Solutions to some exercises

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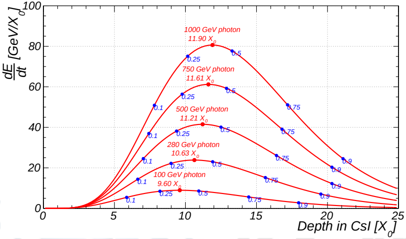
Quantity	Value
LAT side	1.50 m
LAT total thickness on axis	10.1 X_0
TKR layer side	36 cm
Spacing between two consecutive TKR layers	3.5 cm
TKR total depth on axis	1.5 X_0
TKR strip pitch	228 μm
CAL log dimensions (length $\times$ width $\times$ depth)	32.6 $\times$ 2.7 $\times$ 2.0 cm^3
CAL total depth on axis	8.6 X_0
CsI critical energy E_c	11.2 MeV
Earth radius	6400 km
Earth magnetic field at the equator	0.25 gauss

Exercise: What is the fraction of energy escaping out the back of the CAL for a 500 GeV photon on-axis?



The longitudinal shower profile can be parametrized with a gamma function:

$$\frac{dE}{dt} = \frac{E_0 b}{\Gamma(a)} (bt)^{(a-1)} e^{(-bt)} \tag{1}$$

($t = x/X_0$, $b \approx 0.5$ and a is given by $t_{\max} = (a-1)/b = \ln(E/E_c) + 0.5$, with $E_c \approx 11.2$ MeV in CsI).

The fraction of energy escaping for a 500 GeV photon on-axis is of the order of 70%.

Exercise: Estimate the high-energy on-axis effective area of the LAT.

The geometric area of the LAT is approximately:

$$A_{\rm geo} \approx 1.5 \times 1.5 = 2.25\ {\rm m}^2 \tag{2}$$

Not all the area is active, though (look at the gaps between CAL modules in one of the event displays to get an idea). We can give a rough estimate of the active (CAL) surface based on the length $L_{\rm log}$ of a CAL log:

$$A_{\rm active} \approx 16 \times L_{\rm log}^2 = 16 \times 0.326 \times 0.326 = 1.70\ {\rm m}^2 \tag{3}$$

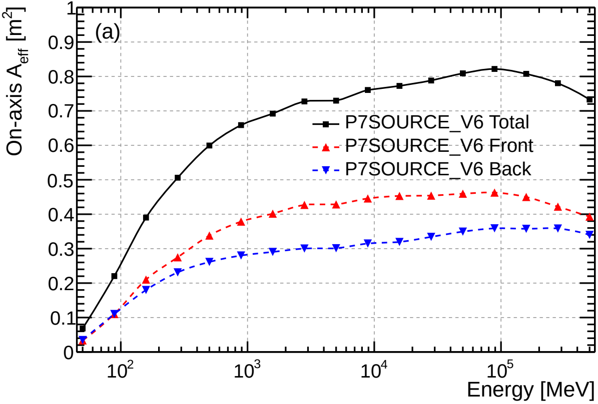
(16, here, is the number of tower modules in the LAT). The probability of converting in the TKR (which is 1.5 X_0 thick on axis) is:

$$\epsilon_{\rm conv} \approx 1 - e^{-\frac{7}{9}\times 1.5} = 0.69 \tag{4}$$

We'll assume 100% triggering and filtering efficiency ($\epsilon_{\rm trg} = \epsilon_{\rm obf} = 1$) and a 70% selection efficiency in the background rejection stage ($\epsilon_{\rm sel} = 0.7$):

$$A_{\rm eff} = A_{\rm active} \times \epsilon_{\rm conv} \times \epsilon_{\rm trg} \times \epsilon_{\rm obf} \times \epsilon_{\rm sel} = 0.82\ {\rm m}^2 \tag{5}$$

Compare this figure with the `P7SOURCE` on-axis effective area. Why does this discussion only apply to high energies?

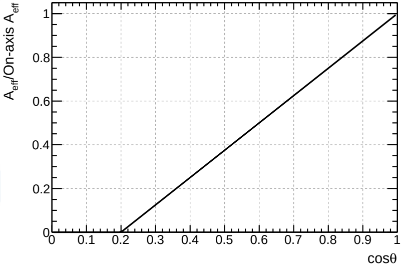

On-axis A_{eff} [m^2]
(a)
P7SOURCE_V6 Total
P7SOURCE_V6 Front
P7SOURCE_V6 Back
Energy [MeV]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10^2
10^3
10^4
10^5

Exercise: Estimate the high-energy acceptance of the LAT.

The maximum off-axis angle a particle can trigger the LAT is fixed by the three-in-a-row trigger condition and depends on the spacing $\Delta z \approx 3.5$ cm between two consecutive TKR layers and the side $s \approx 36$ cm of a TKR Si plane:

$$\theta_{\rm max} = \frac{2\Delta z}{s} \approx 79^\circ \qquad \cos\theta_{\rm max} \approx 0.2 \tag{6}$$

(it is not a coincidence that IRFs are tabulated up to $\cos\theta = 0.2$). Above that angle the effective area is zero. We can't calculate analytically the effective area as a function of θ, but we can *connect the dots*, as shown in figure:



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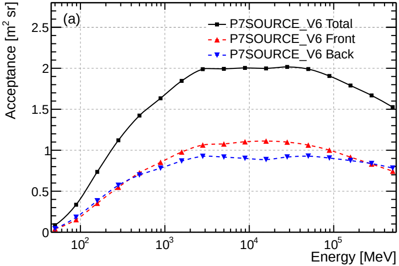
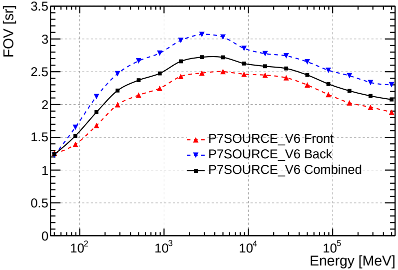
At this point (assuming $A_{\rm eff}$ does not depend on ϕ) the acceptance integral is trivial:

$$\mathcal{A} = \int A_{\rm eff}(\theta, \phi)\, d\Omega = 2\pi \int_{0.2}^{1} A_{\rm eff}(\theta)\, d(\cos\theta) =$$
$$= 2\pi A_{\rm eff}(\theta = 0) \frac{(1 - 0.2)}{2} \approx 2.08 \text{ m}^2 \text{ sr} \qquad (7)$$

Compare this figure with the `P7SOURCE` acceptance (again: this is only relevant at high energy).
The FoV is easy to calculate:

$$\text{FoV} = \frac{\mathcal{A}}{A_{\rm eff}(\theta = 0)} \approx 2.54 \text{ sr} \qquad (8)$$


(a)
Acceptance [m^2 sr]
Energy [MeV]
P7SOURCE_V6 Total
P7SOURCE_V6 Front
P7SOURCE_V6 Back


FOV [sr]
Energy [MeV]
P7SOURCE_V6 Front
P7SOURCE_V6 Back
P7SOURCE_V6 Combined

Exercise: Estimate the asymptotic high-energy PSF for front- and back-converting events. Why are they different?

(The following estimation is so rough that I'll happily give you a factor of 2/3 at each step. The main point is really to understand the orders of magnitude.)

The high-energy PSF is dictated by the strip pitch in the TKR ($p = 228\ \mu$m) and the spacing $\Delta z \approx 3.5$ cm between two consecutive TKR layers. The readout being digital, the hit resolution is

$$\sigma_x = \sigma_y = \frac{p}{\sqrt{12}} \approx 66\ \mu\text{m} \tag{9}$$

If we had to measure a slope (i.e. in the x–z or y–z plane) based on two measurements only, the error on theta, on axis, would be:

$$\sigma_{\theta_{xz}} = \sigma_{\theta_{yz}} = \frac{\sqrt{2}p}{\sqrt{12}\Delta z}. \tag{10}$$

For the space angle we need an extra $\sqrt{2}$:

$$\sigma_\theta = \frac{2p}{\sqrt{12}\Delta z} \tag{11}$$

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Though the track reconstruction is much more complicated than this, we shall assume that the error on the direction scales with the number of measurements s like $1/\sqrt{n}$ as in a least squares fit. For the front (back) section we can have anything between 7 and 18 (3 and 6) point, therefore we'll assume:

$$\langle n \rangle^{\rm Front} = \frac{18+7}{2} = 12.5 \quad \langle n \rangle^{\rm Back} = \frac{3+6}{2} = 4.5 \qquad (12)$$

Putting all together:

$$\sigma_\theta^{\rm Front} = \frac{2p}{\sqrt{12 \langle n \rangle^{\rm Front}} \Delta z} \approx 0.06^\circ$$
$$\sigma_\theta^{\rm Back} = \frac{2p}{\sqrt{12 \langle n \rangle^{\rm Back}} \Delta z} \approx 0.10^\circ \qquad (13)$$

Compare this with the Monte Carlo PSF. Though the hit resolution is the same, the PSF for the front section is better because we have more measurements.

Exercise: Estimate the rollover energy of the transition between the two regimes.

(This is even more crude than the previous two slides.)

We shall take the average angular errors for front and back we just calculated:

$$\sigma_\theta = \frac{\sigma_\theta^{\text{Front}} + \sigma_\theta^{\text{Back}}}{2} \approx 0.08^\circ \tag{14}$$

An electron (positron) traverses $\langle t \rangle = 1.5/2 \approx 0.75\ X_0$ on average. The corresponding multiple scattering angle is

$$\Delta\theta_{ms} \approx \sqrt{2}\frac{0.0136}{E\ [\text{GeV}]}\sqrt{\langle t \rangle}\ \text{rad} \tag{15}$$

therefore the transition between the two regimes happens at

$$\sigma_\theta \approx \Delta\theta_{ms} \tag{16}$$

or

$$E \approx \frac{0.0136\sqrt{2\langle t \rangle}}{\sigma_\theta} \approx 12\ \text{GeV} \tag{17}$$

(you might want to multiply by 2 since this is the electron or the positron, but at this point a factor of 2 is noise).

High-energy PSF