

# HIGH ENERGY CONSTRAINTS ON LORENTZ SYMMETRY VIOLATIONS

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Lorentz violation at high energies might lead to non linear dispersion relations for the fundamental particles. We analyze observational constraints on these without assuming any *a priori* equality between the coefficients determining the amount of Lorentz violation for different particle species. We focus on constraints from three high energy processes involving photons and electrons: photon decay, photo-production of electron-positron pairs, and vacuum Čerenkov radiation. We find that cubic momentum terms in the dispersion relations are strongly constrained.

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## 1 Introduction

There are several reasons to suspect that Lorentz invariance may be only a low energy symmetry. This possibility is suggested by the divergences of local quantum field theory, as well as by tentative results in various approaches to quantum gravity. Moreover, the unboundedness of the boost parameter makes experimental verification of exact Lorentz symmetry impossible in principle.

One can study the possibility of Lorentz violation, without a particular fundamental theory in hand, by considering its manifestation in dispersion relations for matter. It is natural to assume that such dispersion relations  $E^2(p)$  can be characterized by an expansion with integral powers of momentum,

$$E^2 = p^2 + m^2 + [Ap^2 + Bp^3/K_0 + Cp^4/K_0^2 + O(p^5)]. \quad (1)$$

Here  $A, B, C$  are dimensionless coefficients which might be positive as well as negative and  $K_0$  is the “quantum gravity” scale (often identified with the inverse Planck length,  $K_0 = 2\pi/L_{Pl}$ ). [Throughout this paper  $p$  denotes the absolute value of the 3-momentum vector  $\mathbf{p}$ , and we use units with the low energy speed of light in vacuum equal to unity.]

Different approaches to quantum gravity suggest different leading order Lorentz violating terms. The terms with coefficients  $A, B, C$  have mostly been considered so far. Since the  $p^2$  term is not suppressed by the Planck scale, it might be thought to be largest, however observations severely limit  $A$  to be

much less than unity (see e.g.<sup>1</sup> and references therein). The higher order terms are naturally small, with coefficients of order unity.

Our strategy here is to take a purely phenomenological stance and consider the constraints that high energy observations impose on dispersion relations of the form

$$E_a^2 \approx p_a^2 + m_a^2 + \eta_a p_a^n / K_0^{n-2}, \quad (2)$$

where  $a$  labels different fields and  $n \geq 3$ . In the absence of a fundamental theory one has no reason to expect any particular relation between the coefficients  $\eta_a$  for different particles, except perhaps that they should all be of the same order of magnitude.

Dispersion relations of the kind (2) produce kinematic constraints from energy-momentum conservation that differ from the usual Lorentz invariant case. As a result reactions can take place that are normally forbidden, and thresholds for reactions are modified. Observational consequences may seem out of reach because of the Planck scale suppression of the Lorentz violating terms (assuming that, as generally expected,  $K_0$  is of order the Planck scale). However this is not so. One can expect deviations from standard kinematics when the the last two terms of (2) are of comparable magnitude.<sup>a</sup> Assuming  $\eta$  is of order unity this yields the condition  $p_{\text{dev}} \sim (m^2 K_0^{n-2})^{1/n}$ , which is  $\sim (m/m_e)^{2/3} \times 10$  TeV for  $n = 3$  and  $\sim (m/m_e)^{1/2} \times 10^4$  TeV for  $n = 4$ . Although these energies are currently not achievable in particle accelerators (except in the case of the massive neutrinos which however are weakly coupled) they are in the range of current astrophysical observations.

In fact, it has been suggested by several authors<sup>1,2,3,4,5,6,7</sup> (see also<sup>8</sup> and references therein) that we may already be observing deviations from Lorentz invariance via the existence of two puzzles in modern astrophysics: the missing GZK cut-off on cosmic ray protons with ultra high energy greater than  $7 \times 10^{19}$  eV, and the apparent overabundance of gamma rays above 10 TeV from the BL Lac system Mkr 501<sup>9</sup>. Here we shall mostly not consider the constraints imposed by asking Lorentz violation to explain these puzzles. Instead we restrict our attention to constraints imposed by consistency with known phenomena (or lack thereof). (See also<sup>1,3,5</sup> for a similar discussion.)

## 2 Observational constraints

To find the strongest observational constraints without assuming *a priori* relations between the coefficients  $\eta_a$  of (2) we focus here on processes involving

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<sup>a</sup>In general of course one must look at the specific reaction in order to estimate the energies at which deviations from standard behavior can be expected. For example in the case of the reaction  $\gamma\gamma \rightarrow e^+e^-$  it will be the electron mass that sets the scale.

just two fundamental particles, photons and electrons. We also restrict to the case  $n = 3$ , since this should be most tightly constrained. If it can be ruled out, one can then move on to the  $n = 4$  case.

The modified dispersion relations for photons and electrons in general allow two normally forbidden interactions: photon decay,  $\gamma \rightarrow e^+e^-$ , and vacuum Čerenkov radiation,  $e^- \rightarrow e^-\gamma$ . If allowed these processes happen very rapidly. In addition the threshold for photon annihilation,  $\gamma\gamma \rightarrow e^+e^-$ , is shifted. We consider constraints that follow from three observations: (i) Gamma rays up to  $\sim 50$  TeV of cosmological origin arrive on earth<sup>10</sup>, so photon decay does not occur up to this energy. (ii) Electrons of energy  $\sim 100$  TeV are believed to produce observed X-ray synchrotron radiation coming from supernova remnants. Assuming these electrons are actually present, vacuum Čerenkov radiation must not occur up to that energy. (iii) Cosmic gamma rays below 10 TeV are believed to be absorbed in a manner consistent with photon annihilation off the IR background with the standard threshold. Observation (iii) is not model independent, so the corresponding constraint is tentative and subject to future verification.

To derive the observational constraints one needs to determine the threshold for each the process, i.e. the lowest energy for which the process occurs. The details concerning our determination of the thresholds are reported elsewhere<sup>11</sup>. Assuming monotonicity of  $E(p)$  (for the relevant momenta  $p \ll K_0$ ) we have shown that all thresholds for processes with two particle final states occur when the final momenta are parallel. Moreover for two particle initial states the incoming momenta are antiparallel. This geometry has been assumed in previous works but to our knowledge it was not shown to be necessary. (In fact it is not necessary if  $E(p)$  is not monotonic.)

To eliminate the subscript  $a$  we introduce  $\xi := \eta_\gamma$  and  $\eta := \eta_e$ . The constraints will restrict the allowed region of the  $\eta$ - $\xi$  plane. For the rest of the paper we assume  $K_0$  is the Planck energy and we use units with  $K_0 = 1$ .

### 2.1 Photon decay

Photon decay is allowed only above a broken line in the  $\eta$ - $\xi$  plane given by  $\xi = \eta/2$  in the quadrant  $\xi, \eta > 0$  and by  $\xi = \eta$  in the quadrant  $\xi, \eta < 0$ . Above this line, the threshold is given by

$$k_{\text{th}} = \left( \frac{8m^2}{2\xi - \eta} \right)^{1/3} \quad \text{for } \xi \geq 0, \quad (3)$$

$$k_{\text{th}} = \left( \frac{-8m^2\eta}{(\xi - \eta)^2} \right)^{1/3} \quad \text{for } \eta < \xi < 0. \quad (4)$$

The first relation (3) arises when the electron and positron momenta are equal at threshold. In standard Lorentz invariant kinematics such “equipartition” of momentum always holds at threshold for pair production. In all previous work on Lorentz violating dispersion it has been assumed to hold. Surprisingly, however, in the present case the threshold may occur with an asymmetric distribution of momentum. The second relation (4) applies in those cases. The constraint we impose is that the threshold is above 50 TeV, the energy of highest observed gamma rays from the Crab nebula<sup>10</sup>.

## 2.2 Photon annihilation

The threshold relations for a gamma ray to annihilate with an IR background photon of energy  $\epsilon$  take approximately the same form as for photon decay, with the replacement  $\xi \rightarrow \xi'$ , where  $\xi' \equiv \xi + 4\epsilon/k_{\text{th}}^2$ . (Here we have used the fact that  $\epsilon$  is much smaller than any other scale in the problem.) The two different relations arise for the same reason as in the case of photon decay, and they correspond respectively to cubic and quartic polynomial equations for  $k_{\text{th}}$ .

For the observational consequences it is important to recognize that the threshold shifts are much more significant at higher energies than at lower energies. To exhibit this dependence, it is simplest to fix a gamma ray energy  $k$  and to solve for the corresponding soft photon threshold energy  $\epsilon_{\text{th}}$ . Taking the ratio with the usual threshold  $\epsilon_{\text{th},0}$ , we find a dependence on  $k$  at least as strong as  $k^{3/2}$ . Introducing  $k_{10} := k/(10 \text{ TeV})$ , we have

$$\frac{\epsilon_{\text{th}}}{\epsilon_{\text{th},0}} = 1 + 0.05 (\eta - 2\xi) k_{10}^3 \quad \text{for } \xi' \geq 0, \quad (5)$$

$$\frac{\epsilon_{\text{th}}}{\epsilon_{\text{th},0}} = 0.1 (\eta - \xi) k_{10}^3 + \sqrt{-0.2 \eta k_{10}^3} \quad \text{for } \eta < \xi' < 0. \quad (6)$$

High energy TeV gamma rays from the blazars Markarian 421 and Markarian 501 have been detected out to 17 TeV and 24 TeV respectively. Although the sources are not well understood, and the intergalactic IR background is also not fully known, detailed modeling shows that the data are consistent with some absorption by photon annihilation off the IR background (see e.g.<sup>12,13,14</sup> and references therein). However, while the inferred source spectrum for Mrk 501 is consistent with expectations for energies less than around 10 TeV, above this energy there are far more photons than expected according to some IR background models<sup>9,14</sup>.

The uncertainty in the blazar and IR background models currently precludes sharp constraints from photon annihilation. Instead, we just determine the range of parameters  $\xi, \eta$  for which the threshold  $k_{\text{th}}$  lies between 10 TeV

and 20 TeV for an IR photon with which a 10 TeV photon would normally be at threshold. We choose this range since (i) lowering the threshold would make the overabundance problem worse, and (ii) raising this threshold by a factor of two might explain the potential overabundance of photons over 10 TeV<sup>2</sup>. If the overabundance puzzle exists and is to be resolved in this way, the effect of the threshold shifts must be enhanced for photons above 10 TeV relative to those below 10 TeV (since the latter seem to be well accounted for). This enhancement could arise partly from the shape of the IR background spectrum, but the shift itself is also enhanced, as seen in the energy dependence of equations (5,6). The hypothesis<sup>2,5,9</sup> that the potential overabundance is due to Lorentz violating dispersion with  $n = 3$  therefore appears to be consistent with current observations.

### 2.3 Vacuum Čerenkov radiation

An electron can emit Čerenkov radiation in the vacuum if  $\eta > 0$  or if  $\eta < 0$  and  $\xi < \eta$ . In this case there are also two threshold relations, depending on whether the threshold occurs with emission of a zero-energy photon or with emission of a finite energy photon. These two cases correspond to the two following relations, respectively:

$$p_{\text{th}} = \left(\frac{m^2}{2\eta}\right)^{1/3} \quad \text{for } \eta > 0 \text{ and } \xi \geq -3\eta, \quad (7)$$

$$p_{\text{th}} = \left(-\frac{4m^2(\xi + \eta)}{(\xi - \eta)^2}\right)^{1/3} \quad \text{for } \xi < -3\eta < 0 \text{ or } \xi < \eta \leq 0. \quad (8)$$

The vacuum Čerenkov process is extremely efficient, leading to an energy loss rate that goes like  $E^2$  well above threshold. Thus any electron known to propagate must lie below the threshold. Electrons of energy  $\sim 100$  TeV are believed to produce observed X-ray synchrotron radiation coming from supernova remnants. Thus for example in the region of the parameter plane where (7) holds we obtain the constraint  $\eta < m^2/2p_{\text{th}}^3 \sim 10^{-3}$ .

### 3 Combined constraints

Putting together all the constraints and potential constraints we obtain the allowed region in the  $\eta$ - $\xi$  plane (see Figure 1). The photon decay and Čerenkov constraints exclude the horizontally and vertically shaded regions, respectively. The allowed region lies in the lower left quadrant, except for an exceedingly small sliver near the origin with  $0 < \eta \lesssim 10^{-3}$  and a small triangular region

( $-0.16 \lesssim \eta < 0$ ,  $0 < \xi \lesssim 0.08$ ) in the upper left quadrant. The range of the photon annihilation threshold discussed in subsection 2.2 falls between the two roughly parallel diagonal lines. This intersects the otherwise allowed region in a finite, narrow wedge where  $\xi$  and  $\eta$  are negative and of order unity (apart from a minuscule invisible region near the origin with  $\eta > 0$ ). If future

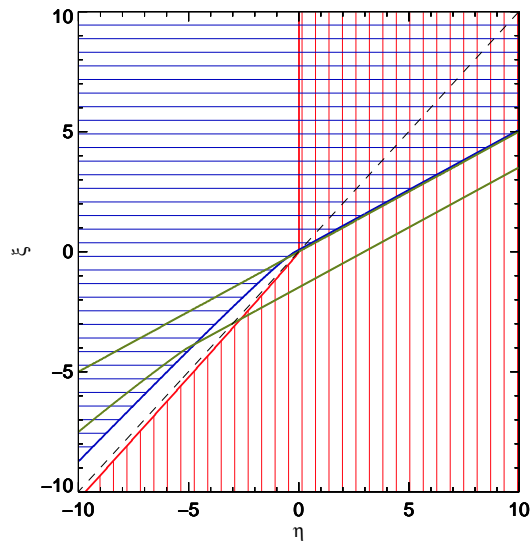


Figure 1: Combined constraints on the photon and electron parameters for the case  $n = 3$ . The regions excluded by the photon decay and Čerenkov constraints are lined horizontally in blue and vertically in red respectively. The photon annihilation threshold limits fall between the two diagonal green lines. The dashed line is  $\xi = \eta$ .

observations of the blazar fluxes and the IR background confirm agreement with standard Lorentz invariant kinematics, the region allowed by the photon annihilation constraint will be squeezed toward the upper line ( $k_{\text{th}} \approx k_s$ ). If the overabundance of gamma rays from Mrk 501 is indeed due to Lorentz violation of the sort we are considering, the region will be squeezed toward the lower line ( $k_{\text{th}} \approx 2k_s$ ) or may even shift toward lower values of  $\xi$  and  $\eta$  if a yet larger threshold shift is indicated.

## 4 Conclusions

We have seen that a conservative interpretation of observations puts strong constraints on the possibility of Planck scale cubic modifications to the electron and photon dispersion relations. The allowed region includes  $\xi = \eta = -1$ , which has been a focus of previous work<sup>2,5,6</sup>. The negative quadrant has most of the allowed parameter range. It is interesting to note that in this quadrant all group velocities are less than the low energy speed of light.

To completely rule out the cubic case would require new observations. Finding higher energy electrons would not help much, while finding higher energy undecayed photons would squeeze the allowed region onto the line  $\xi = \eta$ . To further shrink the allowed segment of this line would require observations allowing the usual threshold for photon annihilation to be confirmed to higher precision. Perhaps other processes could be used as well. If *a priori* relations among the coefficients in the dispersion relations for different particles are hypothesized, stronger constraints can of course be obtained.

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