The retarded multipolar magnetic field of MSP J0030+0451

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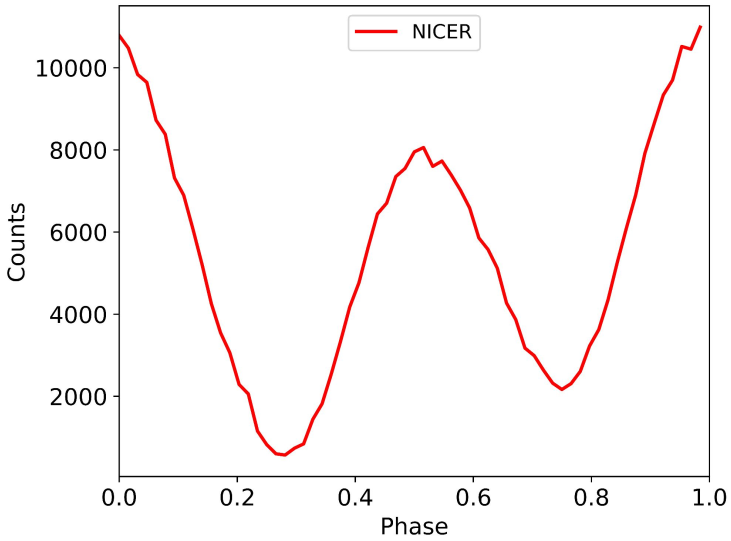
Introduction: PSR J0030+0451

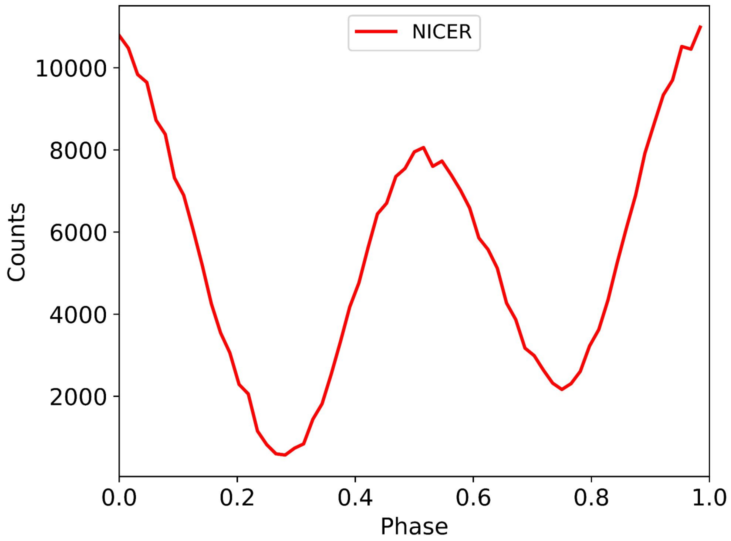
- Isolated MSP, spin period 4.865 ms
- Thermal X-ray light curve (LC) of J0030: two peaks
- Possible origin: hotspots on surface



Bogdanov et al. 2019

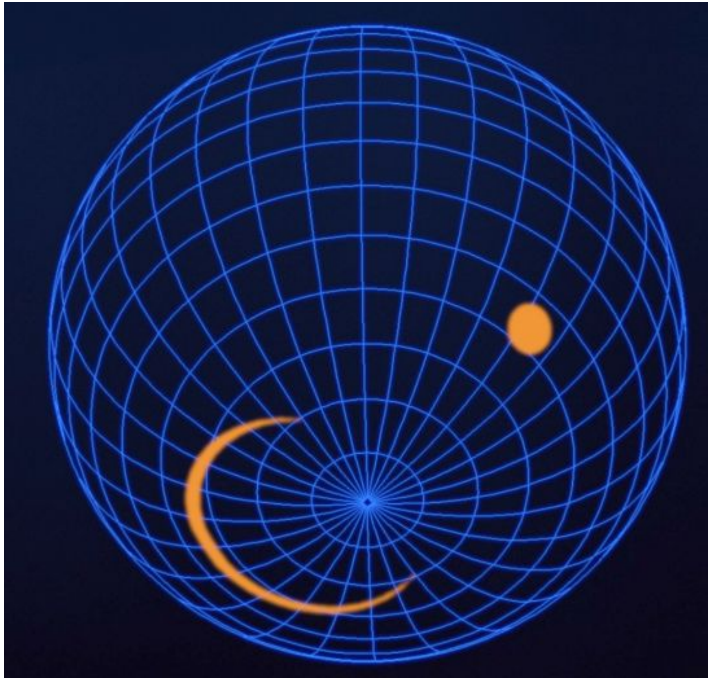
Introduction: PSR J0030+0451

- Isolated MSP, spin period 4.865 ms
- Thermal X-ray light curve (LC) of J0030: two peaks
- Possible origin: hotspots on surface
- Model the hotspots
- Constrain the neutron star Equation of State (mass and radius)

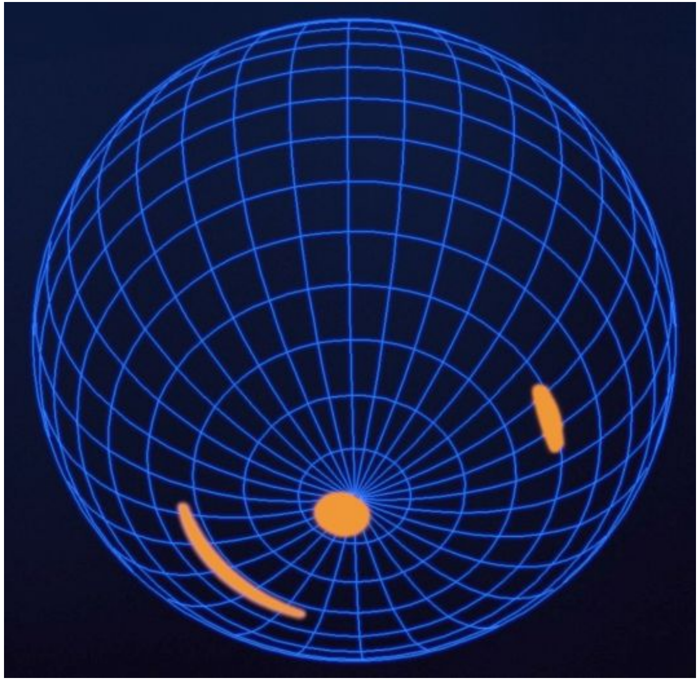


Bogdanov et al. 2019

NICER view of J0030



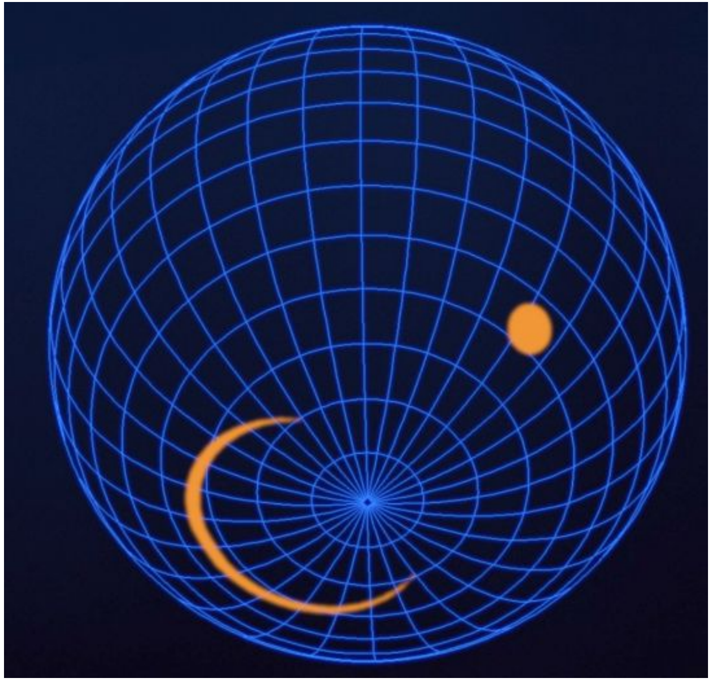
Riley et al. 2019



Miller et al. 2019

Credit: NASA GFSC

NICER view of J0030



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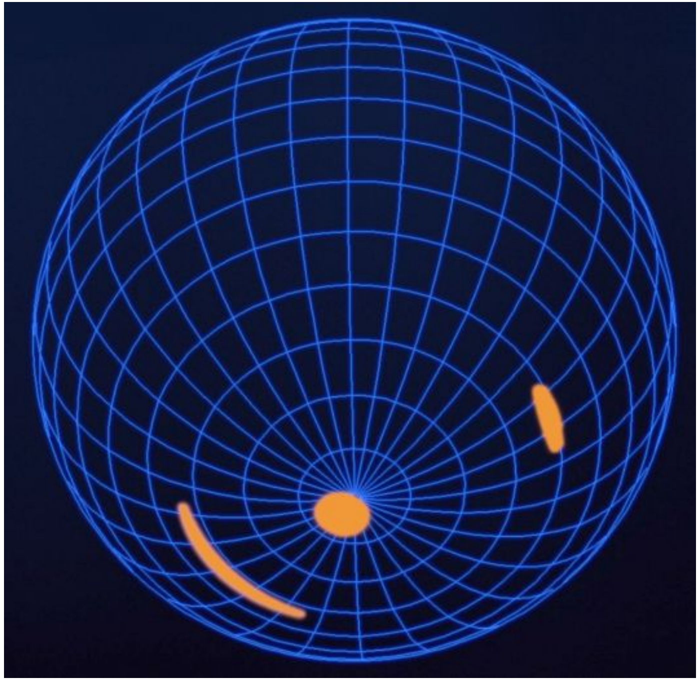
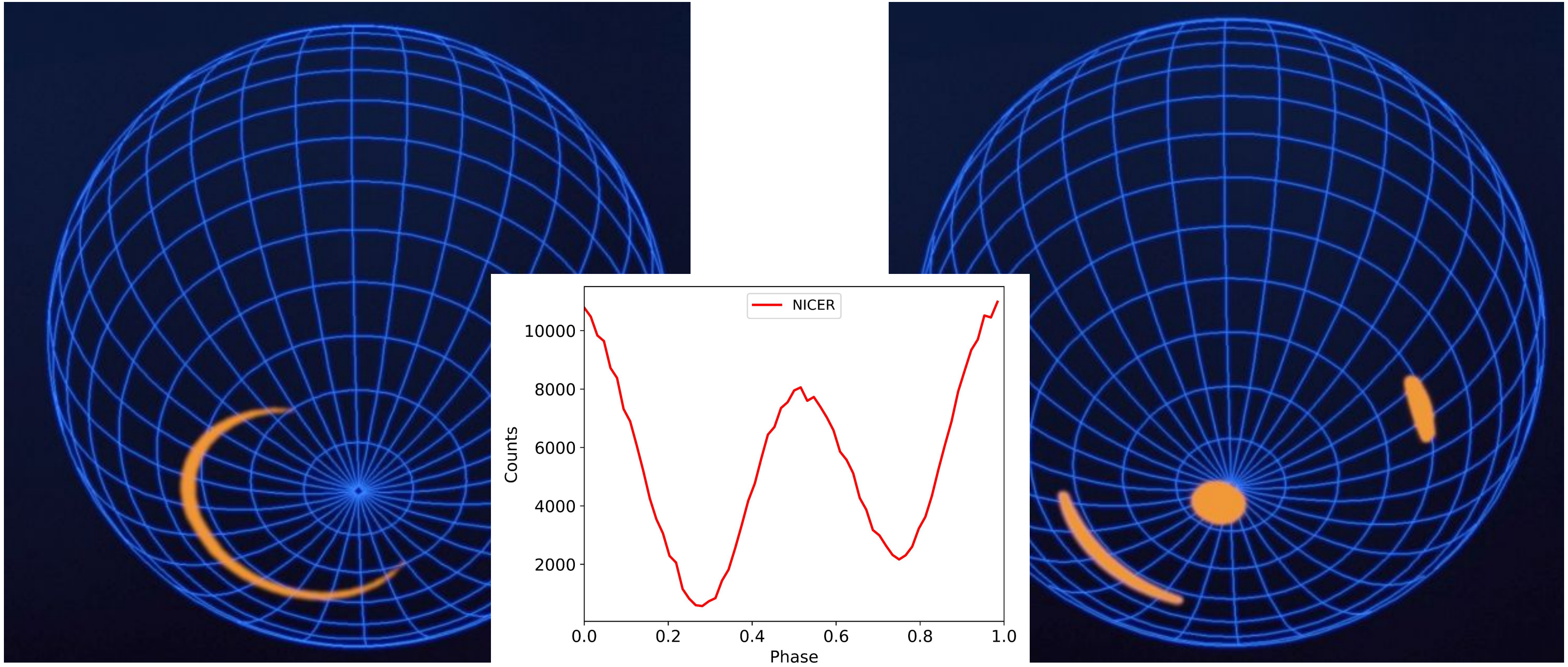


Miller et al. 2019

Hotspots: Two or three, oval, arc-shaped.
Same hemisphere.

Credit: NASA GFSC

NICER view of J0030



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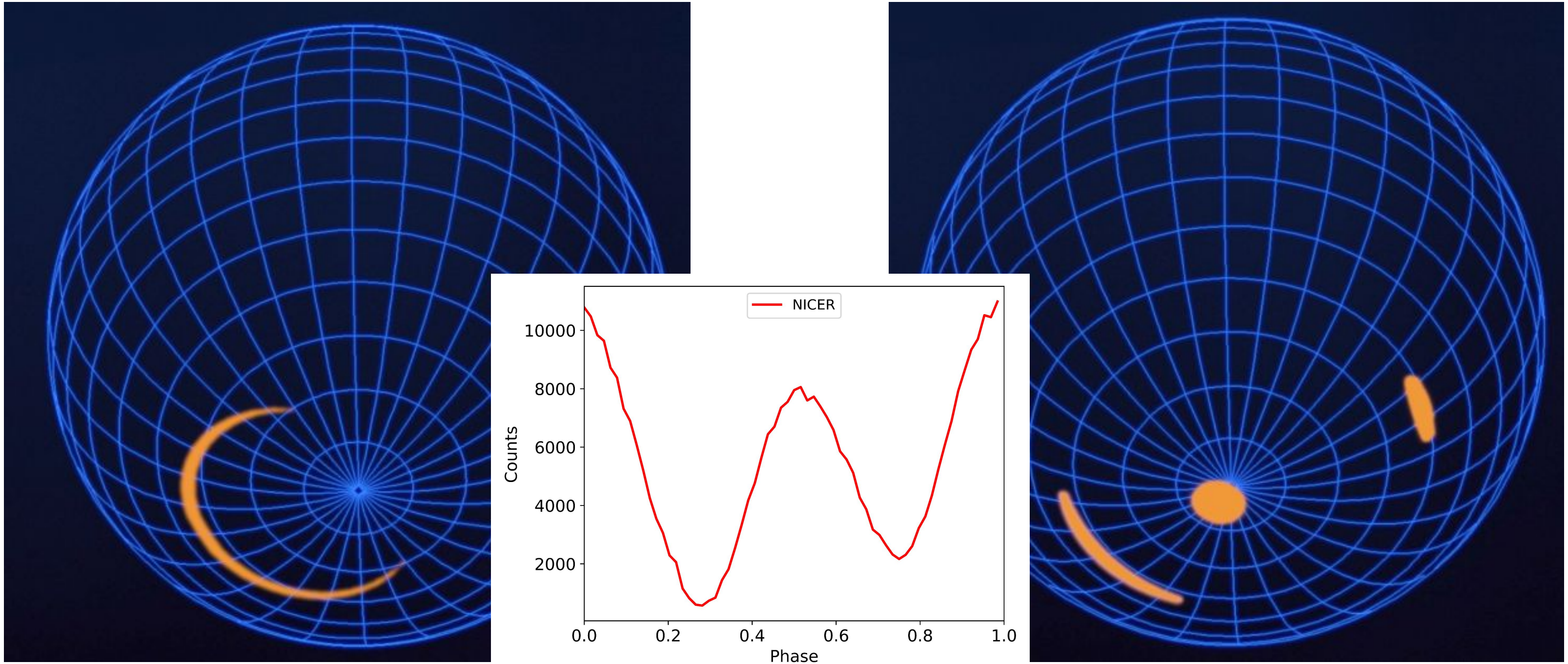
It has to be a non-dipolar magnetic field!

Credit: NASA GFSC

NICER view of J0030



Riley et al. 2019

Miller et al. 2019

It has to be a non-dipolar magnetic field!

Non-physical and computationally heavy.

Credit: NASA GFSC

Magnetic field of J0030

- Implemented static vacuum magnetic field

	Kalapotharakos et al. 2021
l = 1: Dipole l = 2: Quadrupole	l1 + l2
m = - l to l (Sub components)	m = 0
Centered: Rotation axis and magnetic axis centred at physical centre of the star	Offset
Static: non-rotating star	Static

Magnetic field of J0030

- Implemented retarded vacuum multipolar magnetic field

	Kalapotharakos et al. 2021	Current study
l = 1: Dipole l = 2: Quadrupole l = 3: Octopole	l1 + l2	l1 + l2 + l3
m = - l to l (Sub components)	m = 0	m = -l to l
Centered: Rotation axis and magnetic axis centred at physical centre of the star	Offset	Centered
Static: non-rotating star Retarded: rotating star	Static	Retarded

Pétri 2015

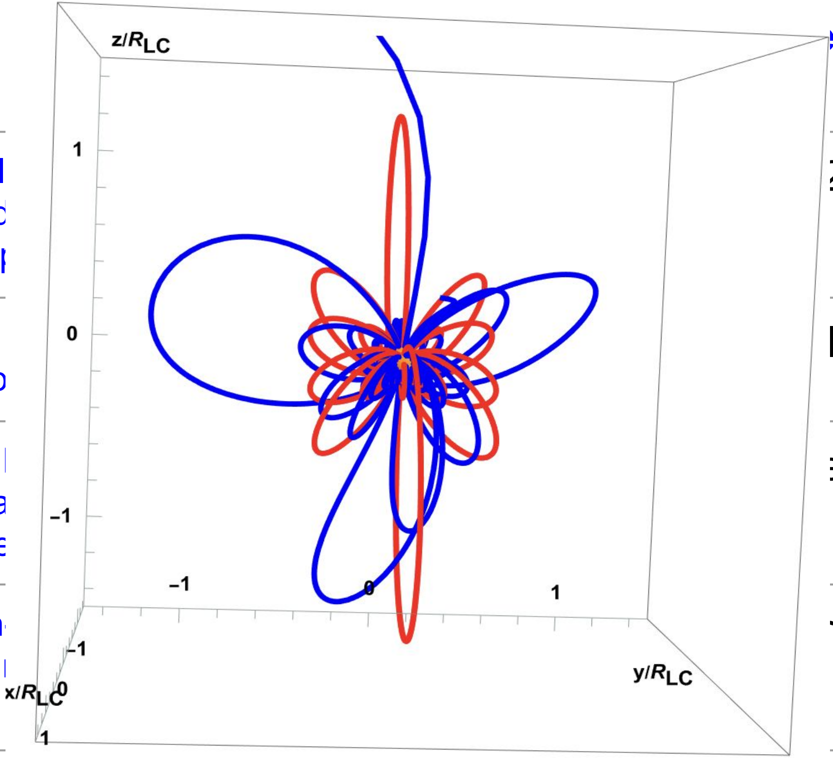
Magnetic field of J0030

Static dipole

Retarded dipole

- Implemented retarded vacuum multipolar magnetic field

	›nt study
l = 1: Dipol l = 2: Quac l = 3: Octo|	2 + l3
m = - l to l (Sub comp	l to l
Centered: l magnetic a physical ce	›ered
Static: non Retarded:	›ded



Pétri 2015

Retarded vacuum multipolar magnetic field

Multipolar magnetic field outside neutron star

$$\boldsymbol{B}(r,\theta,\phi,t) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}[\boldsymbol{B_r}(r,\theta,\phi,t) + \boldsymbol{B_\theta}(r,\theta,\phi,t) + \boldsymbol{B_\phi}(r,\theta,\phi,t)]$$

l = 1: Dipole
l = 2: Quadrupole
l = 3: Octopole
...
m = - l to l : different orientations for the corresponding multipolar component

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l = 1: Dipole
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m = - l to l : different orientations for the corresponding multipolar component

$$\boldsymbol{B_r} = -\frac{\sqrt{l(l+1)}}{r} f_{lm}^{B} Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$\boldsymbol{B_\theta} = -\frac{\partial_r (r f_{lm}^{B})}{r\sqrt{l(l+1)}} \partial_\theta (Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t} + \frac{i\mu_0 m\Omega f_{lm}^{D}}{\sin\theta\sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$\boldsymbol{B_\phi} = -\frac{\partial_r (r f_{lm}^{B})}{r\sin\theta\sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t} - \frac{i\mu_0 m\Omega f_{lm}^{D}}{\sqrt{l(l+1)}} \partial_\theta (Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t}$$

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Terms with r dependence

$$\boldsymbol{B_r} = -\frac{\sqrt{l(l+1)}}{\boxed{r}}\boxed{f^B_{lm}}Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}$$

$$\boldsymbol{B_\theta} = -\frac{\boxed{\partial_r(rf^B_{lm})}}{\boxed{r}\sqrt{l(l+1)}}\partial_\theta(Y_{lm}(\theta))e^{im\phi}e^{-im\Omega t} + \frac{i\mu_0 m\Omega \boxed{f^D_{lm}}}{\sin\theta\sqrt{l(l+1)}}imY_{lm}(\theta)e^{im\phi}e^{-im\Omega t}$$

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Retarded vacuum multipolar magnetic field

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$$\boldsymbol{B}(r,\theta,\phi,t) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\left[\boldsymbol{B_r}(r,\theta,\phi,t) + \boldsymbol{B_\theta}(r,\theta,\phi,t) + \boldsymbol{B_\phi}(r,\theta,\phi,t)\right]$$

Terms with r, (ϑ, φ) dependence

$$\boldsymbol{B_r} = -\frac{\sqrt{l(l+1)}}{\boxed{r}}\boxed{f_{lm}^B}\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}}$$

$$\boldsymbol{B_\theta} = -\frac{\boxed{\partial_r(rf_{lm}^B)}}{\boxed{r}\sqrt{l(l+1)}}\boxed{\partial_\theta(Y_{lm}(\theta))e^{im\phi}e^{-im\Omega t}} + \frac{i\mu_0 m\Omega \boxed{f_{lm}^D}}{\boxed{\sin\theta}\sqrt{l(l+1)}}im\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}}$$

$$\boldsymbol{B_\phi} = -\frac{\boxed{\partial_r(rf_{lm}^B)}}{\boxed{r}\boxed{\sin\theta}\sqrt{l(l+1)}}im\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}} - \frac{i\mu_0 m\Omega \boxed{f_{lm}^D}}{\sqrt{l(l+1)}}\boxed{\partial_\theta(Y_{lm}(\theta))e^{im\phi}e^{-im\Omega t}}$$

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Retarded vacuum multipolar magnetic field

Multipolar magnetic field outside neutron star

$$\boldsymbol{B}(r,\theta,\phi,t)=\sum_{l=1}^{\infty}\sum_{m=-l}^{l}\left[\boldsymbol{B_r}(r,\theta,\phi,t)+\boldsymbol{B_\theta}(r,\theta,\phi,t)+\boldsymbol{B_\phi}(r,\theta,\phi,t)\right]$$

l = 1: Dipole
l = 2: Quadrupole
l = 3: Octopole
...
m = - l to l : different orientations for the corresponding multipolar component

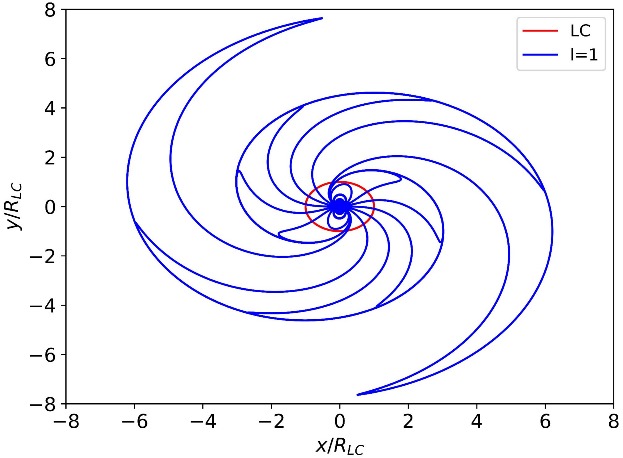
Terms with r, (ϑ, φ), (l,m) dependence

$$\boldsymbol{B_r}=-\frac{\sqrt{l(l+1)}}{\boxed{r}}\boxed{f_{lm}^{B}}\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}}$$

$$\boldsymbol{B_\theta}=-\frac{\boxed{\partial_r(rf_{lm}^{B})}}{\boxed{r}\sqrt{l(l+1)}}\boxed{\partial_\theta(Y_{lm}(\theta))e^{im\phi}e^{-im\Omega t}}+\frac{i\mu_0 m\Omega \boxed{f_{lm}^{D}}}{\boxed{\sin\theta}\sqrt{l(l+1)}}im\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}}$$

$$\boldsymbol{B_\phi}=-\frac{\boxed{\partial_r(rf_{lm}^{B})}}{\boxed{r}\,\boxed{\sin\theta}\sqrt{l(l+1)}}im\boxed{Y_{lm}(\theta)e^{im\phi}e^{-im\Omega t}}-\frac{i\mu_0 m\Omega \boxed{f_{lm}^{D}}}{\sqrt{l(l+1)}}\boxed{\partial_\theta(Y_{lm}(\theta))e^{im\phi}e^{-im\Omega t}}$$

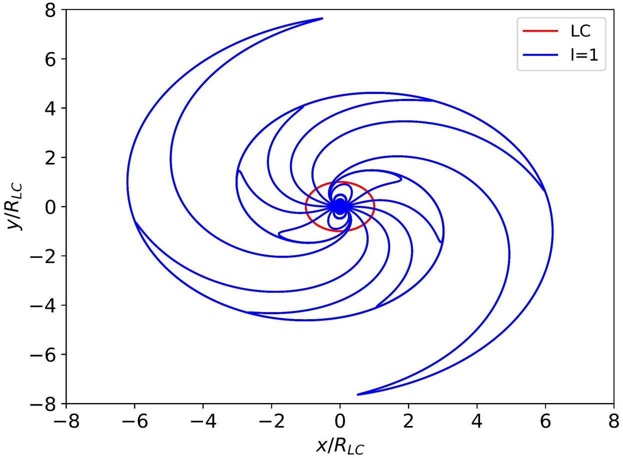
Multipolar magnetic field: field lines

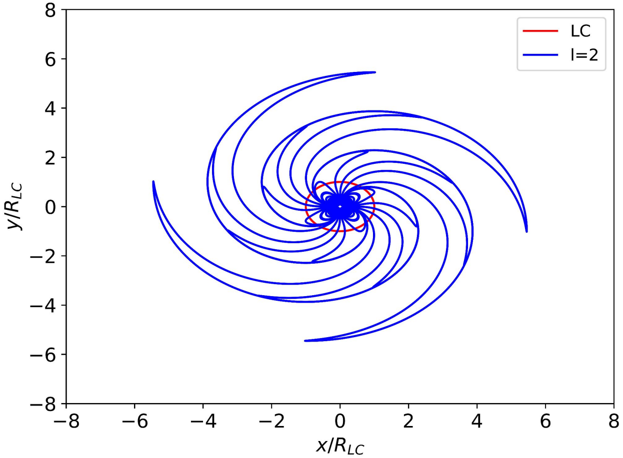


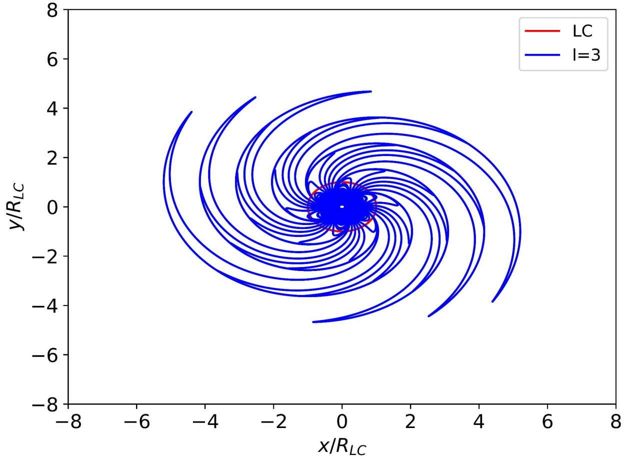
Spiral arms, a feature of equatorial lines

LC stands for ‘Light Cylinder’ in this slide

Multipolar magnetic field: field lines







Spiral arms, a feature of equatorial lines

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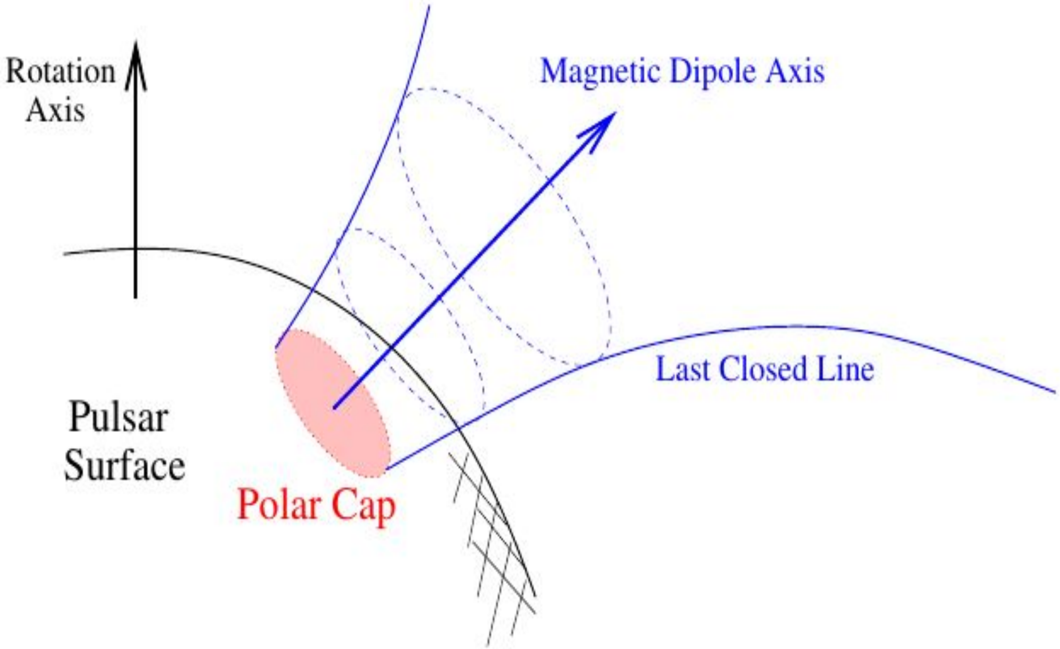
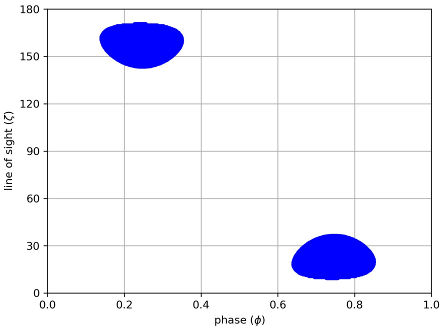
Multipolar magnetic field: polar caps



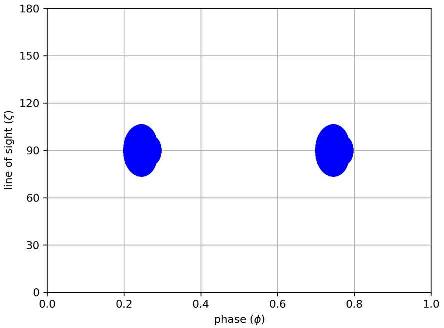
Saito 2011

Multipolar magnetic field: polar caps

$\chi_D = 30°$

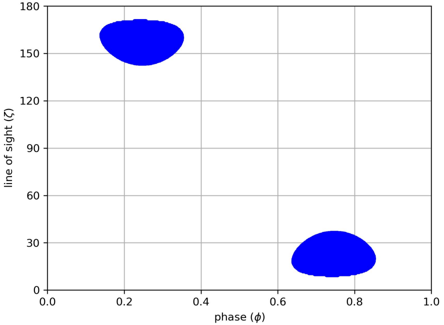


$\chi_D = 90°$

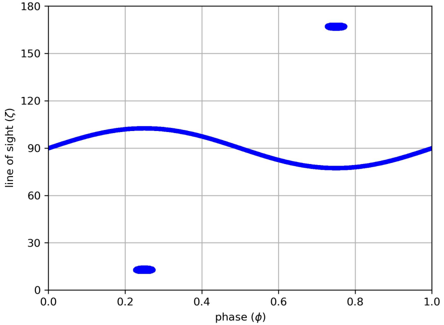


Multipolar magnetic field: polar caps

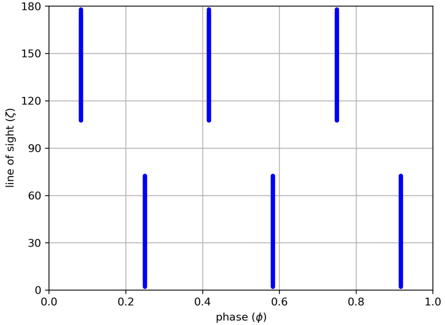
$\chi_D = 30°$



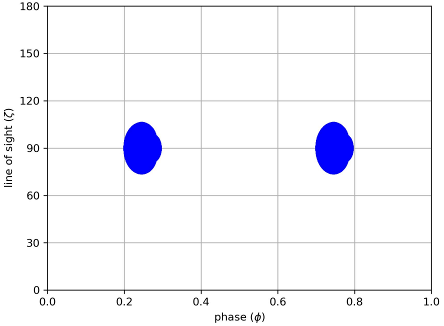
$\chi_{Q1}, \chi_{Q2} = 30°, 0°$



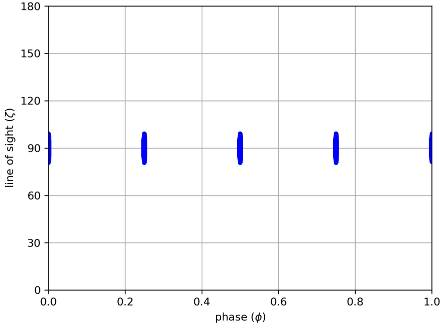
$\chi_{O1}, \chi_{O2}, \chi_{O3} = 60°, 90°, 90°$



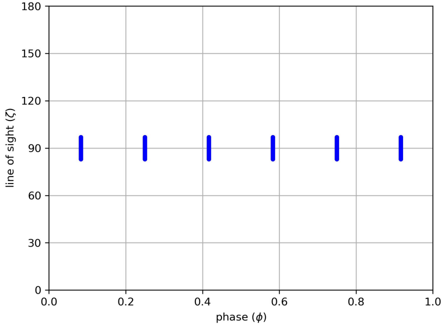
$\chi_D = 90°$



$\chi_{Q1}, \chi_{Q2} = 90°, 90°$



$\chi_{O1}, \chi_{O2}, \chi_{O3} = 90°, 90°, 90°$



Methods

- Markov chain Monte Carlo (MCMC) code (Kalapotharakos et al. 2021):
  - Input M, R, photon trajectories (ray-tracing code)

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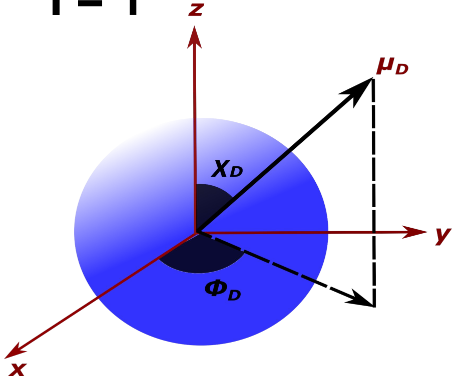
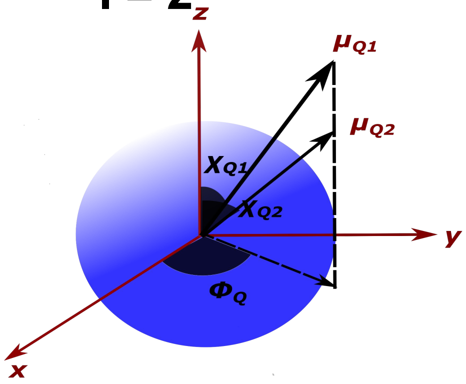
  - Input M, R, photon trajectories (ray-tracing code)
  - Module for retarded multipolar magnetic field: parameter space
    - Phase - 3 parameters: $\varphi_{D,}\ \varphi_{Q,}\ \varphi_{O}$
      Angular - 6 parameters: $\chi_{D,}\chi_{Q1,}\chi_{Q1,}\chi_{O1,}\chi_{O2,}\chi_{O3}$
      Field strength - 2 parameters: B_Q/B_D, B_O/B_D

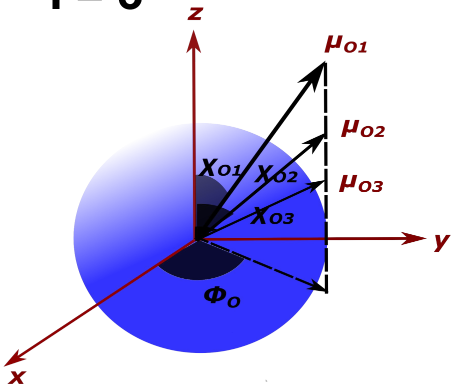
l = 1



l = 2



l = 3



Methods

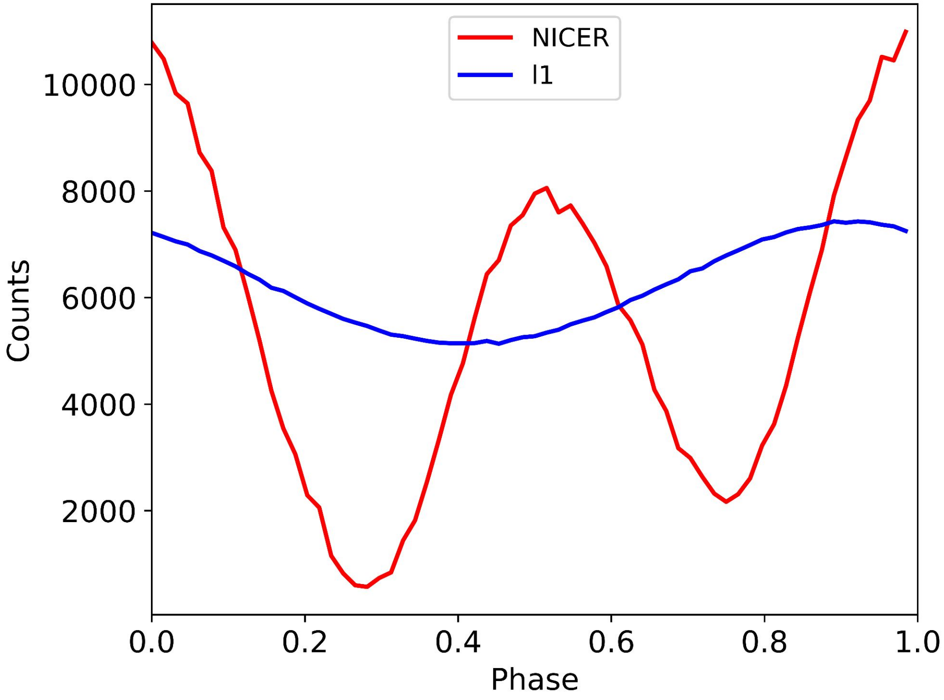
- Markov chain Monte Carlo (MCMC) code (Kalapotharakos et al. 2021):
  - Input M, R, photon trajectories (ray-tracing code)
  - Module for retarded multipolar magnetic field: parameter space
  - Calibration (field lines; polar caps calculation -> hotspots!)

Methods

- Markov chain Monte Carlo (MCMC) code (Kalapotharakos et al. 2021):
  - Input M, R, photon trajectories (ray-tracing code)
  - Module for retarded multipolar magnetic field: parameter space
  - Calibration (field lines; polar caps calculation -> hotspots!)
  - Produce thermal X-ray LCs
  - Compare with *NICER* LC, best-fit via log-likelihood test

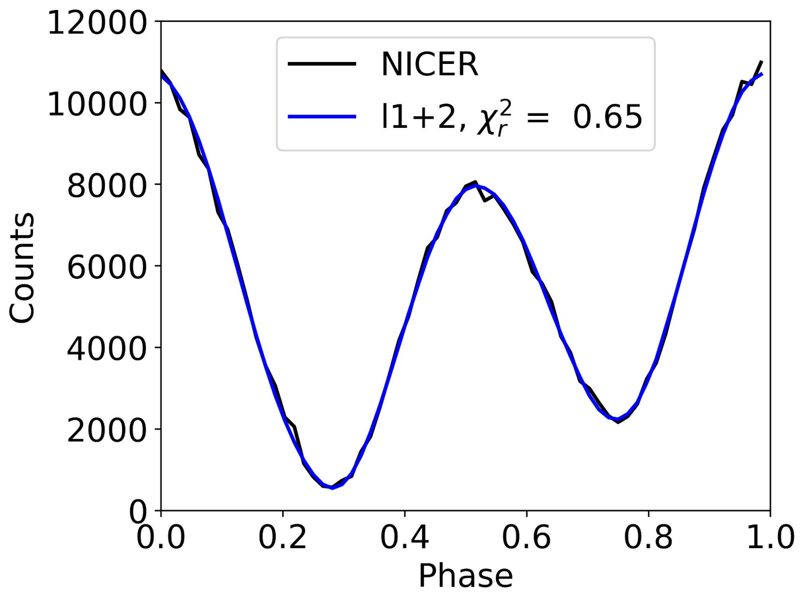
Results

Results: l1

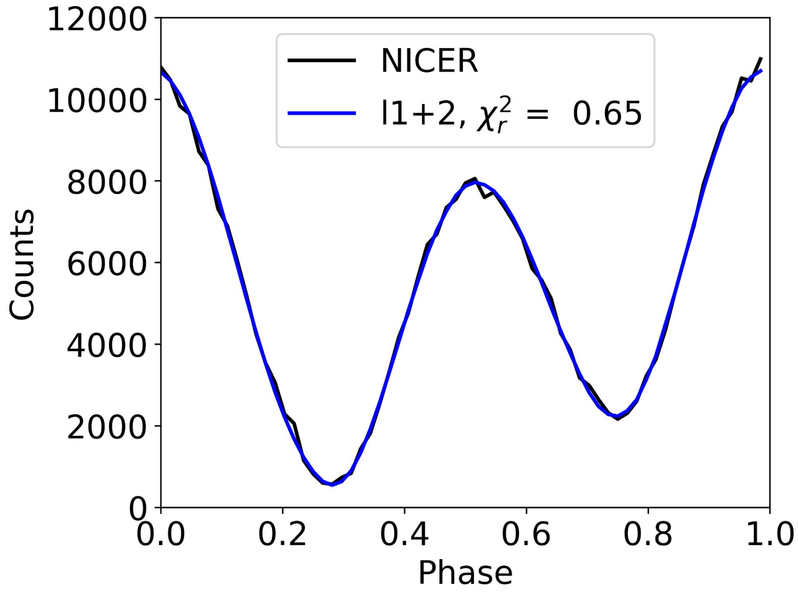


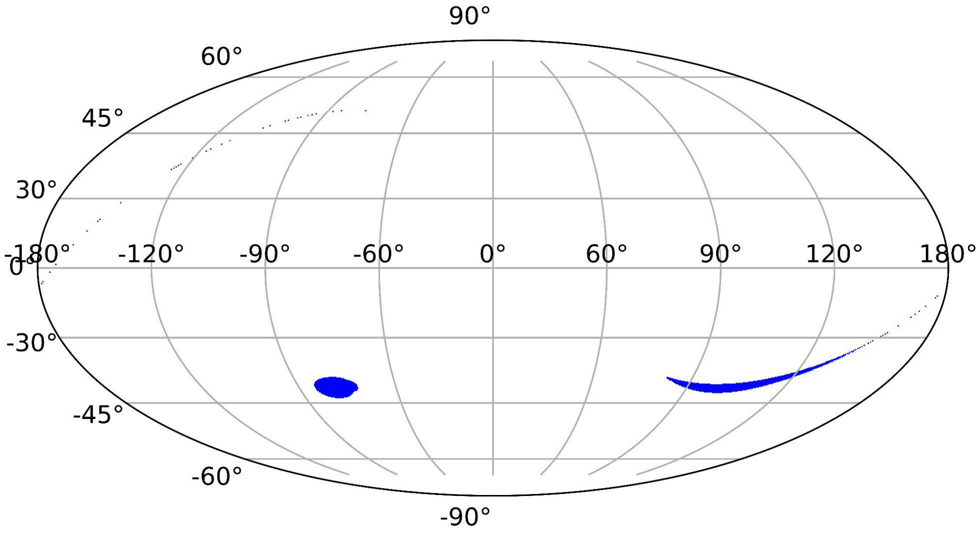
Standard centered dipole is not enough.

Results: l1+l2

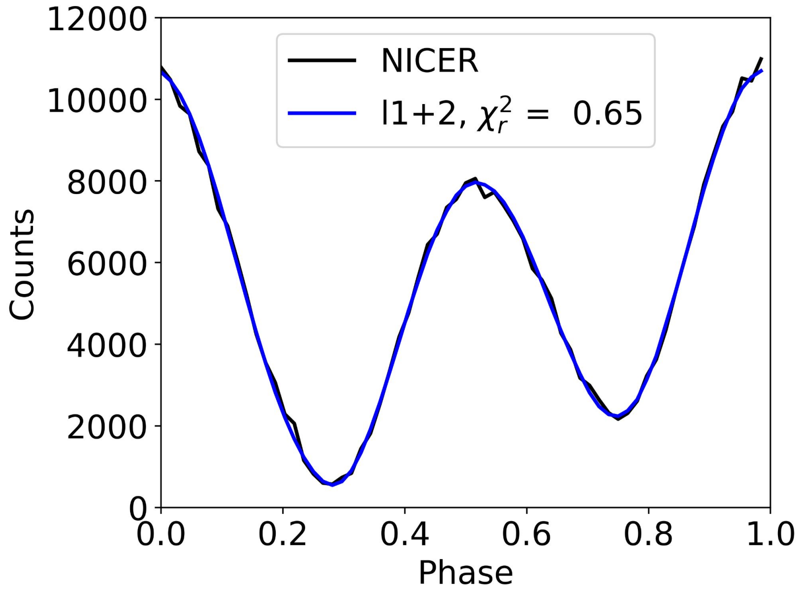


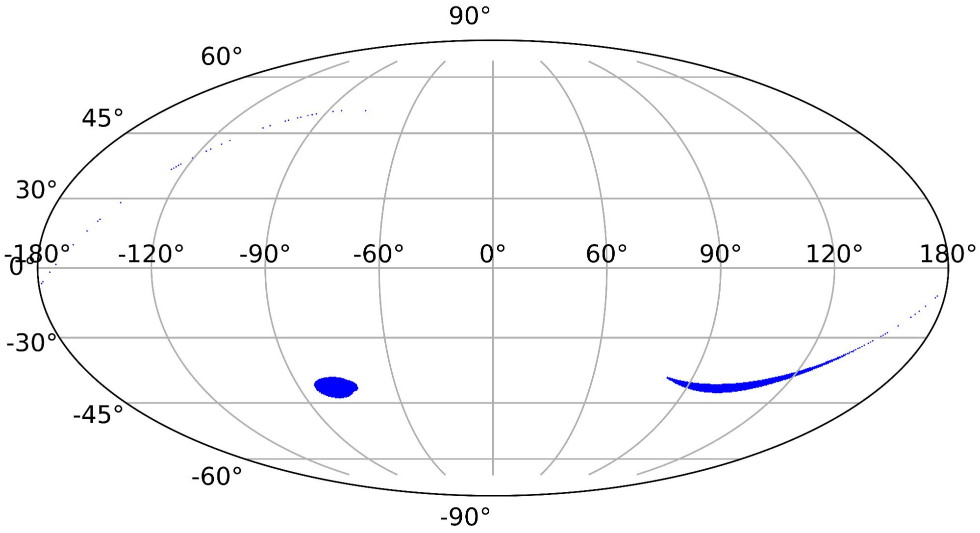
Results: I1+I2





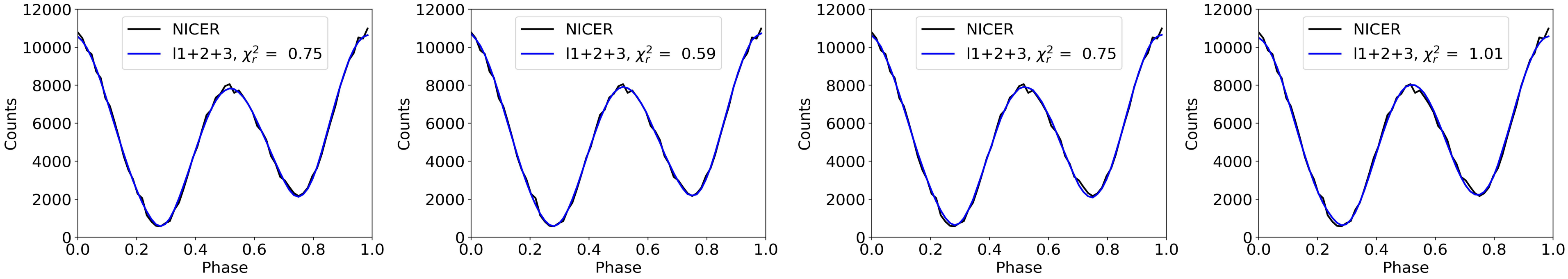
Results: I1+I2



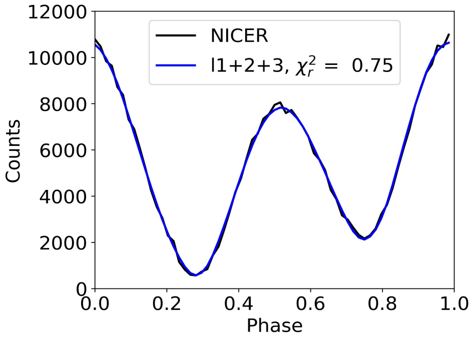


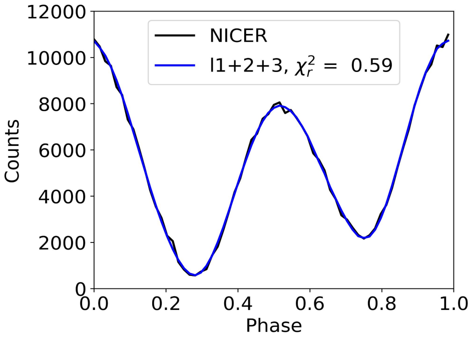
Centered field with all sub components…
Asymmetries like offset (equivalence…)

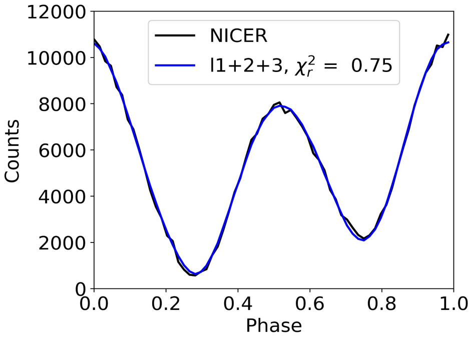
Results: l1+l2+l3

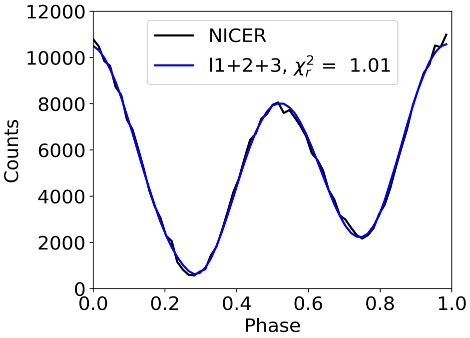


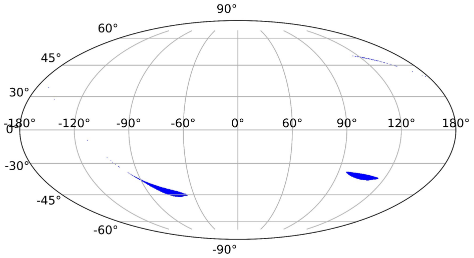
Results: I1+I2+I3

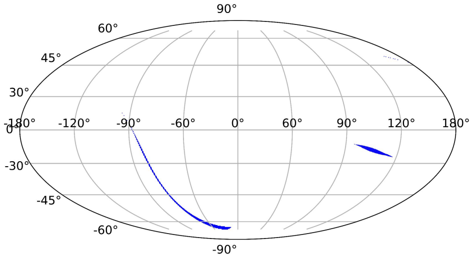


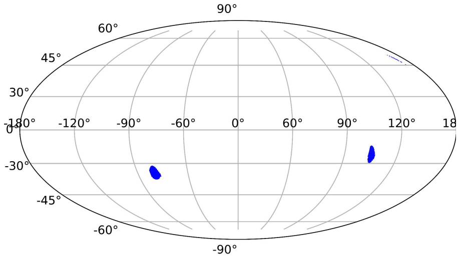


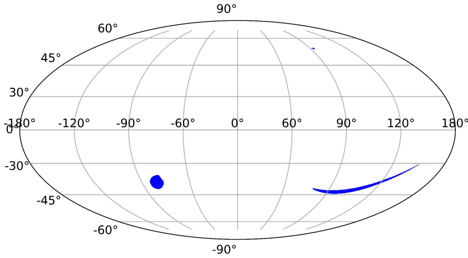




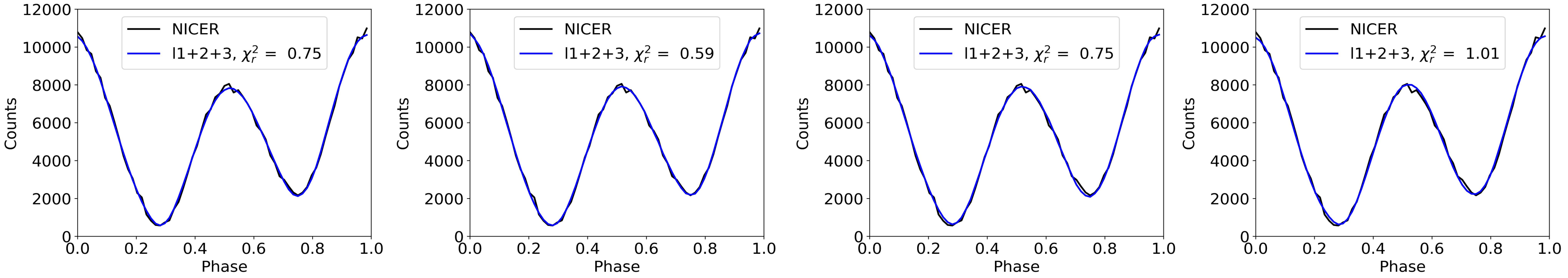


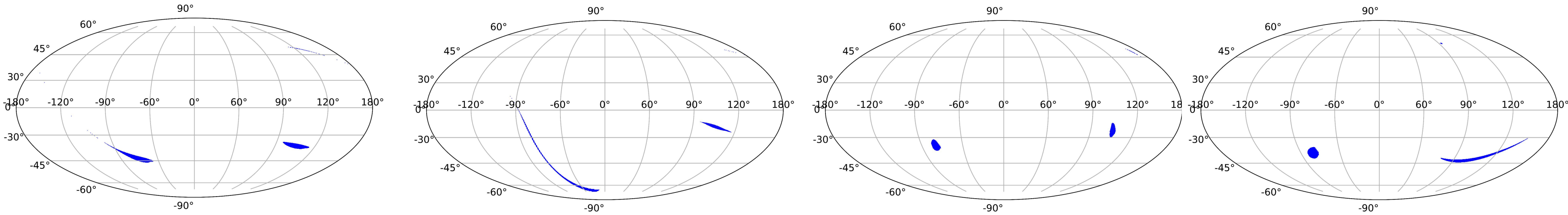






Results: l1+l2+l3





Multiple solutions…
Field degeneracies…

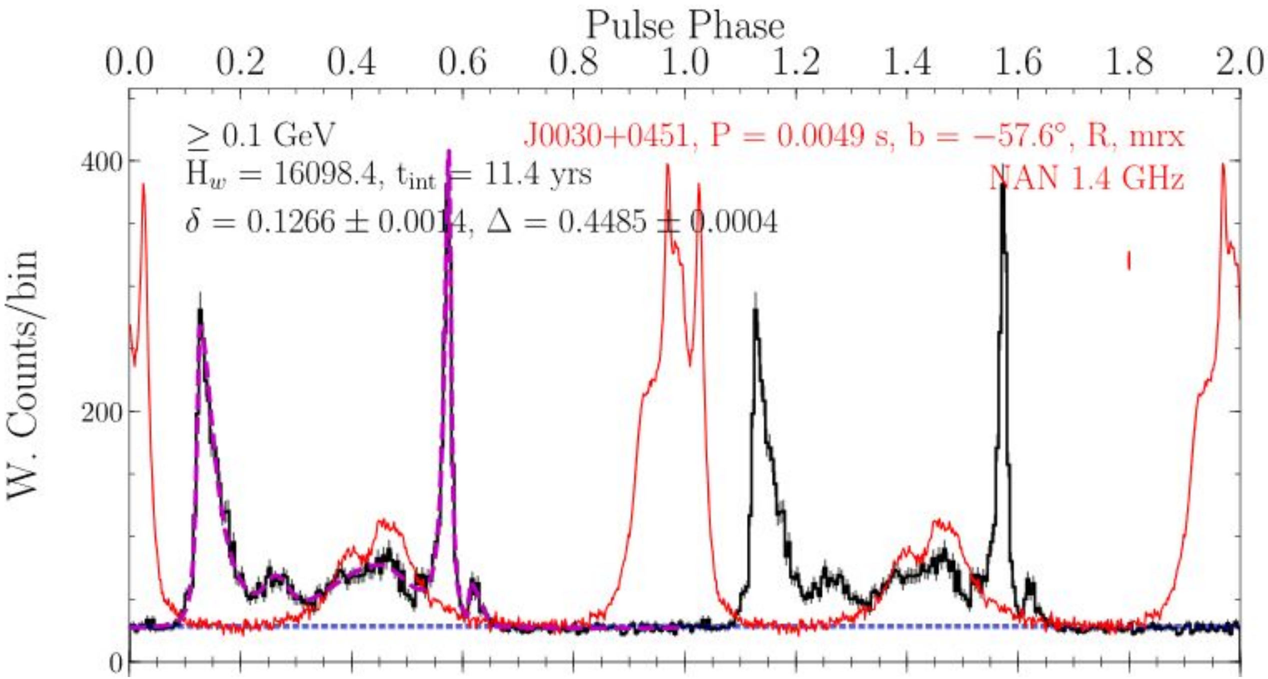
Discussion

Why retarded fields?

Discussion

Why retarded fields?

- Force-free (FF) to fit *Fermi* gamma-ray LC



Fermi 3PC (Smith et al. 2024)

Discussion

Why retarded fields?

- Force-free (FF) to fit *Fermi* gamma-ray LC

- Vacuum to FF:
  - Closer to FF than static
  - Save computation time for FF runs
  - Neural network training: more advanced (see talk by Kalapotharakos)

Discussion

Why retarded fields?

- Force-free (FF) to fit *Fermi* gamma-ray LC
- Vacuum to FF:
    - Closer to FF than static
    - Save computation time for FF runs
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Comparing different field configurations:

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Comparing different field configurations:

- Completeness: “Matching” static to retarded
- Number of parameters, physical
- Uniqueness of solutions

Summary

- Physically-founded model, using magnetic field
  - More realistic multipolar field
  - A complete description
  - No assumptions about sub-components

- Fit *NICER* data for MSPs, predict hotspot morphology

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What's next?

- FF for *Fermi* LC

- Self-consistent mass, radius determination

- Apply to other *NICER* MSPs, e.g., J0740+6620 (Riley et al. 2021, Miller et al. 2021)

- Ultimate aim: Constraining neutron star Equation of State

Thank you!