

TeV halo physics with HAWC -Theory and Observation

a probe for cosmic ray propagation

11th Fermi Symposium, College Park, MD

Matt Roth

Los Alamos National Laboratory
Intelligence and Space Research

LA-UR-24-29722

The High Altitude Water Cherenkov (HAWC) Observatory (in one slide)

Located in Mexico on the flanks of Sierra Negra next to Citlaltépetl/Pico de Orizaba at ~4100m

300 main tanks + 345 outriggers

4 PMTs in each main tank

1 PMT in each outrigger

2sr FOV, high duty cycle

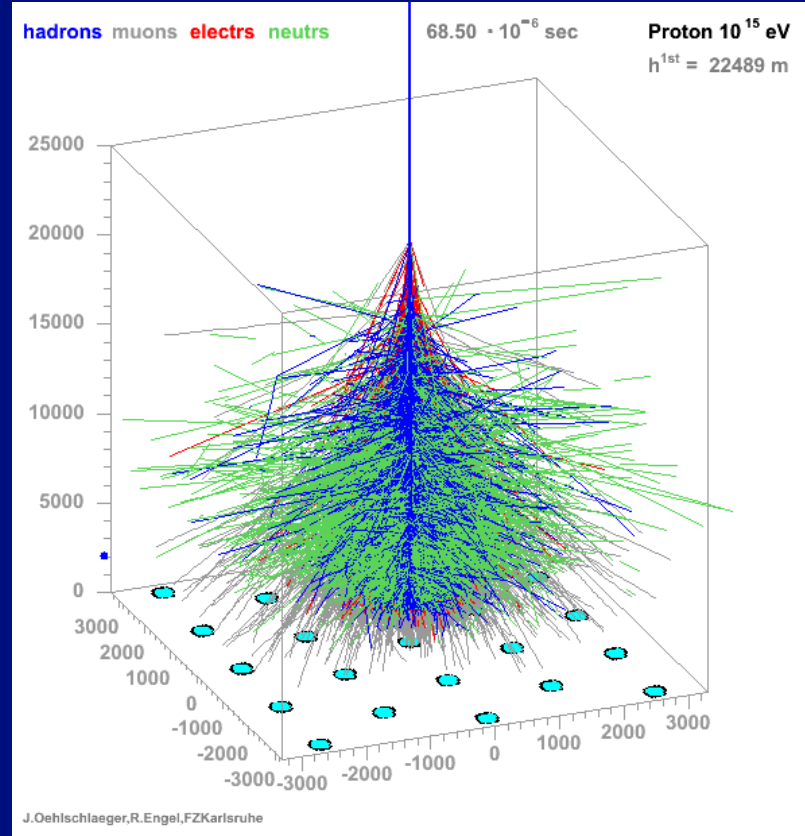
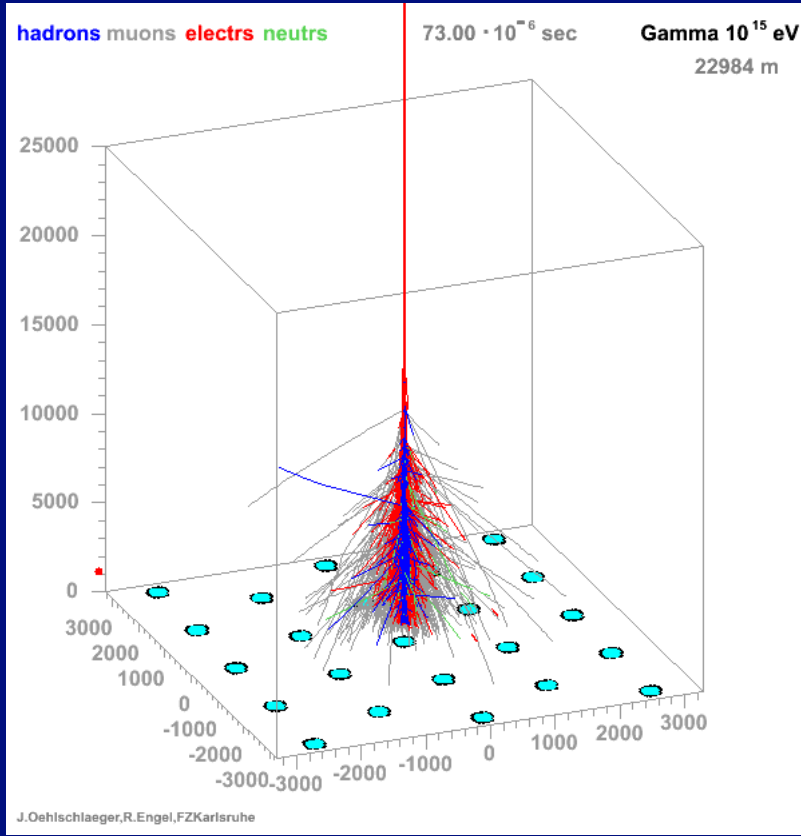
~300 GeV to > 100 TeV

How does it work?

- detect Cherenkov light from particles produced in extensive air showers
- shower geometry and morphology allow us to determine energy + direction + γ -hadron separation



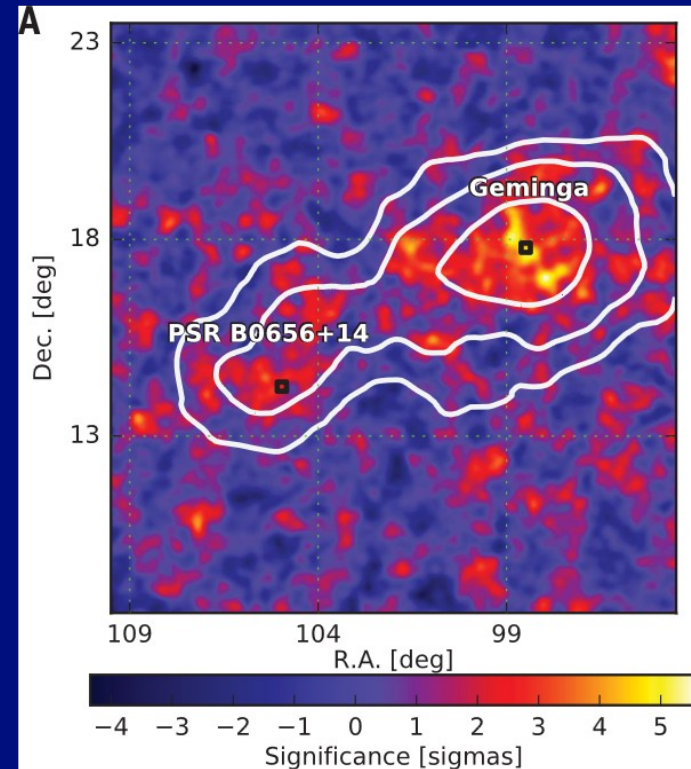
Extensive Air Showers



animations by J Oehlschläger and R. Engel <https://www.iap.kit.edu/corsika/71.php>

TeV Halos

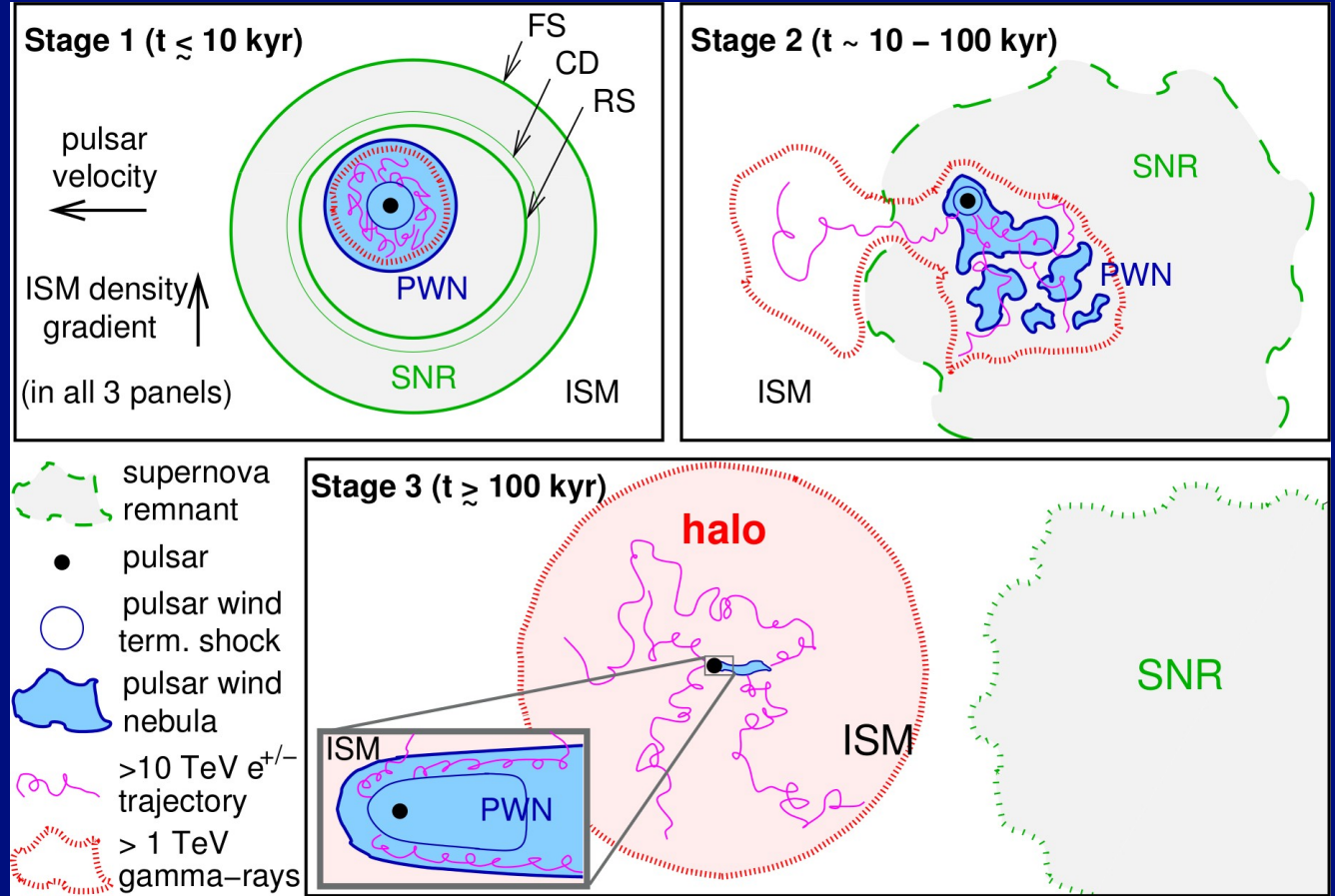
- source of extended O(10s pc) TeV emission
- middle- around aged pulsars O(100s kyr)
- Milagro (HAWCs predecessor) saw extended TeV emission around Geminga
- HAWC confirmed + more halos
- halos also observed by LHAASO, HESS
- large FOV to observe extended sources



HAWC collab., Science 358, 911-914 (2017)

TeV Halos

→ a simple model



TeV Halos and HAWC

Gaussian Width

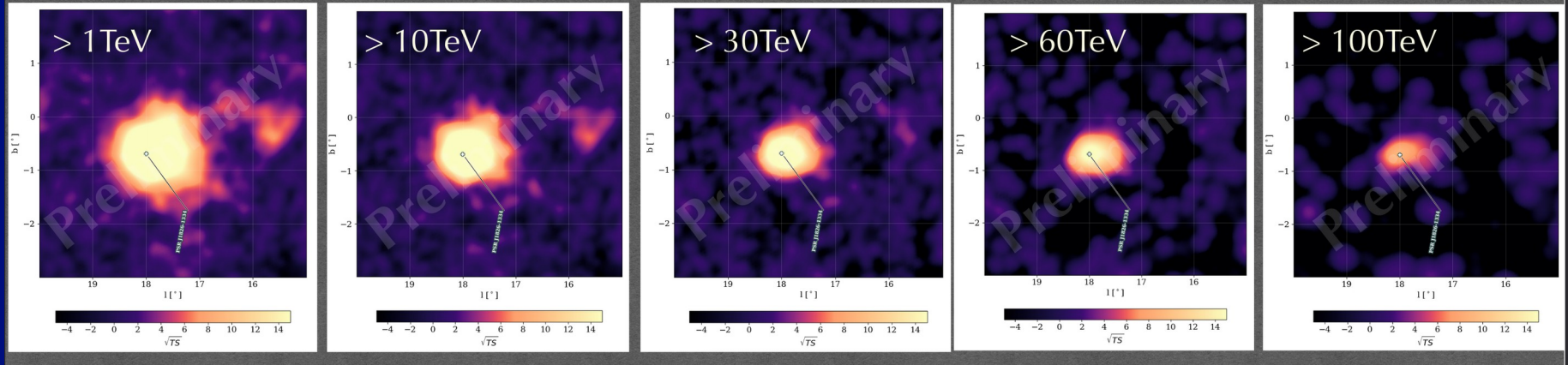
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J1825-134, a nascent TeV halo powered by PSR J1826-1334?
(Dezhi Huang's TeVPA 2024 talk)

TeV Halos – what’s the problem?

$$t_{\text{cool}} \sim 300 \text{ Myr} ([\text{GeV}]/E_{\text{CR}}) ([\text{eV cm}^{-3}]/u_{\text{B}} + u_{\text{rad}})$$

For the MW (roughly) $u_{\text{B}} \sim u_{\text{rad}} \sim 1 \text{ eV cm}^{-3}$

$$D \sim 3e27 \text{ cm}^2 \text{ s}^{-1} (E_{\text{CR}}/[\text{GeV}])^{0.5} \text{ (for a Kraichnan cascade)}$$

diffusion distance

$$L_{\text{D}} \sim (D t_{\text{cool}})^{0.5} \sim 1727 \text{ pc} (E_{\text{CR}}/[\text{GeV}])^{-0.25} (u_{\text{B}} + u_{\text{rad}}/[\text{eV cm}^{-3}])^{-0.5}$$

→ for $E_{\text{CR}} = 1 \text{ TeV}$ we get $L_{\text{D}} \sim 217 \text{ pc}$ and for $E_{\text{CR}} = 100 \text{ TeV}$ we have $L_{\text{D}} \sim 97 \text{ pc}$

(n.b. we set the effective $u_{\text{rad}} \sim 0$ as we are well into the Klein-Nishina regime)

Would not be able to see this!

TeV Halos – what's the problem?

Two possible solutions:

→ diffusion is suppressed close to source

Low required injection efficiency $O(\text{few } \%)$ of spin-down power.

Chose your scenario:

additional turbulence (Alfvén waves, HII regions), multi-zonal D , anisotropic D , convection, etc. → find a model to fit the data

(e.g. works by Profumo, Hooper, Evoli, Fang, Di Mauro,...)

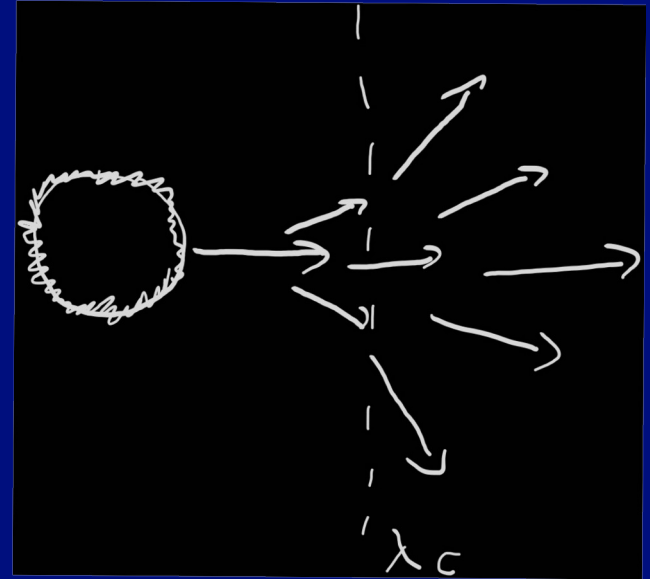
TeV Halos – what’s the problem?

Two possible solutions:

→ initial transport is not diffusive in nature but ballistic

More “realistic” model? (Recchia et al. 2021)
reproduces morphology
emission is beamed in the ballistic regime

However:
requires “excessive” injection efficiency close to and sometimes
well above available spin-down power → depends on injection
index



Modelling emission: How do we use TeV halo observations to constrain CR transport models?

CR injection by a central source (pick your application...)

- allow for ballistic and diffusive propagation
- explore different configurations of ISM conditions (n , B , ISRF) and diffusion coefficients
- use an appropriate model for beamed emission

(code solves as a function of time – enables study of transient phenomena in future)

Modelling emission

Solve the full kinetic equation:

$$\begin{aligned} \frac{\partial q_e}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial q_e}{\partial r} \right) \left(1 - e^{-(r/2\lambda_c)} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r q_e \right) e^{-(r/2\lambda_c)} \\ & - \frac{\partial}{\partial E} \left(\dot{E} q_e (E, r, t) \right) \\ & - q_e(E) \int_{m_e c^2}^E \frac{d\Gamma}{dk}(E, k) dk + \int_E^\infty q_e(k) \frac{d\Gamma}{dE}(k, E) dk + Q \end{aligned}$$

- obtain in-situ CR spectra
- calculate broadband emission SY/BS/IC

Full treatment of IC losses/emission

Preliminary results

Test case:

isotropic $B = 3 \mu\text{G}$

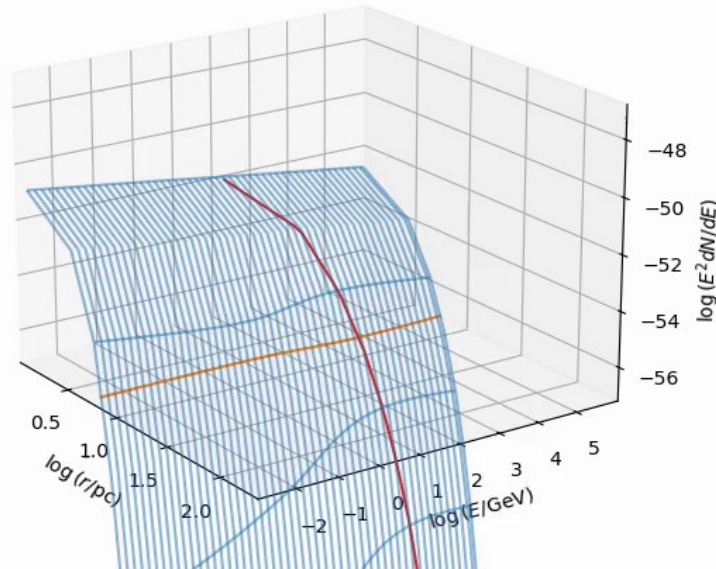
Draine MW ISRF

$D = 3.e27 (E/[\text{GeV}])^\delta$ with $\delta = 1/2$ (Kraichnan
turbulent cascade – $1/3$ for Kolmogorov)

injection normalised to unity

evolved for 2kyr

close to steady-state for $r \sim \lambda_c$

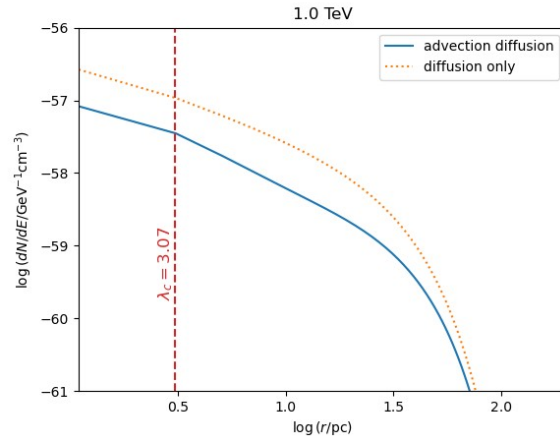
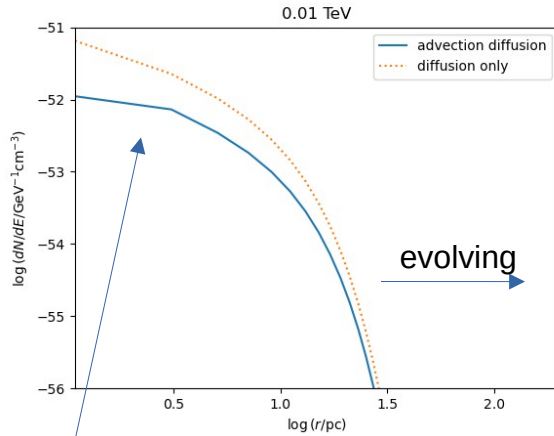


$r = \lambda_c$

xy-plane intersect (just for orientation)

Preliminary results

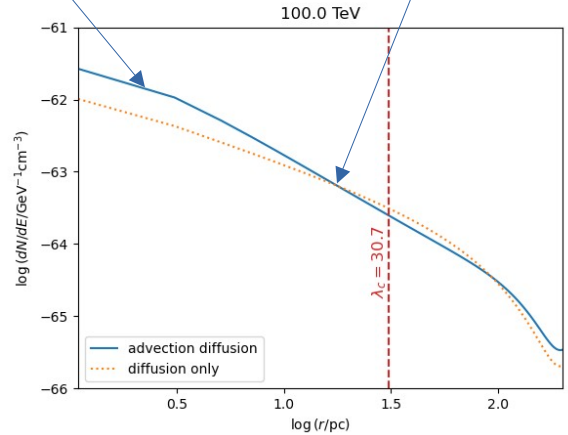
CR density at 10 GeV, 1 TeV, 100 TeV



transport “suppressed” for
adv/diff, emission beamed

ballistic-diffusive transition
marginally isotropised

highly beamed



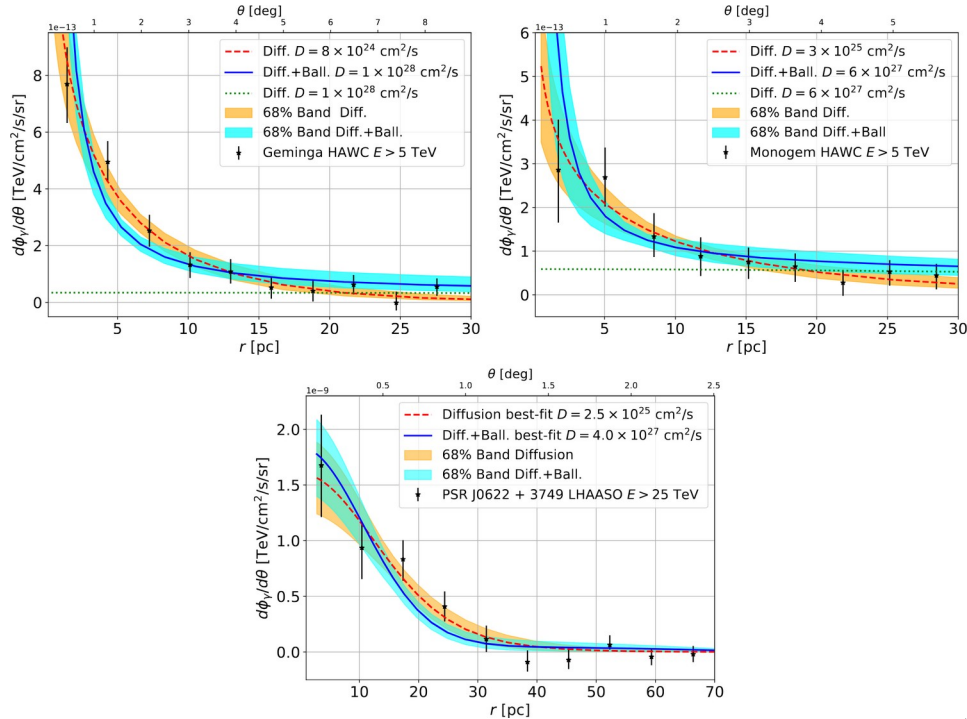
slow diffusion

Fang 2024: slow diff ~30-70pc around Geminga

beamed emission for $r < \approx \lambda_c$

Preliminary results:

No emission yet, stand by for more..... but:



Recchia+ 2021

→ achieve an ok fit, but suppressed diffusion models still appear to work better

→ efficiency:

Geminga	180-200%
Monogem	60-100%
PSR J0622+3749	40-100%

→ warrants further investigation as ball/diff transport is a physical process that needs to be considered in models

Applying models

- growing population of observed TeV halos and candidates, analyse extension as a function of energy
- broadband spectrum, e.g. X-ray constraints, Fermi
- constrain CR transport models/parameter space
- do we actually need to postulate “slow-diffusion”

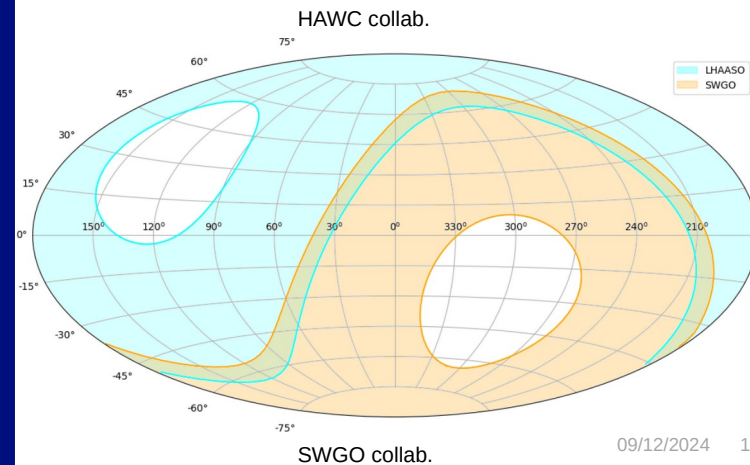
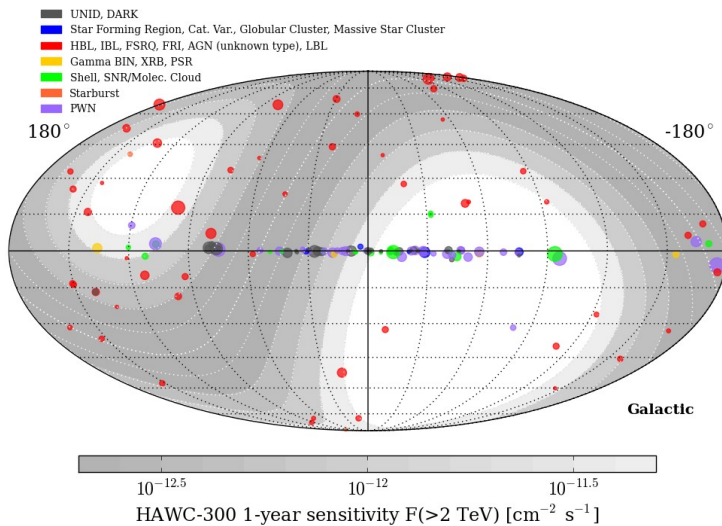
3HWC catalogue lists 12 TeV halo candidates, several observational studies in progress

LHAASO has seen 4+

TeVcat currently lists 8 TeV halo-like objects

The future

- HAWC is collecting data
- SWGO will open up a complementary view of the Galactic plane (to HAWC & LHAASO)
- CTA!



Summary

- strictly assuming MW-like diffusion around the source suggests TeV emission should be much more extended than the halos we observe, this motivates “slow” diffusion scenarios.
- comes down to solving particle transport (ballistic/transition to diffusive) and emission (beamed/isotropic) close to source, i.e. need more physics in our models
- need population studies: detections will be as important as non-detections
- new facilities (SWGGO/CTA) and more data (HAWC) will provide larger populations to study

Thanks for listening!

How do we do this

Solve the kinetic equation numerically

$$\begin{aligned} \frac{\partial q_e}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial q_e}{\partial r} \right) \left(1 - e^{-(r/2\lambda_c)} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r q_e \right) e^{-(r/2\lambda_c)} \\ & - \frac{\partial}{\partial E} \left(\dot{E} q_e (E, r, t) \right) \\ & - q_e(E) \int_{m_e c^2}^E \frac{d\Gamma}{dk}(E, k) dk + \int_E^\infty q_e(k) \frac{d\Gamma}{dE}(k, E) dk + Q \end{aligned}$$

The $d\Gamma/dE(E_{\text{initial}}, E_{\text{final}})$ functions are differential transition rates (bremsstrahlung/inverse Compton) from and initial energy E_{initial} to E_{final} . Transitions can be “catastrophic”.

\dot{E} encodes loss processes that are “smooth”, i.e. where on each interaction only a small fraction of the particle energy is lost (synchrotron, ionisation losses). Compare this to “drift”.

Numerical scheme

$$\begin{aligned}
 \frac{\partial \epsilon_i}{\partial t} = & \tilde{D} (1 - e^{-fx}) \frac{\partial^2 \epsilon_i}{\partial x^2} \\
 & + \left(\left(\frac{2}{x} \tilde{D} + \frac{\partial \tilde{D}}{\partial x} \right) (1 - e^{-fx}) - \tilde{v}_r e^{-fx} \right) \frac{\partial \epsilon_i}{\partial x} \\
 & + \left(\frac{\dot{E}}{E} \Big|_i - \frac{2}{x} \tilde{v}_r e^{-fx} \right) \epsilon_i \\
 & - \frac{\frac{\dot{E}}{E} \Big|_{i+1/2} \frac{E_{i+1/2}}{E_{i+1}} \epsilon_{i+1} - \frac{\dot{E}}{E} \Big|_{i-\frac{1}{2}} \frac{E_{i-1/2}}{E_i} \epsilon_i}{\Delta \ln E} \\
 & - \epsilon_i \left(\sum_{j=0}^i \int_{E_{i-1/2}}^{E_{i+1/2}} \frac{d \ln E}{\Delta \ln E} \int_{E_{j-\frac{1}{2}}}^{E_{j+1/2}} \frac{d\Gamma}{d \ln k}(E) d \ln k + \int_{m_e c^2}^{E_{0-1/2}} \frac{d\Gamma}{d \ln k}(E) d \ln k \right) \\
 & + \sum_{j=i}^n \epsilon_j \int_{E_{i-\frac{1}{2}}}^{E_{i+\frac{1}{2}}} \frac{d \ln E}{\Delta \ln E} \int_{E_{j-\frac{1}{2}}}^{E_{j+\frac{1}{2}}} \frac{E}{k} \frac{d\Gamma}{dE}(k) dk + E^2 Q
 \end{aligned}$$

Use a 8-stage 4th order strong stability preserving Runge-Kutta scheme SSPRK(8,

“Limitation”: Need to satisfy the CFL (Courant–Friedrichs–Lewy) condition for advection and the diffusive part of the PDE:

$$\text{CFL} \approx 1 > \Delta t (|u_x/\Delta x| + |u_E/\Delta E|)$$

and

$$\text{CFL} \approx 1 > \Delta t D_x/\Delta x^2$$

for our purposes this often implies hideously small time steps!



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2sr FOV, high duty cycle

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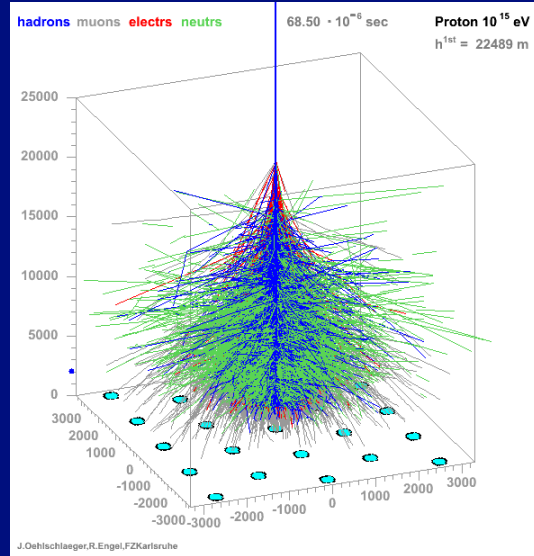
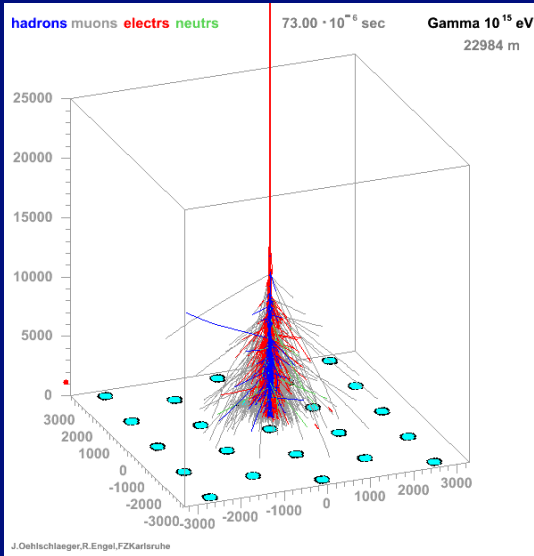
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Key takeaways is the large FOV and the high duty cycle.

Extensive Air Showers

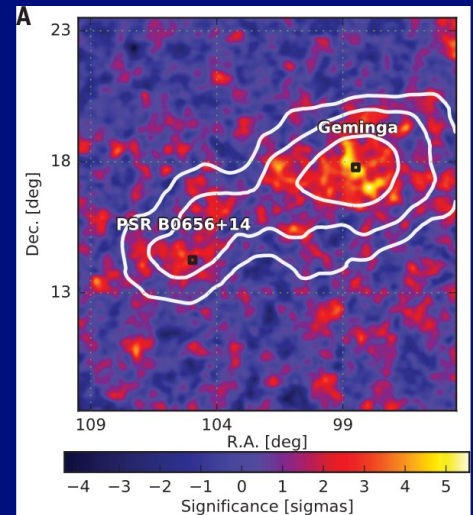


animations by J Oehlschläger and R. Engel <https://www.iap.kit.edu/corsika/71.php>

EAS morphology is quite different for hadrons and gamma-rays → we don't see neutrons (green in the proton shower), however hadrons produce substantially more muons, this allows us to distinguish between gamma-ray and hadronic showers.

TeV Halos

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middle- around aged pulsars O(100s kyr)
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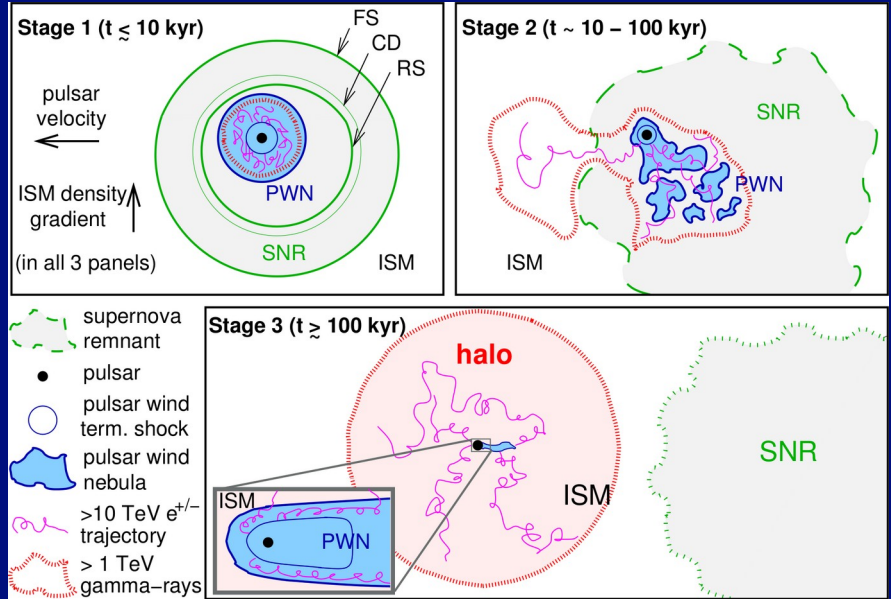


HAWC collab., Science 358, 911-914 (2017)

HAWC is uniquely positioned to observe TeV halos due its large field of view. We are continuously collecting data and several studies of potential TeV halos are in the works.

TeV Halos

→ a simple model



Classic picture of how TeV halos are believed to be formed. Escape due to the initial kick from the SN allows the NS to escape the SNR, this is important when considering models such as “slow diffusion” scenarios.

TeV Halos and HAWC

Gaussian Width

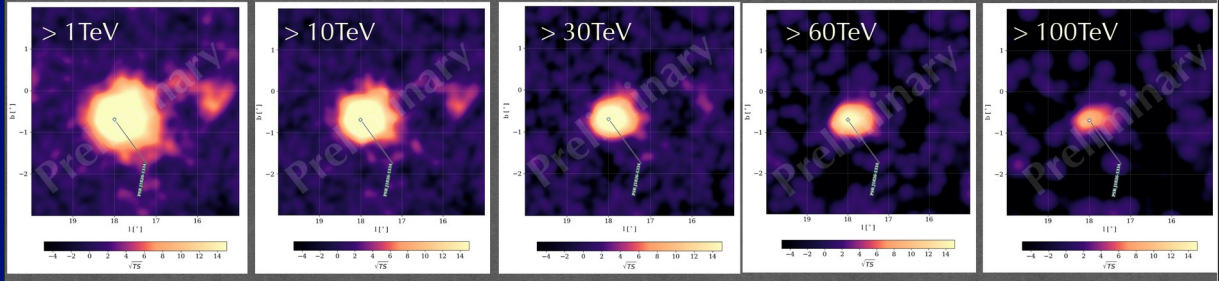
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J1825-134, a nascent TeV halo powered by PSR J1826-1334?
(Dezhi Huang's TeVPA 2024 talk)

This is an example of what is possible with HAWC data, note that this shows extension above a certain energy rather than in bins of energy. This is also a peculiar source as it is powered by a very young ($\sim 20\text{kyr}$) pulsar – remember TeV halos are usually observed around pulsars of age $O(\sim 100\text{kyr})$. This might be a TeV halo in its early stages.

TeV Halos – what’s the problem?

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(n.b. we set the effective $u_{\text{rad}} \sim 0$ as we are well into the Klein-Nishina regime)

Would not be able to see this!

Takeaway from this little back-of-the-envelope calculation: assuming purely diffusive transport, TeV emission would be much more spread out than we observe. The surface brightness of the emission would be so low that this would not be observable easily. So we need some process that “constrains” emission around the pulsar.

TeV Halos – what’s the problem?

Two possible solutions:

→ diffusion is suppressed close to source

Low required injection efficiency $O(\text{few } \%)$ of spin-down power.

Chose your scenario:

additional turbulence (Alfvén waves, HII regions), multi-zonal D, anisotropic D, convection, etc. → find a model to fit the data

(e.g. works by Profumo, Hooper, Evoli, Fang, Di Mauro,...)

One “simple” solution is to postulate suppressed diffusion given some physical process. E.g. streaming instability driven turbulence, left-over turbulence from the SNR, etc.

This can reproduce the morphology of sources such as Geminga.

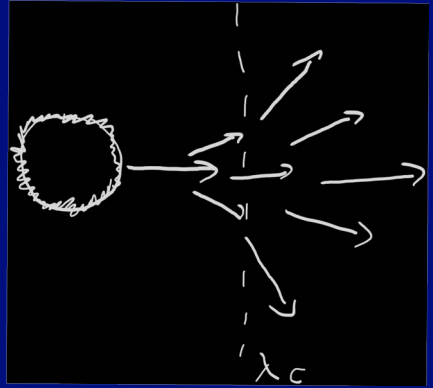
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More “realistic” model? (Recchia et al. 2021)
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However:
requires “excessive” injection efficiency close to and sometimes
well above available spin-down power → depends on injection
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Another process that will lead to an apparent ‘constraining’ of emission close to the source due to relativistic beaming is ballistic transport. This assumes that post escape from the source, CRs travel ballistically until they start to isotropise due to scattering. Scattering lengths can vary from very close to the source for low E and low D to several 10s of pc for high E, high D CRs.

Modelling emission: How do we use TeV halo observations to constrain CR transport models?

CR injection by a central source (pick your application...)

- allow for ballistic and diffusive propagation
- explore different configurations of ISM conditions (n , B , ISRF) and diffusion coefficients
- use an appropriate model for beamed emission

(code solves as a function of time – enables study of transient phenomena in future)

Key takeaway up to now: This warrants further study, so we create some detailed models that allow us to test various scenarios and include all relevant physics.

Modelling emission

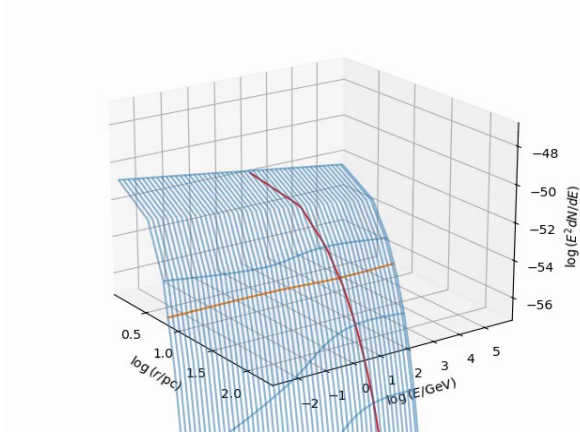
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- obtain in-situ CR spectra
- calculate broadband emission SY/BS/IC

Full treatment of IC losses/emission

Solve the kinetic equation in its full form, where we have introduced the transition terms between ballistic and diffusive transport and solve the integro-differential version to properly account for bremsstrahlung and inverse Compton losses.



$r = \lambda_c$

xy-plane intersect (just for orientation)

Preliminary results

Test case:

isotropic $B = 3 \mu\text{G}$

Draine MW ISRF

$D = 3.e27 (E/[\text{GeV}])^\delta$ with $\delta = 1/2$ (Kraichnan
turbulent cascade – $1/3$ for Kolmogorov)

injection normalised to unity

evolved for 2kyr

close to steady-state for $r \sim \lambda_c$

This is an example of a CR spectrum around a central source as a function of radius and energy. This is normalised to unity injection.

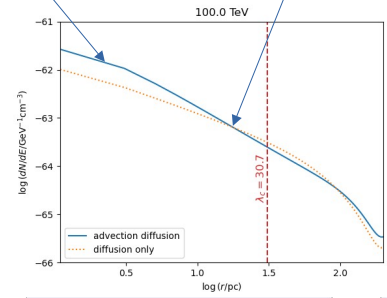
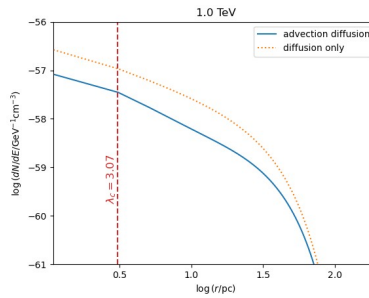
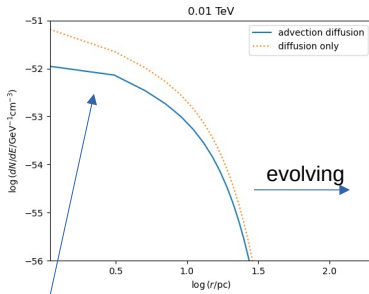
Preliminary results

CR density at 10 GeV, 1 TeV, 100 TeV

transport "suppressed" for adv/diff, emission beamed

ballistic-diffusive transition marginally isotropised

highly beamed



slow diffusion

beamed emission for $r < \approx \lambda_c$

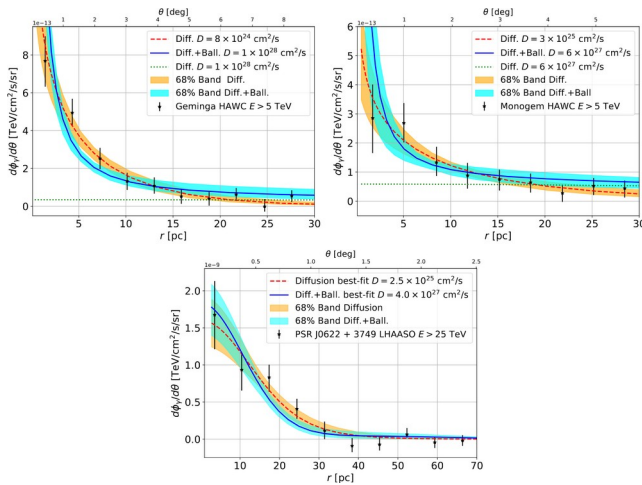
Fang 2024: slow diff $\sim 30\text{-}70\text{pc}$ around Geminga

Takeaway:

Can clearly see how diffusion is slower at low energies (hence we have higher steady state spectrum) but predicts an excessively fast transport above streaming at c at high energies. The part of the spectrum at and below $\sim \lambda_c$ is relativistically beamed towards the observer.

Preliminary results:

No emission yet, stand by for more..... but:



Recchia+ 2021

→ achieve an ok fit, but suppressed diffusion models still appear to work better

→ efficiency:

Geminga	180-200%
Monogem	60-100%
PSR J0622+3749	40-100%

→ warrants further investigation as ball/diff transport is a physical process that needs to be considered in models

The spectra from our code are still in the works, however Recchia et al. have shown that they can achieve decent fits to the data of the three halos studied in their work.

Key takeaway: we should study this on a larger population as ballistic transport is clearly something that needs to be considered in models. i.e. it is unphysical to assume that the source injects an isotropised distribution of CRs.

Applying models

→ growing population of observed TeV halos and candidates, analyse extension as a function of energy

→ broadband spectrum, e.g. X-ray constraints, Fermi

→ constrain CR transport models/parameter space

→ do we actually need to postulate “slow-diffusion”

3HWC catalogue lists 12 TeV halo candidates, several observational studies in progress

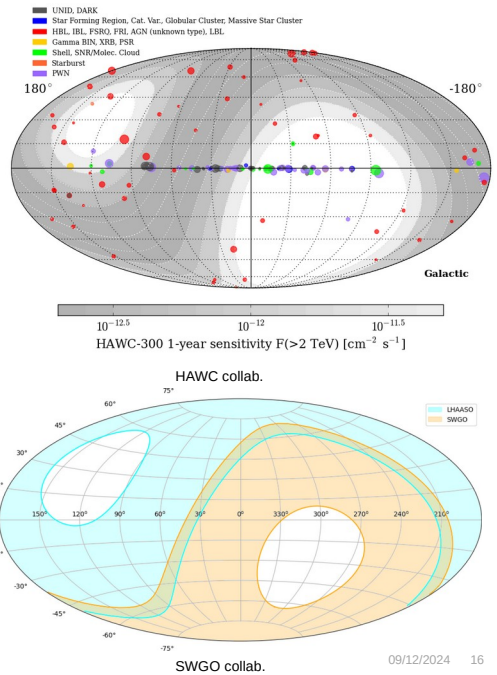
LHAASO has seen 4+

TeVcat currently lists 8 TeV halo-like objects

Takeaway: We have plenty of sources to study and observations from other instruments, such as Fermi and X-ray observatories will provide constraints for the models and will allow us to specifically test certain theories (e.g. certain parameters for a streaming instability damped by non-linear Landau damping predict TeV halos that are not observable at GeV energies due to fast diffusion at low E).

The future

- HAWC is collecting data
- SWGO will open up a complementary view of the Galactic plane (to HAWC & LHAASO)
- CTA!



SWGO in particular will be important for studying large extended sources in the high-energy sky and will open up a different part of the Galactic plane at greater sensitivity than HAWC. There is bound to be a plethora of new sources to be discovered. It is worth funding!

Summary

- strictly assuming MW-like diffusion around the source suggests TeV emission should be much more extended than the halos we observe, this motivates “slow” diffusion scenarios.
- comes down to solving particle transport (ballistic/transition to diffusive) and emission (beamed/isotropic) close to source, i.e. need more physics in our models
- need population studies: detections will be as important as non-detections
- new facilities (SWGOC/CTA) and more data (HAWC) will provide larger populations to study

Thanks for listening!

You can contact me on Slack or by email at mattroth@lanl.gov with any questions. Thanks!

How do we do this

Solve the kinetic equation numerically

$$\begin{aligned} \frac{\partial q_e}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial q_e}{\partial r} \right) \left(1 - e^{-(r/2\lambda_c)} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r q_e \right) e^{-(r/2\lambda_c)} \\ & - \frac{\partial}{\partial E} \left(\dot{E} q_e(E, r, t) \right) \\ & - q_e(E) \int_{m_e c^2}^E \frac{d\Gamma}{dk}(E, k) dk + \int_E^\infty q_e(k) \frac{d\Gamma}{dE}(k, E) dk + Q \end{aligned}$$

The $d\Gamma/dE(E_{\text{initial}}, E_{\text{final}})$ functions are differential transition rates (bremsstrahlung/inverse Compton) from and initial energy E_{initial} to E_{final} . Transitions can be “catastrophic”.

\dot{E} encodes loss processes that are “smooth”, i.e. where on each interaction only a small fraction of the particle energy is lost (synchrotron, ionisation losses). Compare this to “drift”.

Numerical scheme

$$\begin{aligned} \frac{\partial \epsilon_i}{\partial t} = & \tilde{D} (1 - e^{-fx}) \frac{\partial^2 \epsilon_i}{\partial x^2} \\ & + \left(\left(\frac{2}{x} \tilde{D} + \frac{\partial \tilde{D}}{\partial x} \right) (1 - e^{-fx}) - \tilde{v}_r e^{-fx} \right) \frac{\partial \epsilon_i}{\partial x} \\ & + \left(\frac{\dot{E}}{E} \Big|_i - \frac{2}{x} \tilde{v}_r e^{-fx} \right) \epsilon_i \\ & - \frac{\frac{\dot{E}}{E} \Big|_{i+1/2} \frac{E_{i+1/2}}{E_{i+1}} \epsilon_{i+1} - \frac{\dot{E}}{E} \Big|_{i-1/2} \frac{E_{i-1/2}}{E_i} \epsilon_i}{\Delta \ln E} \\ & - \epsilon_i \left(\sum_{j=0}^i \int_{E_{i-1/2}}^{E_{i+1/2}} \frac{d \ln E}{\Delta \ln E} \int_{E_{j-1/2}}^{E_{j+1/2}} \frac{d \Gamma}{d \ln k}(E) d \ln k + \int_{m_e c^2}^{E_{0-1/2}} \frac{d \Gamma}{d \ln k}(E) d \ln k \right) \\ & + \sum_{j=i}^n \epsilon_j \int_{E_{i-1/2}}^{E_{i+1/2}} \frac{d \ln E}{\Delta \ln E} \int_{E_{j-1/2}}^{E_{j+1/2}} \frac{E}{k} \frac{d \Gamma}{d E}(k) dk + E^2 Q \end{aligned}$$

Use a 8-stage 4th order strong stability preserving Runge-Kutta scheme SSPRK(8,

“Limitation”: Need to satisfy the CFL (Courant–Friedrichs–Lewy) condition for advection and the diffusive part of the PDE:

$$\text{CFL} \approx 1 > \Delta t (|u_x/\Delta x| + |u_{\bar{r}}/\Delta E|)$$

and

$$\text{CFL} \approx 1 > \Delta t D_x/\Delta x^2$$

for our purposes this often implies hideously small time steps!