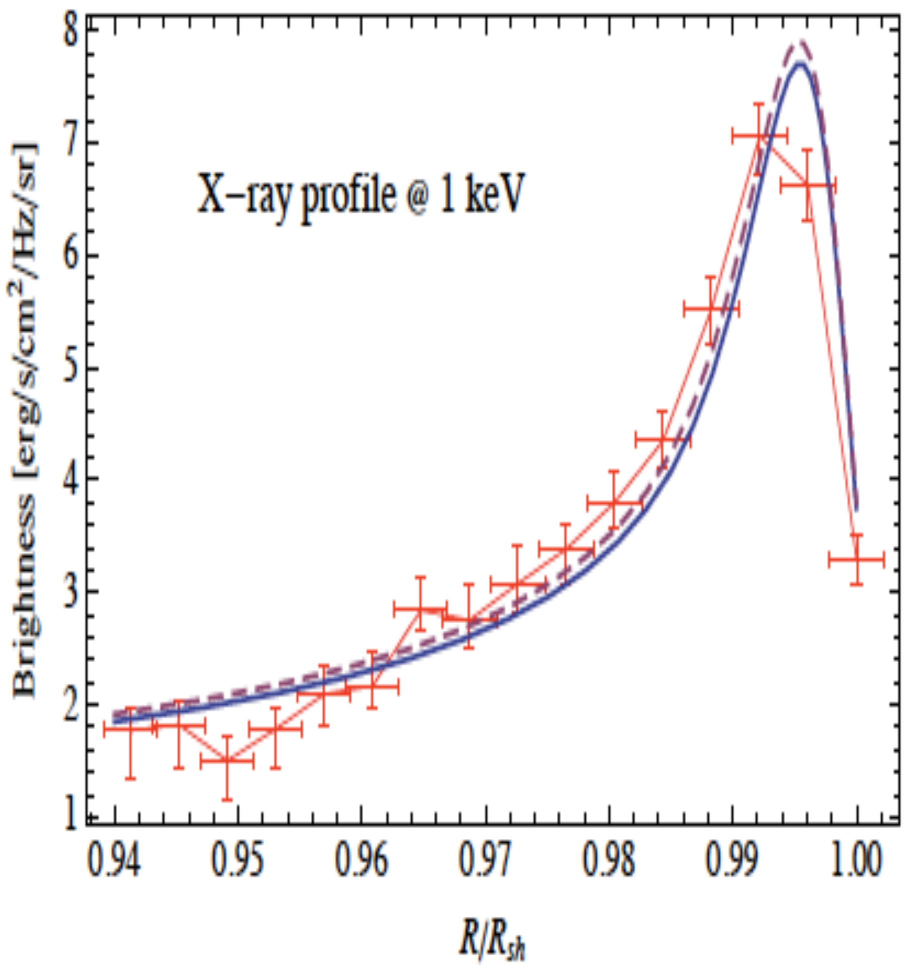
The case of Tycho

Morlino&Caprioli 2011

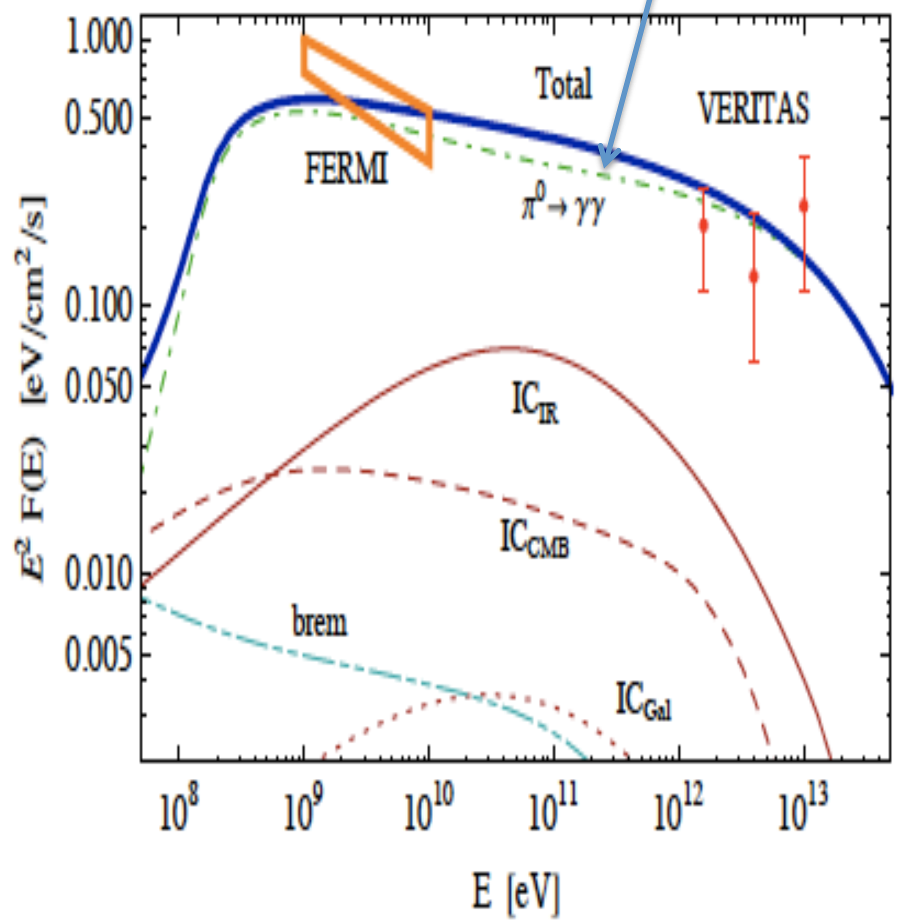


The case of Tycho

Morlino&Caprioli 2011

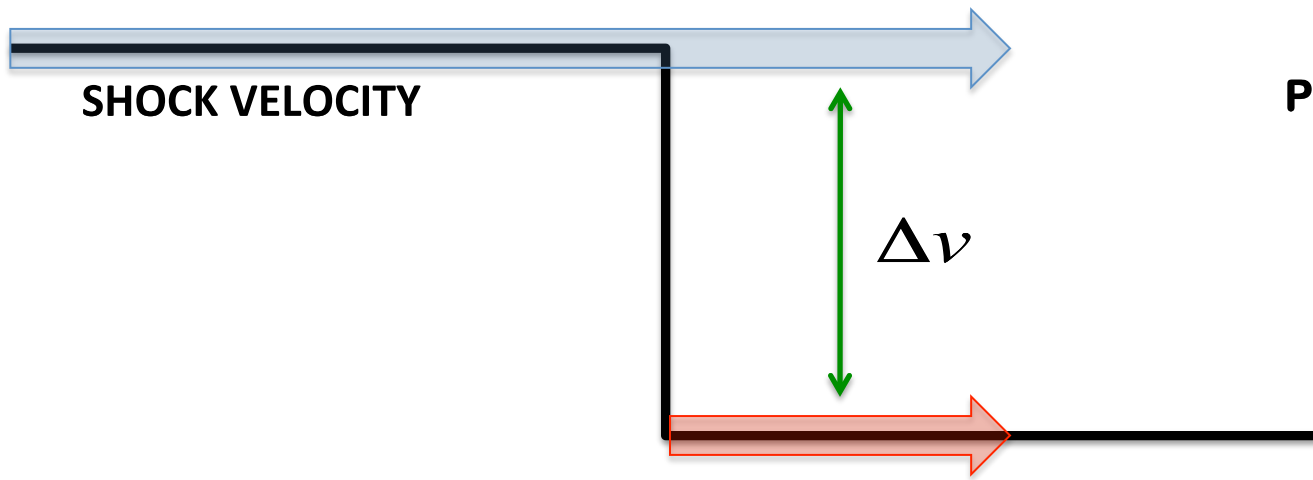


STEEP SPECTRUM BASICALLY IMPOSSIBLE TO EXPLAIN WITH LEPTONS



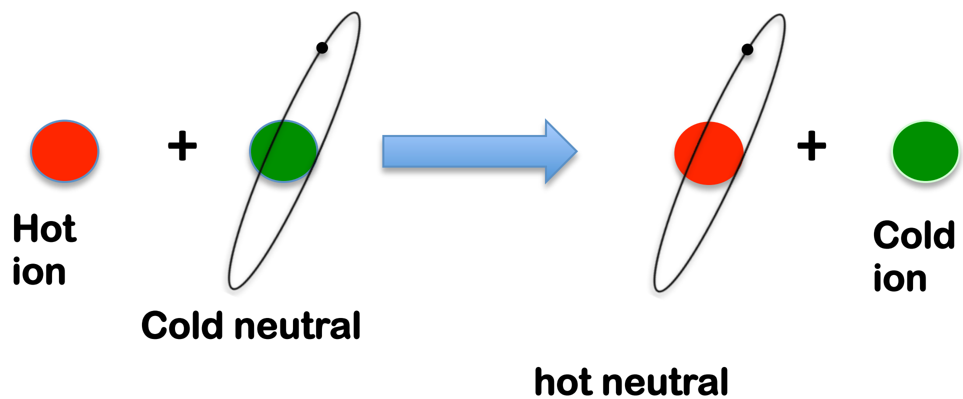
MODERN ASPECTS OF ACCELERATION AT A COLLISIONLESS SHOCK

NEUTRALS
AND IONS



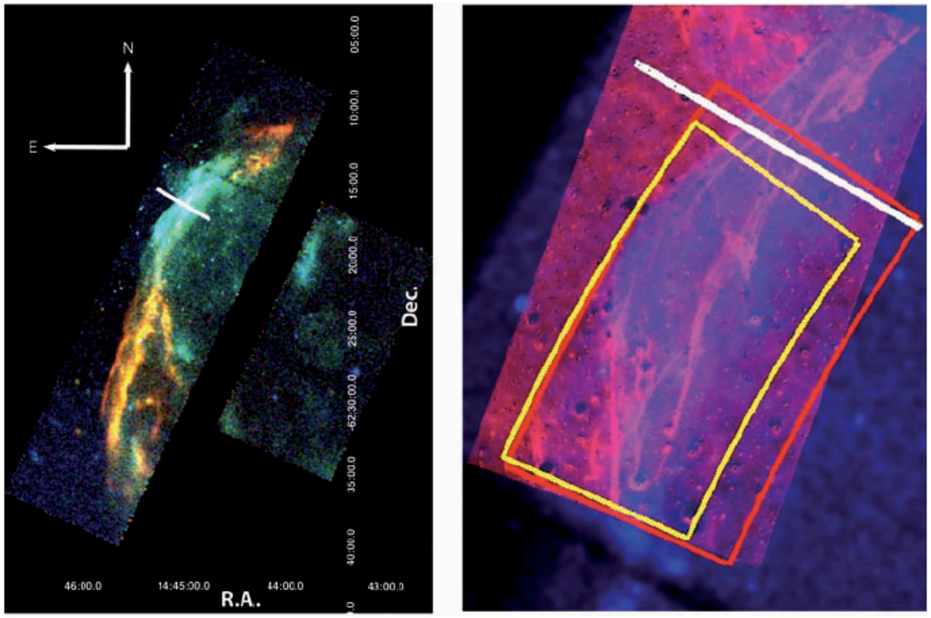
PB+, 2011

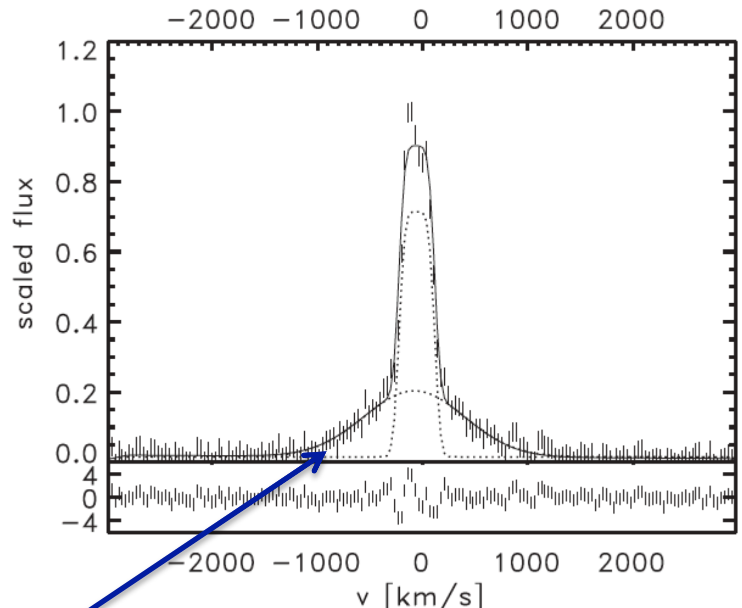
CHARGE EXCHANGE → BROAD BALMER LINE (NEUTRALS THAT MADE CHARGE EXCHANGE) REFLECTING THE TEMPERATURE OF IONS…



BUT THE LATTER AFFECTED BY EFFICIENT CR ACCELERATION

BROAD BALMER LINES NARROWER THAN FOR UNMODIFIED SHOCKS





Helder et al. 2009

$$W_{broad} = \sqrt{8\ln 2\frac{kT_2}{m}} \approx 1.02\ \mathrm{v_{sh}}$$

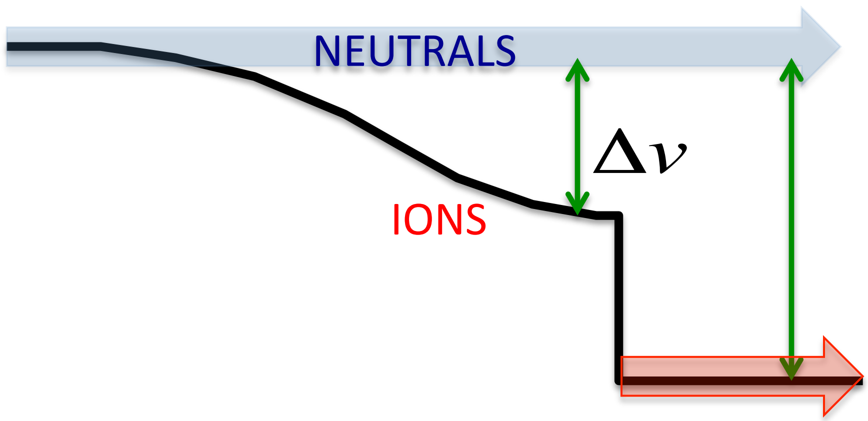
$$\boxed{W_{broad} = \ 1100 \pm 63\, km/s \ \rightarrow \ T_2 = \ 2.3 \pm 0.3\, keV}$$

Shock speed from proper motion

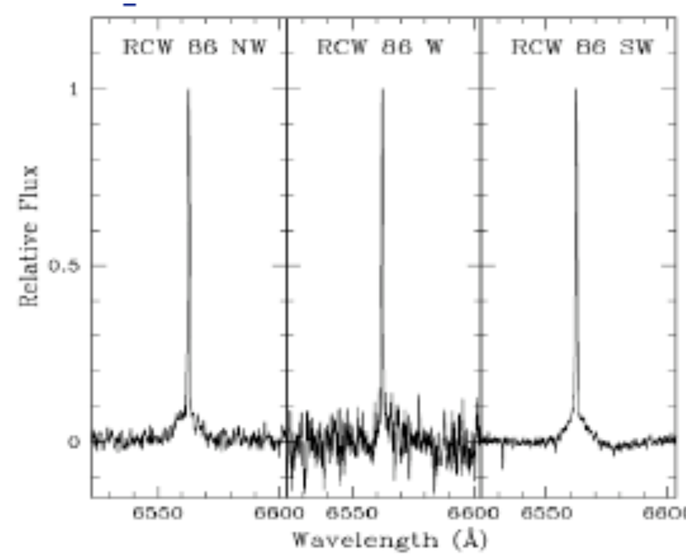
$$\boxed{v_{shock} = \ 6000 \pm 2800\, km/s \left(\frac{d}{2.5 \pm .5 kpc}\right)\left(\frac{\dot{\theta}_{obs}}{0.5 \pm .2''\, yr^{-1}}\right) \rightarrow T_2 = \begin{matrix} 20-150\, keV\,(no\ equilibration) \\ 12-90\, keV\,(equilibration) \end{matrix}}$$

INFERRED EFFICIENCY of CR ACCELERATION 50-60% !!! (BUT model dependent)

NARROW BALMER LINES BROADER THAN FOR UNMODIFIED SHOCKS



Sollerman et al. 2003



CHARGE EXCHANGE OCCURS NOW IN THE CR INDUCED PRECURSOR

$$W_{broad} = \sqrt{8\ln 2\frac{kT_0}{m}} \approx 21\ \mathrm{km/s}\left(\frac{T_0}{10^4 K}\right)^{1/2}$$

$$\boxed{W_n \sim 30-50\, km/s \rightarrow T \sim 2-6\, 10^4\, K}$$

NARROW BALMER LINE BROADER THAN FOR AN UNMODIFIED SHOCK

APPENDIX 1
Hydrodynamics and Shocks

There are many instances of astrophysical systems that result in explosive phenomena in which large amounts of mass and energy are released in the surrounding medium (interstellar medium or intergalactic medium) at high speed. The ejected material behaves as a fluid, though often the importance of magnetic fields cannot be neglected.

Here I will discuss the basic laws that govern the dynamics of such a fluid, under ideal conditions in which the fluid evolves adiabatically and the effects of thermal conductivity can be neglected.

I will show how the laws that govern the motion of such a fluid lead to conclude that in some conditions *shock waves* can develop in the fluid.

These concepts are of particular importance in supernova explosions, which Are likely to play an important role for particle acceleration in the universe.

I will restrict the attention to fluid that move subrelativistically, so that only Newtonian dynamics applies.

I will also comment upon the *collisionless nature* of the shock waves that develop in astrophysics (with some exceptions).

Conservation of mass

Let us consider a fixed infinitesimal volume dV where the matter density is r. The mass in the volume rdV remains constant unless mass is allowed to Flow in and out of the volume dV. The total mass is

$$\int \rho dV$$

and changes in time because of the flux of mass per unit time and volume across the surface dA that surrounds dV:

$$-\frac{d}{dt}\int \rho dV = \oint \rho\vec{v}\cdot d\vec{A} \equiv \int \nabla\cdot(\rho\vec{v})dV$$

Gauss Theorem

It follows that:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

Conservation of momentum

An element of surface suffers a pressure p and a force over the volume:

$$-\oint_{d\vec{A}} p d\vec{A} = -\int_{dV} \nabla p dV$$

The force exerted on the fluid element of mass rdV is:

$$\rho dV \frac{D\vec{v}}{Dt} = -\nabla p dV \rightarrow \rho \frac{D\vec{v}}{Dt} = -\nabla p$$

Where D/Dt is the convective derivative. Let us consider a fluid element that is at x at time t and moves with velocity v(x,t). At time t+dt the fluid element is located at x+vdt, therefore the acceleration is

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

It follows that:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho}$$

Conservation of energy

In the assumption of adiabatic evolution of the fluid, the entropy per unit mass s is conserved:

$$\frac{\partial s}{\partial t} + \vec{v} \cdot \nabla s = 0$$

and using conservation of mass, one immediately gets:

$$\frac{\partial(\rho s)}{\partial t} + \nabla \cdot (\rho s \vec{v}) = 0$$

Introducing the specific enthalpy: w=e+p/r, one can write:

$$dw = T ds + \frac{dp}{\rho} = \frac{dp}{\rho}$$

Adiabatic→ds=0

Which leads to:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho} = -\nabla \left(e + \frac{p}{\rho} \right)$$

For a polytropic gas with adiabatic index g one has that the energy density per Unit volume is u=p/(g-1) therefore:

$$u = \rho e = \frac{p}{\gamma - 1} \rightarrow e = \frac{1}{\gamma - 1}\frac{p}{\rho}$$

So that

$$w = e + \frac{p}{\rho} = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$$

And the previous equation becomes:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\gamma}{\gamma - 1} \nabla \left(\frac{p}{\rho} \right)$$

In the one dimensional stationary case one has

$$v\frac{\partial v}{\partial x} + \frac{\gamma}{\gamma - 1}\frac{\partial}{\partial x}\left(\frac{p}{\rho}\right)$$

And using the eqn for conservation of mass one immediately gets:

$$\frac{\partial}{\partial x}\left[\frac{1}{2}\rho v^3 + \frac{\gamma}{\gamma - 1} v p\right] = 0 \rightarrow \frac{1}{2}\rho v^3 + \frac{\gamma}{\gamma - 1} v p = Constant$$

APPENDIX 2: Acceleration Time

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

Let us move to the Laplace transform:

$$g(s,x,p) = \int_0^\infty dt e^{-st} f$$

so that the transport equation becomes:

$$sg + u\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}\left[D\frac{\partial g}{\partial x}\right] + \frac{1}{3}\left(\frac{du}{dx}\right)p\frac{\partial g}{\partial p} + \frac{Q(x,p)}{s}$$

Integrating this equation between $x=0^-$ and $x=0^+$ one gets:

$$0 = \left[D\frac{\partial g}{\partial x}\right]_2 - \left[D\frac{\partial g}{\partial x}\right]_1 + \frac{1}{3}(u_2 - u_1)p\frac{\partial g_0}{\partial p} + \frac{Q_0(p)}{s}$$

Where we have assumed that:

$$Q(x,p) = Q_0(p)\delta(x) \quad and \quad Q_0(p) = A\delta(p - p_{inj})$$

<u>UPSTREAM</u>

$$sg_1 + u_1 \frac{\partial g_1}{\partial x} = \frac{\partial}{\partial x}\left[D_1 \frac{\partial g_1}{\partial x}\right]$$

and assuming that the diffusion coefficient is independent upon location x the solution has the form:

$$g_1(s,p,x) = g_0(s,p)\exp\left[\beta_1 x\right] \qquad x < 0$$

$$\beta_1 = \frac{u_1 + \sqrt{u_1^2 + 4D_1 s}}{2D_1} = \frac{u_1}{2D_1}\left[1 + \sqrt{1 + \frac{4D_1 s}{u_1^2}}\right]$$

$$\left[D_1 \frac{\partial g_1}{\partial x}\right]_1 = \frac{u_1}{2}\left[1 + \sqrt{1 + \frac{4D_1 s}{u_1^2}}\right] g_0 = \sigma_1 g_0$$

<u>DOWNSTREAM</u>

$$sg_2 + u_2 \frac{\partial g_2}{\partial x} = \frac{\partial}{\partial x}\left[D_2 \frac{\partial g_2}{\partial x}\right]$$

Proceeding as in the previous case:

$$g_2(s,p,x) = g_0(s,p)\exp\left[\beta_2 x\right] \qquad x > 0$$

$$\beta_2 = \frac{u_2 - \sqrt{u_2^2 + 4D_2 s}}{2D_2} = \frac{u_2}{2D_2}\left[1 - \sqrt{1 + \frac{4D_2 s}{u_2^2}}\right]$$

$$\left[D_2 \frac{\partial g_2}{\partial x}\right]_2 = \frac{u_2}{2}\left[1 - \sqrt{1 + \frac{4D_2 s}{u_2^2}}\right] g_0 = \sigma_2 g_0$$

Notice that in the long time limit, namely s→0 one gets the well known result:

$$\left[D_1 \frac{\partial g_1}{\partial x}\right]_1 = u_1 g_0 \qquad \left[D_2 \frac{\partial g_2}{\partial x}\right]_2 = 0$$

Notice that one can easily write:

$$\left[D_1 \frac{\partial g_1}{\partial x}\right]_1 = \frac{u_1}{2}\left[1+\sqrt{1+\frac{4D_1 s}{u_1^2}}\right] g_0 - u_1 g_0 + u_1 g_0 = -\frac{u_1}{2} g_0 + \frac{u_1}{2}\sqrt{1+\frac{4D_1 s}{u_1^2}} g_0 + u_1 g_0 =$$

$$= g_0 u_1 A_1 + u_1 g_0 \qquad A_1 = \frac{1}{2}\left[-1+\sqrt{1+\frac{4D_1 s}{u_1^2}}\right]$$

In this way A_1 has the same property as s_1 namely they both vanish in the long time limit $s \rightarrow 0$.

Substituting in the equation at the shock:

$$0 = \left[D \frac{\partial g}{\partial x}\right]_2 - \left[D \frac{\partial g}{\partial x}\right]_1 + \frac{1}{3}\left(u_2 - u_1\right) p \frac{\partial g_0}{\partial p} + \frac{Q_0(p)}{s}$$

$$\Downarrow$$

$$\left(\sigma_2 - u_1 A_1 - u_1\right) g_0 + \frac{1}{3}(u_2 - u_1) p \frac{\partial g_0}{\partial p} + \frac{Q_0(p)}{s} = 0$$

The homogeneous equation associated with this is:

$$\left(\sigma_2 - u_1 A_1 - u_1\right)\tilde{g}_0 + \frac{1}{3}(u_2 - u_1)p\frac{\partial \tilde{g}_0}{\partial p} = 0$$

Which has the solution:

$$\tilde{g}_0 = \exp\left[\int_{p_0}^{p}\frac{dp'}{p'}\frac{3}{u_1 - u_2}(\sigma_2 - u_1 A_1 - u_1)\right] = \left(\frac{p}{p_0}\right)^{-\frac{3u_1}{u_1-u_2}}\exp\left[\int_{p_0}^{p}\frac{dp'}{p'}\frac{3}{u_1 - u_2}(\sigma_2 - u_1 A_1)\right]$$

The general solution of the equation has the form: $g_0 = \tilde{g}_0\lambda$
therefore the equation for l must be:

$$\frac{1}{3}(u_2 - u_1)p\frac{\partial \lambda}{\partial p}\tilde{g}_0 + \frac{Q_0}{s} = 0$$

which is readily solved:

$$\lambda = \int_{p_0}^{p}\frac{dp'}{p'}\frac{3}{u_1 - u_2}\frac{A}{s}\delta(p' - p_0)\left(\frac{p'}{p_0}\right)^{\frac{3u_1}{u_1-u_2}}\exp\left[-\int_{p_0}^{p'}\frac{dp"}{p"}\frac{3}{u_1 - u_2}(\sigma_2 - u_1 A_1)\right] =$$

$$= \frac{A}{s}\frac{3}{u_1 - u_2}$$

It follows that the solution of our equation is:

$$g_0 = \frac{3A}{s(u_1-u_2)}\left(\frac{p}{p_0}\right)^{-\frac{3u_1}{u_1-u_2}} \exp\left[\int_{p_0}^{p} \frac{dp'}{p'}\frac{3}{u_1-u_2}(\sigma_2 - u_1 A_1)\right]$$

and carrying out the Laplace inverse transform:

$$f_0(t,p) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} ds\ e^{ts} g_0(s,p)$$

Note that the pole in the equation for g_0 is at s=0 and it is obvious that in the limit of large times one has:

$$f_0(p) = \frac{3A}{s(u_1-u_2)}\left(\frac{p}{p_0}\right)^{-\frac{3u_1}{u_1-u_2}} \equiv K(p)$$

Therefore one can write:

$$f_0(p,t) = K(p)\exp\left[h(p,s)\right]$$

Let us introduce the function:

$$\tilde{f}(p,t) = K(p) \int_0^t dt' \phi(p,t') \quad with$$

$$\phi(p,t) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} ds \; e^{ts} \exp\left[h(p,s)\right]$$

Taking the Laplace Transform of this new function one has:

$$\int_0^\infty dt \; e^{-st} K(p) \int_0^t dt' \phi(p,t') = \frac{K(p)}{s} \exp\left[h(p,s)\right] = g_0(p,s)$$

This means that the solution of our problem is the spectrum K(p) and infinite time times a probability function that at time t one can have a particle with momentum p. Indeed one has that:

$$\int_0^\infty dt \; \phi(p,t') = 1$$

Namely the function is correctly normalized.

One can now use the obvious property that:

$$\int_0^\infty \mathrm{d}t\, \phi(p,t)\, \mathrm{e}^{-ts} = \exp[h(p,s)]$$

From which it follows that the average time to get particles with momentum p is:

$$\langle t \rangle = -\left[\frac{\partial h}{\partial s}\right]_{s=0}$$

It follows that:

$$t_{acc}(p) = \langle t \rangle = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \frac{dp'}{p'} \left[\frac{D_1(p')}{u_1} + \frac{D_2(p')}{u_2}\right]$$