Fermi
Gamma-ray
Space Telescope

A Little History, but…
in Astronomy its
Resolution,
Resolution,
Resolution

Bill Atwood
SCIPP/UCSC

Gamma Ray Pair Conversion

Energy loss mechanisms

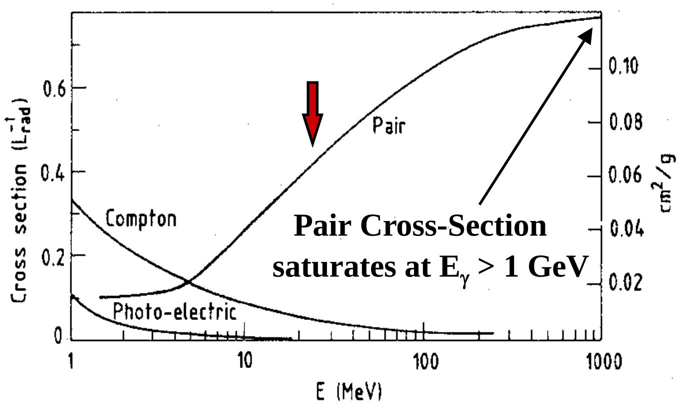
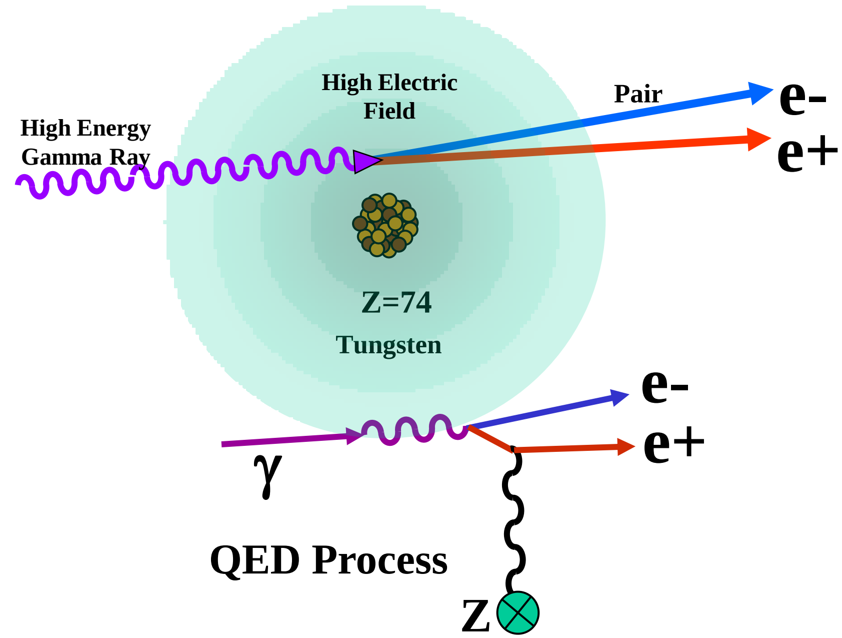
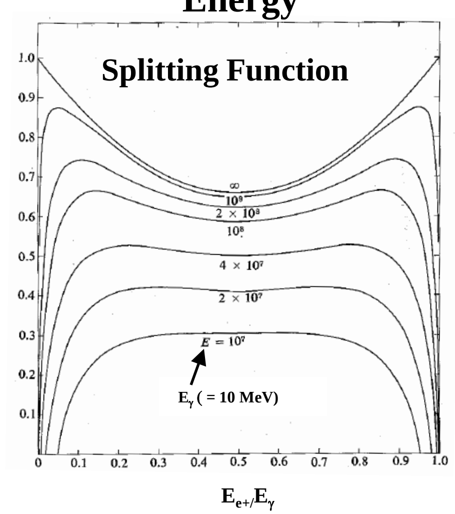


Fig. 2: Photon cross-section σ in lead as a function of photon energy. The intensity of photons can be expressed as $I = I_0 \exp(-\sigma x)$, where x is the path length in radiation lengths. (Review of Particle Properties, April 1980 edition).



Energy



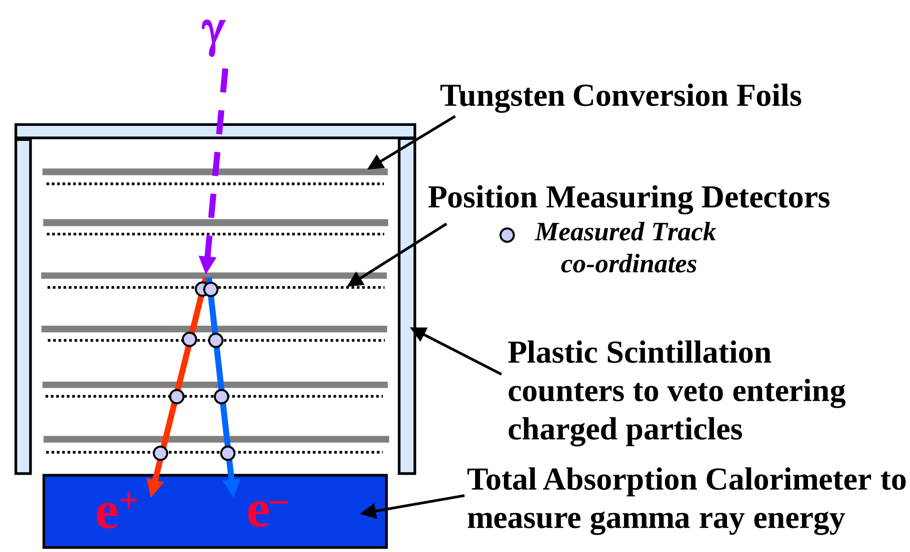
From Rossi, High Energy Particles, 1952

Opening Angle

$$\theta_{Open} \approx \frac{4m_e}{E_\gamma}$$

At 100 MeV

$\theta_{Open} \sim 1^o$



Previous Satellite Detectors

- **1967-1968, OSO-3 Detected Milky Way as an extended γ-ray source**

  621 γ-rays
- **1972-1973, SAS-2,**

  ~8,000 γ-rays
- **1975-1982, COS-B**

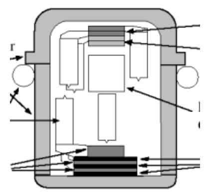
  orbit resulted in a large and variable background of charged particles

  ~200,000 γ-rays
- **1991-2000, EGRET**

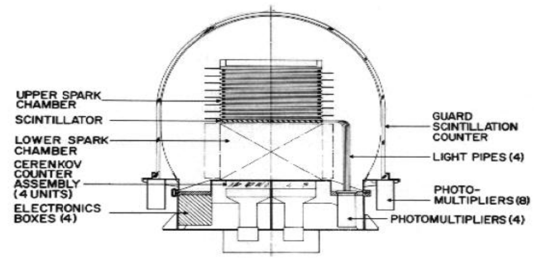
  Large effective area, good PSF, long mission life, excellent background rejection

  $>1.4 \times 10^6$ γ-rays

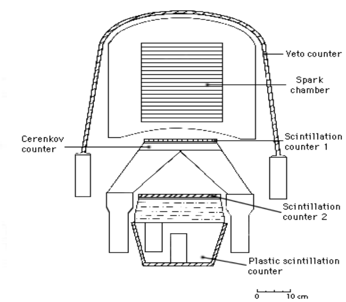
OSO-3



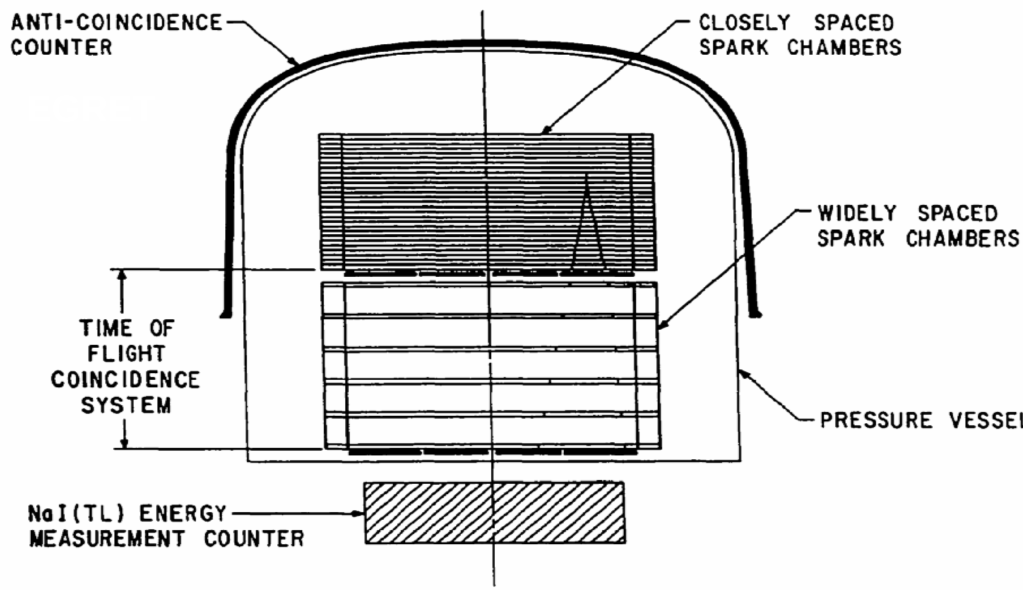
SAS-2



COS-B



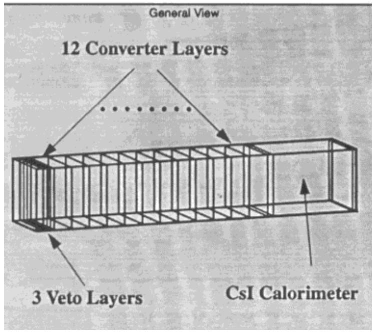
EGRET



Evolution of GLAST

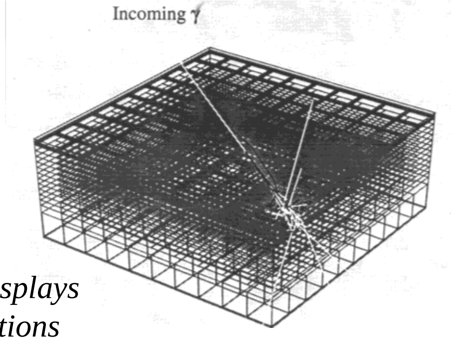
- April, 1991 CGRO (with EGRET on board) Shuttle Launch
- May, 1992 NASA SR & T Proposal Cycle

1. Select the Technologies



Large area SSD systems and CsI Calorimeters resulted from SSC R&D

2. Make it Modular



Another lesson learned in the 1980's: monolithic detectors are inferior to Segmented detectors

Original GISMO 1 Event Displays from the first GLAST simulations

3. Pick the Rocket

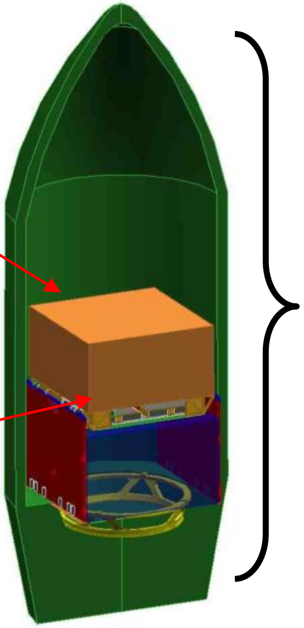
Delta II (launch of GP-B)

Cheap, reliable Communication satellite launch vehicle

4. Fill-it-up!

Diameter sets transverse size

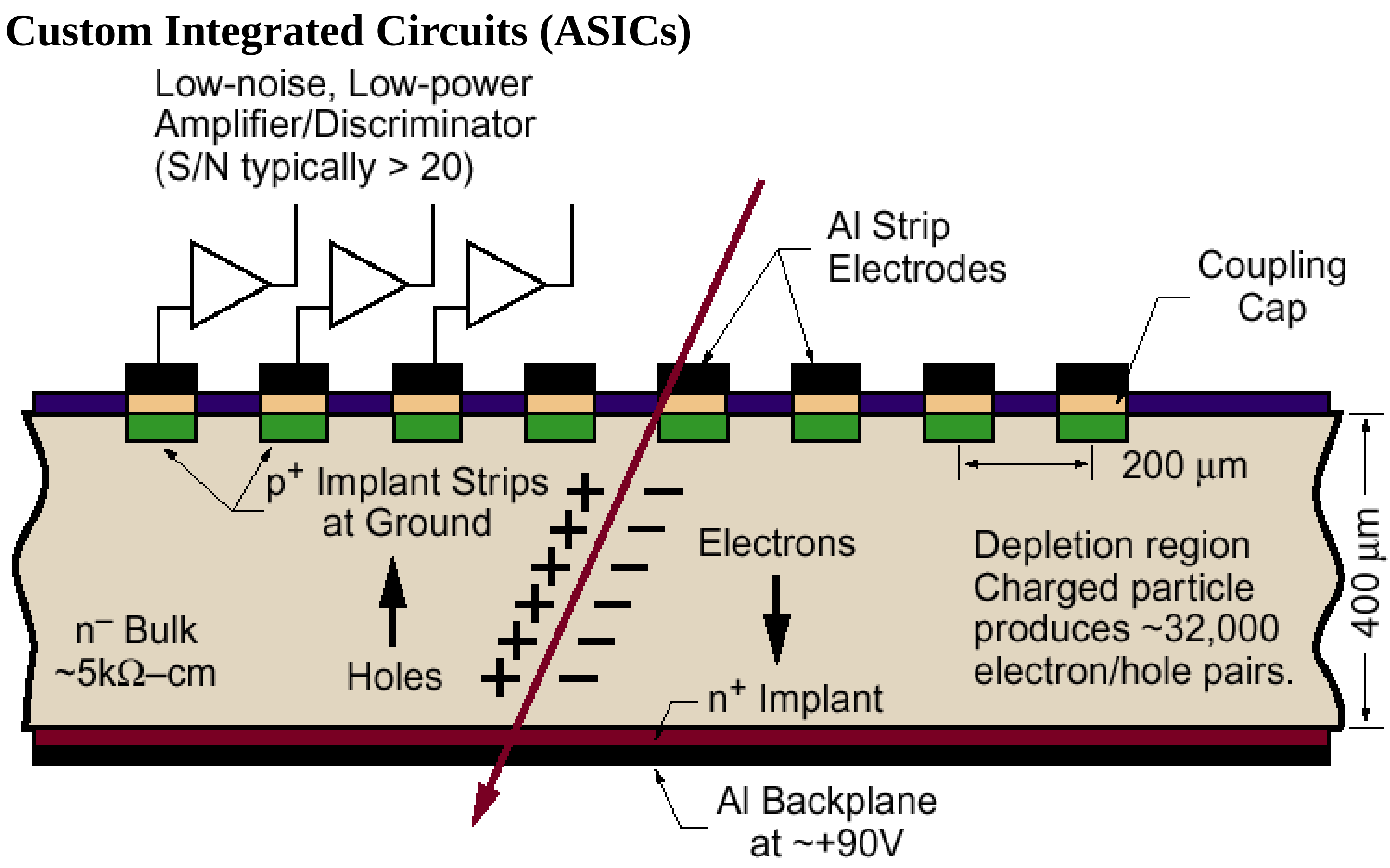
Lift capacity to LEO sets depth of Calorimeter



Rocket Payload Fairing

Technology of Choice: Solid State

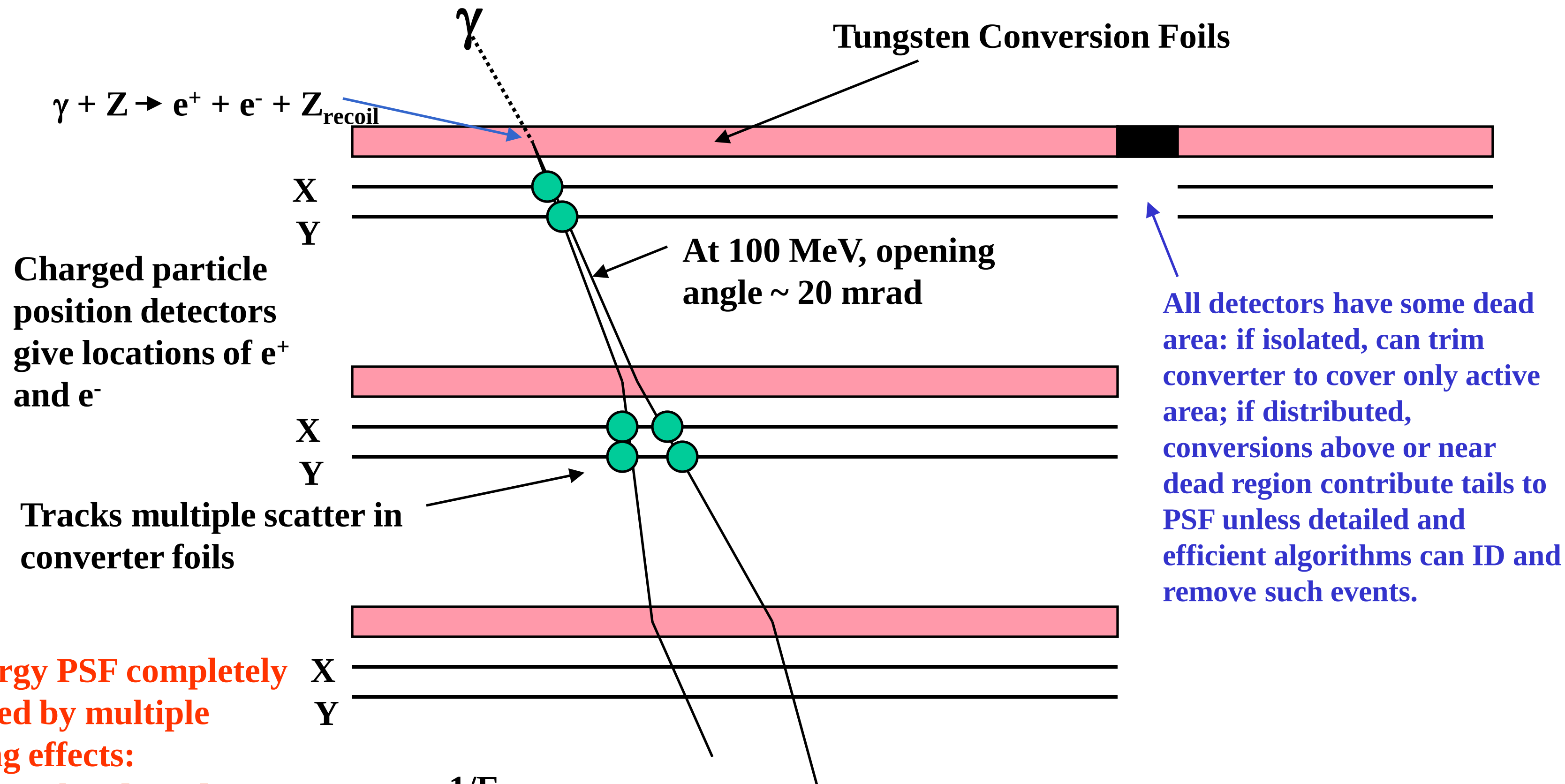
Silicon Strip Detector Principle



GLAST has 884736 channels. Total Tracker Power = 160 Watts!

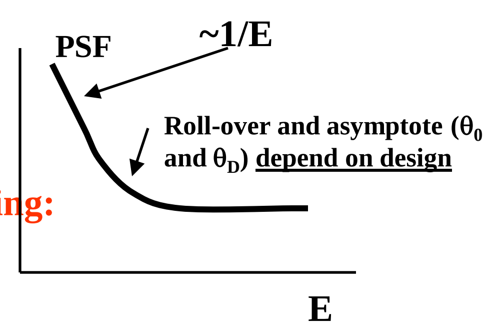
Pair Conversion Telescope

Expanded view of converter-tracker:



Low energy PSF completely dominated by multiple scattering effects:
$\theta_0 \sim 2.9$ mrad / E[GeV]
(scales as $(x_0)^{½}$)

High energy PSF set by hit resolution/plane spacing:
$\theta_D \sim 1.8$ mrad.

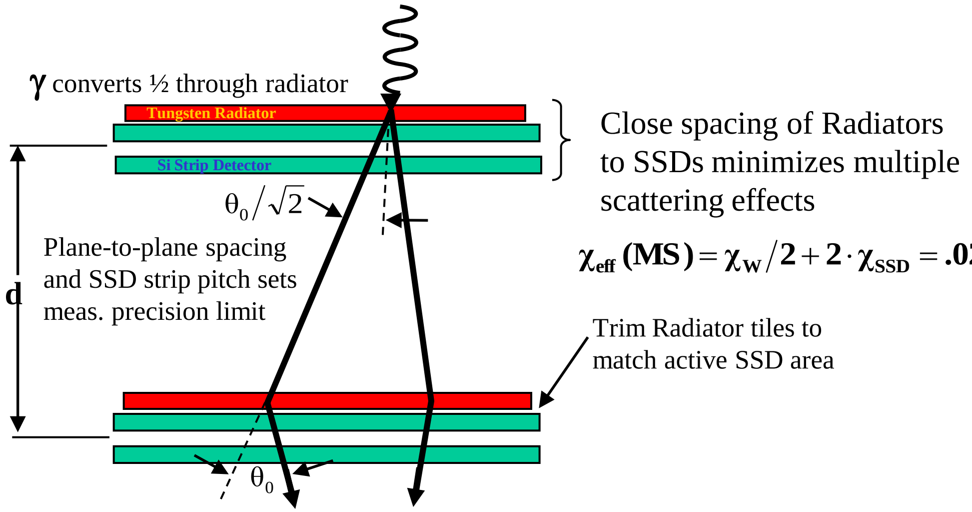


At higher energies, more planes contribute information:

Energy	# significant planes
100 MeV	2
1 GeV	~5
10 GeV	>10

Tracker Design and Analysis

Pair Conversion Telescope Layout



$$\chi_{eff}(MS) = \chi_W/2 + 2\cdot\chi_{SSD} = .025$$

Angular Resolution Parameters

$$\delta\theta_{MS}(100MeV) \cong 38mrad$$

$$\delta\theta_{MS}(Space) \cong \sqrt{2}\cdot 38mrad = 54mrad = 3.1^o$$

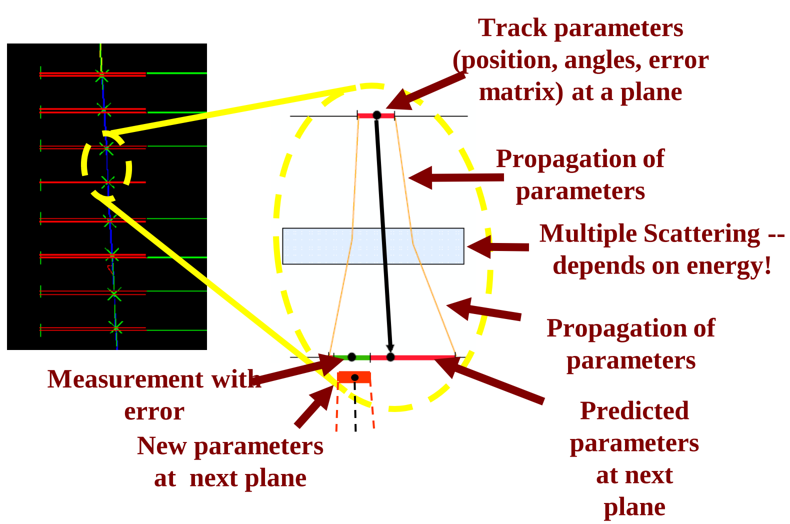
$$\delta\theta_{Det} = \frac{\sigma_{SSD}}{d}\sqrt{2} = \frac{Pitch/\sqrt{12}}{d}\cdot\sqrt{2}$$

$$\delta\theta_{Det} = \frac{228\mu m}{32.9mm\cdot\sqrt{6}} = 2.8mrad = .16^o$$

Multiple Scattering

$$\theta_o = \frac{14mrad-GeV}{p\beta}\sqrt{\chi_{Eff}} \cong \frac{14mrad-GeV}{.5E_\gamma}\sqrt{\chi_{Eff}}$$

Kalman Tracking/Fitting



Data Analysis Techniques for High Energy Physics, R. Fruhwirth et al., (Cambridge U. Press , 2000, 2nd Edition)

Trade Between A_{eff} & PSF

$$N_\gamma \propto \chi_{Rad} \qquad PSF \propto \sqrt{\chi_{Rad}}$$

Source Sensitivity ➡ Photon Density

$$Sens. \approx \frac{N_\gamma}{PSF^2}$$

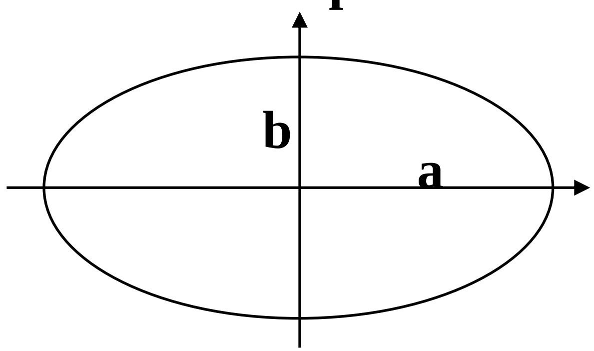
Doesn't depend on χ_{Rad} !

2-Source Separation pushes for thin radiators

Transient sensitivity pushes for thick radiators

Diversion: Review of Covariance

Ellipse



Take a circle – scale the x & y axis:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

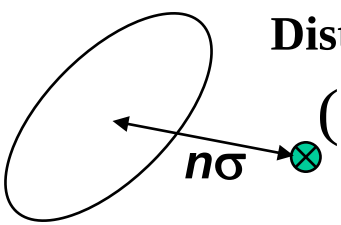
Rotate by θ :
$$x \to x\cos(\theta)+y\sin(\theta)$$
$$y \to y\cos(\theta)-x\sin(\theta)$$

Results:

$$x^2\left(\frac{\cos^2(\theta)}{a^2}+\frac{\sin^2(\theta)}{b^2}\right)+2xy\cos(\theta)\sin(\theta)\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+y^2\left(\frac{\sin^2(\theta)}{a^2}+\frac{\cos^2(\theta)}{b^2}\right)=1$$

Rotations mix *x* & *y*. Major & minor axis plus rotation angle θ complete description.

Error Ellipse described by Covariance Matrix:



Distance between a point with an error and another point measured in σ's:

$(n\sigma)^2 = r^T C^{-1} r$ **where** $r=(\vec{x}-\bar{x})$ **and**

$$C^{-1}=Inverse(C)=\begin{bmatrix} C^{-1}_{xx} & C^{-1}_{xy} \\ C^{-1}_{yx} & C^{-1}_{yy} \end{bmatrix}$$ **and** $C^{-1}_{xy}=C^{-1}_{yx}$

Simply weighting the distance by $1/\sigma^2$

Covariance - 2

Multiplying it out gives:

$$(n\sigma)^2 = r^T C^{-1} r = (x,y)\begin{bmatrix} C_{xx}^{-1} & C_{xy}^{-1} \\ C_{xy}^{-1} & C_{yy}^{-1} \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x^2 C_{xx}^{-1} + 2xyC_{xy}^{-1} + y^2 C_{yy}^{-1}$$

Where I take $\bar{x} = 0$ **without loss of generality.**

This is the equation of an ellipse! Specifically for a 1σ error ellipse (*n*σ = *1*) we identify:

$$C_{xx}^{-1} = \frac{\cos^2(\theta)}{a^2} + \frac{\sin^2(\theta)}{b^2} \qquad C_{yy}^{-1} = \frac{\sin^2(\theta)}{a^2} + \frac{\cos^2(\theta)}{b^2} \qquad C_{xy}^{-1} = \sin(\theta)\cos(\theta)(\frac{1}{a^2} - \frac{1}{b^2})$$

and $C^{-1} = \frac{1}{\det(C)}\begin{bmatrix} C_{yy} & -C_{xy} \\ -C_{xy} & C_{xx} \end{bmatrix}$ **where** $\det(C) = (C_{xx}C_{yy} - C_{xy}^2)$

The correlation coefficient is defined as: $R^2 = \frac{C_{xy}^2}{C_{xx}C_{yy}}$

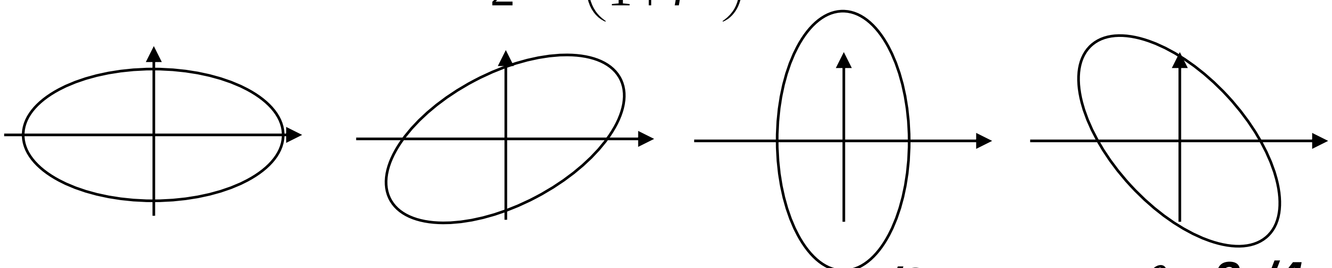
<u>Summary:</u> The *Covariance Matrix* describes an ellipse where the major and minor axis and the rotation angle map directly onto its components!

Covariance - 3

Let the fun begin! To disentangle the two descriptions consider

$$A = \frac{C^{-1}_{xy}}{C^{-1}_{xx} + C^{-1}_{yy}} = \frac{-C_{xy}}{C_{xx} + C_{yy}} = \frac{\cos(\theta)\sin(\theta)(b^2 - a^2)}{a^2 + b^2}$$

thus $A = \frac{\sin(2\theta)}{2}\left(\frac{1-r^2}{1+r^2}\right)$ **where *r* = *a/b***



$\theta = 0$ $\theta = \pi/4$ $\theta = \pi/2$ $\theta = 3\pi/4$

Also det(C) yields (with a little algebra & trig.):

$$a \cdot b = \sqrt{\det(C)} = \frac{Area}{\pi}$$

Now we're ready to continue the story of GLAST (Fermi-LAT)!

Multi-variate Analysis: Kalman Filter

The Kalman filter process is a successive approximation scheme to estimate parameters

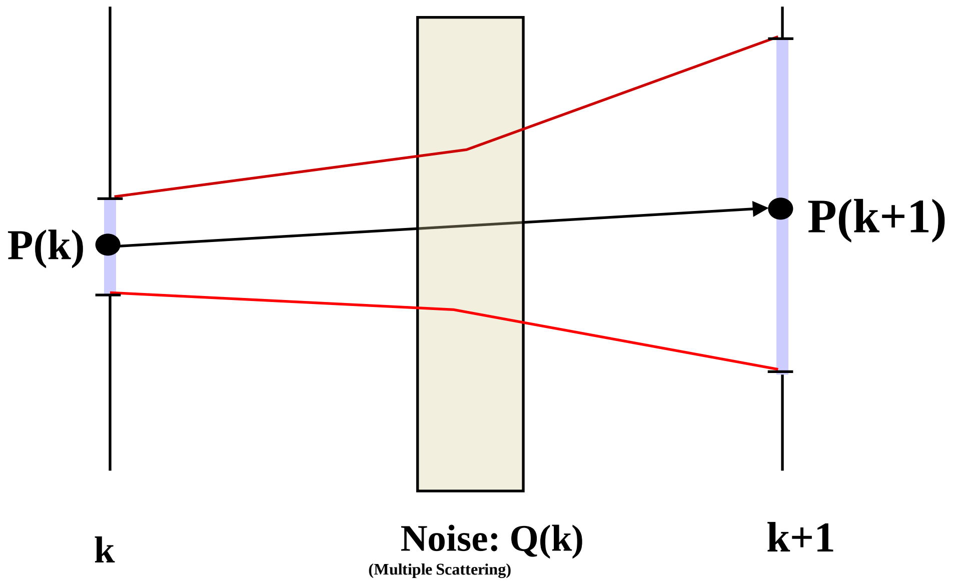
Simple Example: 2 parameters - *intercept* and *slope*: $x = x_0 + S_x * z$; $P = (x_0 , S_x)$

Errors on parameters x_0 & S_x: Covariance Matrix $C = \begin{pmatrix} C_{xx} & C_{xs} \\ C_{sx} & C_{ss} \end{pmatrix}$

$C_{xx} = <(x-x_m)(x-x_m)>$

In general

$C = <(P - P_m)(P-P_m)^T>$



Propagation:

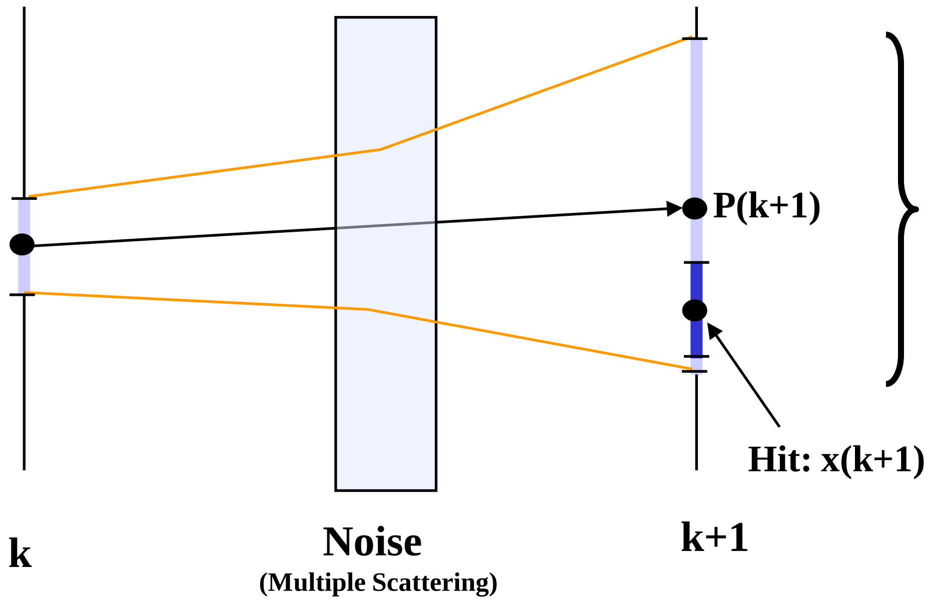
$x(k+1) = x(k)+S_x(k)*(z(k+1)-z(k))$

P (k+1) = F(δz) * P(k) where

$$F(\delta z) = \begin{pmatrix} 1 & z(k+1)-z(k) \\ 0 & 1 \end{pmatrix}$$

$$C(k+1) = F(\delta z) * C(k) * F(\delta z)^T + Q(k)$$

Kalman Filter (2)



Form the weighted average of the k+1 measurement and the propagated track model: Weights given by inverse of Error Matrix: C^{-1}

$$P(k+1) = \frac{C^{-1}(k+1)*P(k+1) + V^{-1}(k+1)*x(k+1)}{C^{-1}(k+1) + V^{-1}(k+1)} \quad \text{and} \quad C(k+1) = \left(C^{-1}(k+1) + V^{-1}(k+1)\right)^{-1}$$

Now its repeated for the k+2 planes and so - on. This is called FILTERING - each successive step incorporates the knowledge of previous steps as allowed for by the NOISE and the aggregate sum of the previous hits.

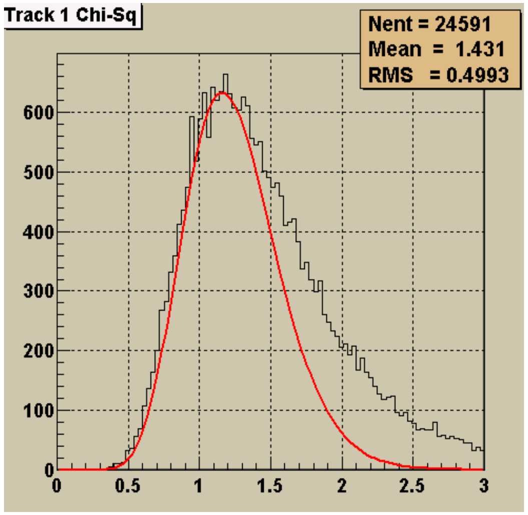
How Well does it work?

Really a game of "Do you get out what you put in?"

1 GeV Muons used for testing
Energy for Kalman Fit = MC Energy
Results: Large Tail on χ^2
Solution: Include energy losses in Tracker (Bethe-Bloch)

$\langle N_{hits} \rangle = 36$

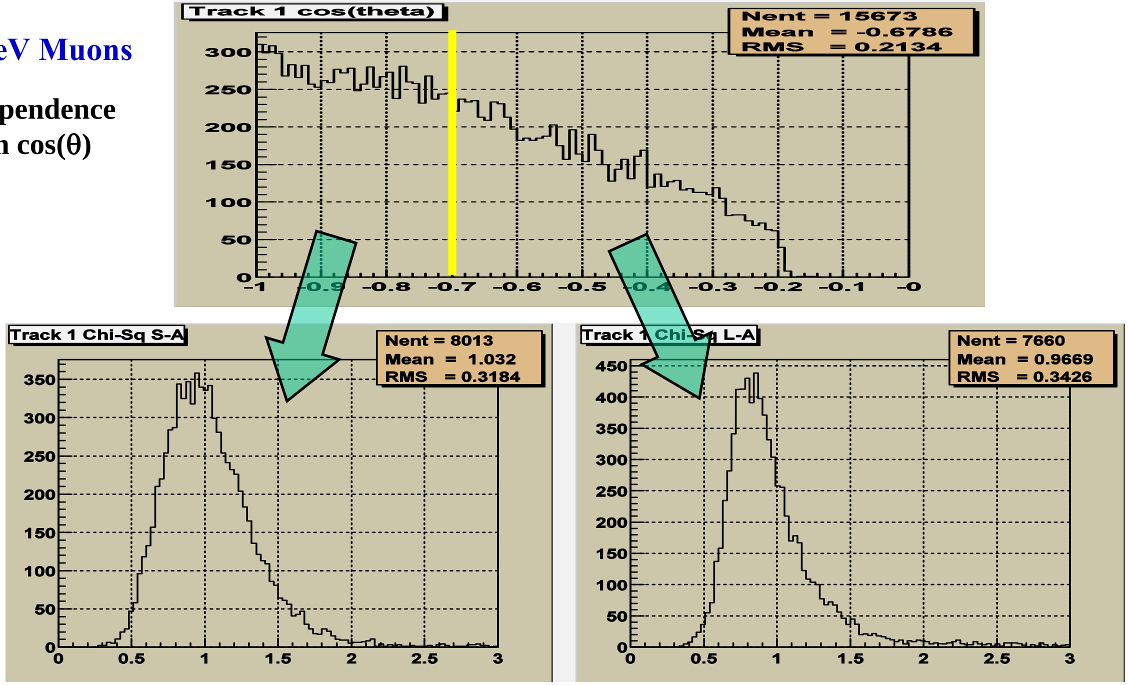
$\langle \chi^2 \rangle = 1.25$



Next: χ^2 Depends on Angle

1 GeV Muons

Dependence on cos(θ)

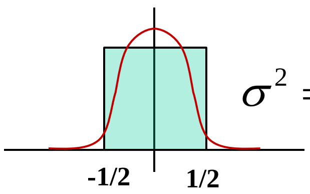


Large Angles ⇨ too Narrow!!

…Suspect Meas. Errors

Re-Work of Meas. Errors

Gaussian Equivalent σ for a Square Distribution

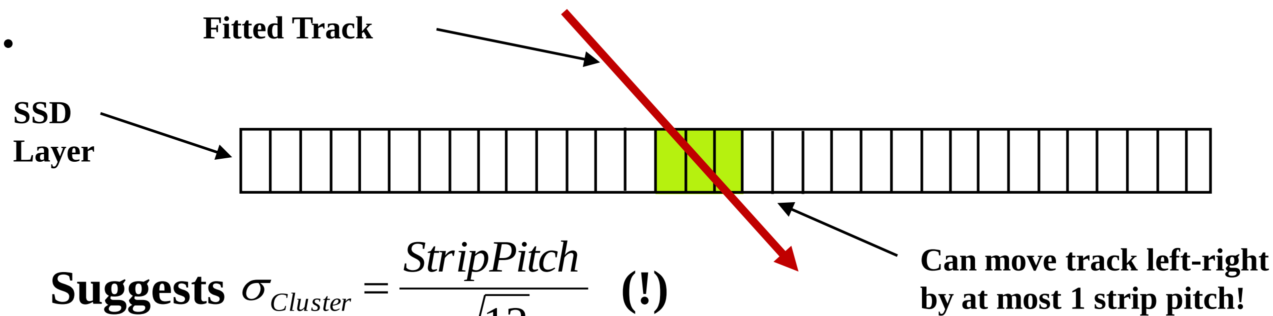


$$\sigma^2 = \int_{-1/2}^{+1/2} x^2 dx = \left.\frac{x^3}{3}\right|_{-1/2}^{1/2} = \frac{1}{12} \qquad \sigma_{Meas.} = \frac{Width}{\sqrt{12}}$$

Hence $$\frac{Strip - Pitch}{\sqrt{12}} = \frac{228\mu m}{\sqrt{12}} = 65.8\mu m$$

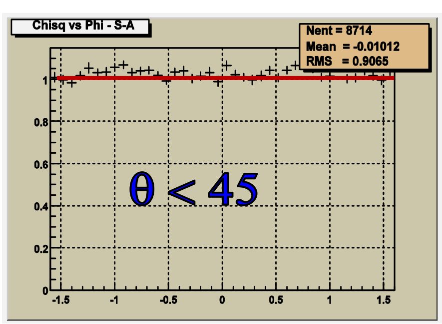
Actual "hits" on tracks are in general Clusters of Strips. Naively expect $\sigma_{Cluster} = \dfrac{ClusterWidth}{\sqrt{12}}$

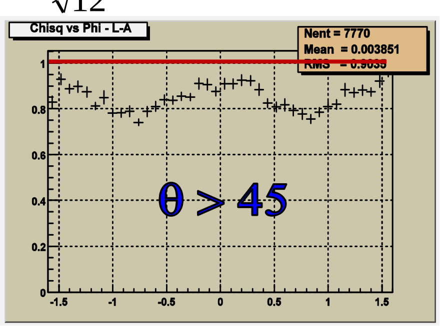
But…

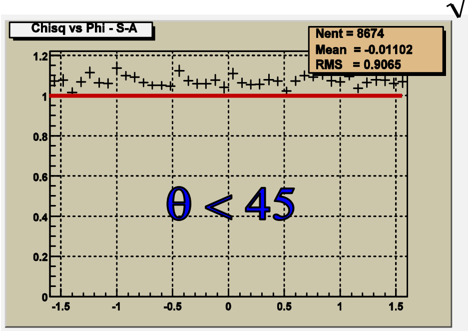


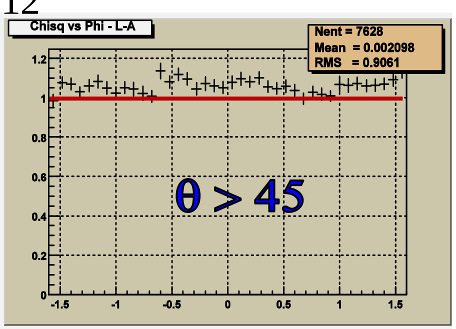
Suggests $\sigma_{Cluster} = \dfrac{StripPitch}{\sqrt{12}}$ **(!)**

$$\sigma_{Cluster} = \frac{ClusterWidth}{\sqrt{12}} \qquad\qquad \sigma_{Cluster} = \frac{StripPitch}{\sqrt{12}}$$







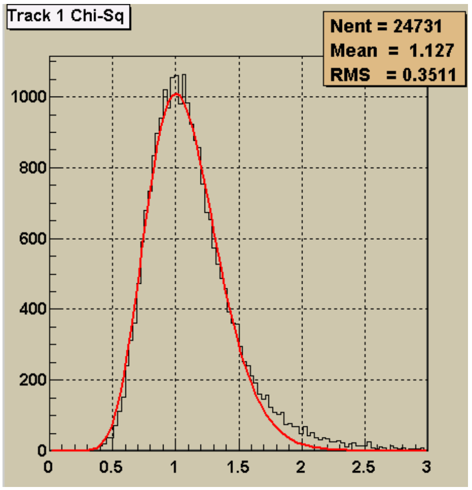


Success! χ^2 Distributions – Text-Book!

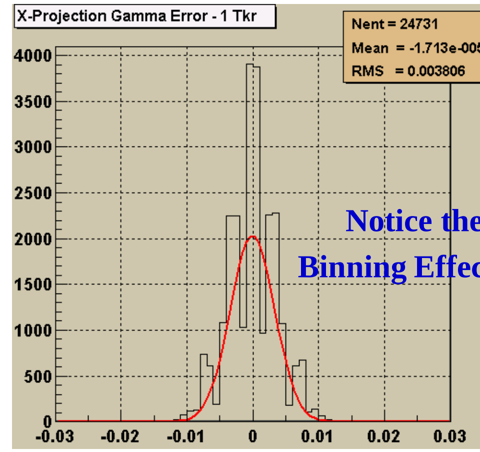
$\cos(\theta) = -1$

$<N_{hits}> = 36$

$<\chi^2> = 1.05$



$<\sigma_{FIT}> = 3.4$ mrad

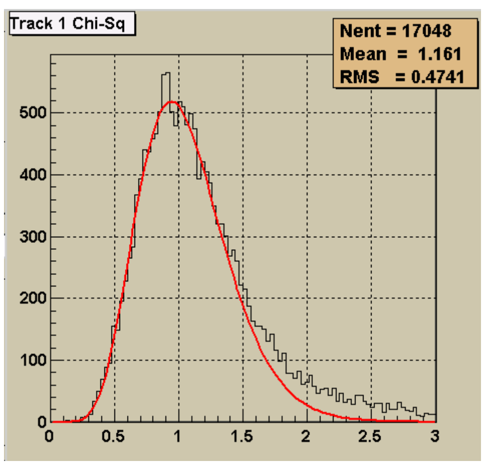


Notice the Binning Effects?

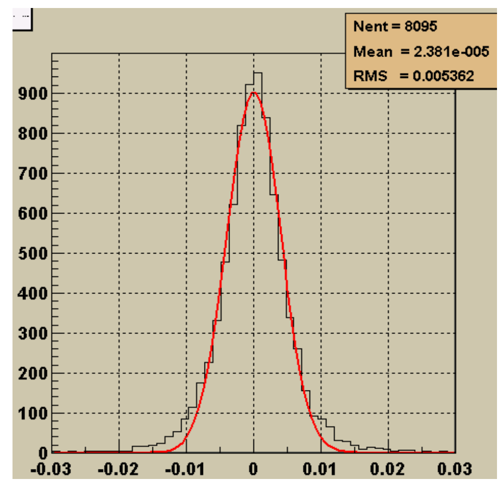
$-1 < \cos(\theta) < 0$

$<N_{hits}> = 22$

$<\chi^2> = 1.06$



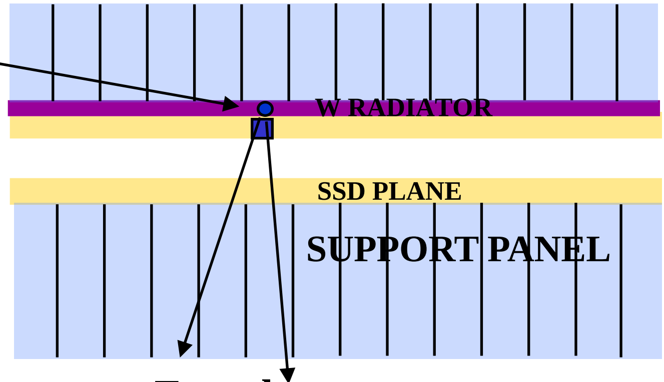
$<\sigma_{FIT}> = 4.0$ mrad



Vertexing: Two Problems

1. Z Location of the Vertex

Put Vertex
at Radiator
Mid-Point

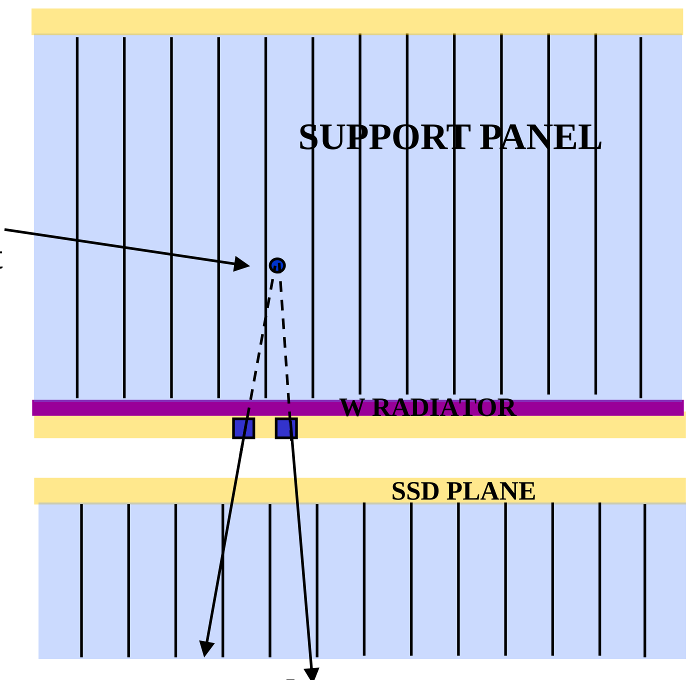


Found
Tracks

Preferred Solution

If 2 Tracks share the same first hit and the Cluster Size is no more then 2 Strips and the first hit directly proceed a W radiator
Z_{VTX} = middle of W Radiator prior to first hit

Put Vertex
DOCA Point



Found
Tracks

Next Best Solution

If DOCA location of 2 tracks lies before first hits but is after the next layer up –
-Z_{VTX} = DOCA – Z

All Other Cases

Put Vertex at Z location
of start of the 1st Track

Vertexing (2)

2. Covariant Averaging of Tracks

Multivariate Averaging: $P_{Pair} = \dfrac{C_1^{-1} \cdot P_1 + C_2^{-1} \cdot P_2}{C_1^{-1} + C_2^{-1}}$

$$P_{Pair} = (C_1^{-1} + C_2^{-1})^{-1} \cdot (C_1^{-1} \cdot P_1 + C_2^{-1} \cdot P_2)$$

$$C_{Pair} = (C_1^{-1} + C_2^{-1})^{-1}$$

where P_i are the parameter vectors of the combination(Pair) and tracks (P_1 and P_2) and C_i are the covariance matrices

And

$$\chi^2 = (P_1 - P_{Pair})^T C_{\mathrm{Res}1}^{-1} (P_1 - P_{Pair}) + (P_2 - P_{Pair})^T C_{\mathrm{Res}2}^{-1} (P_2 - P_{Pair})$$

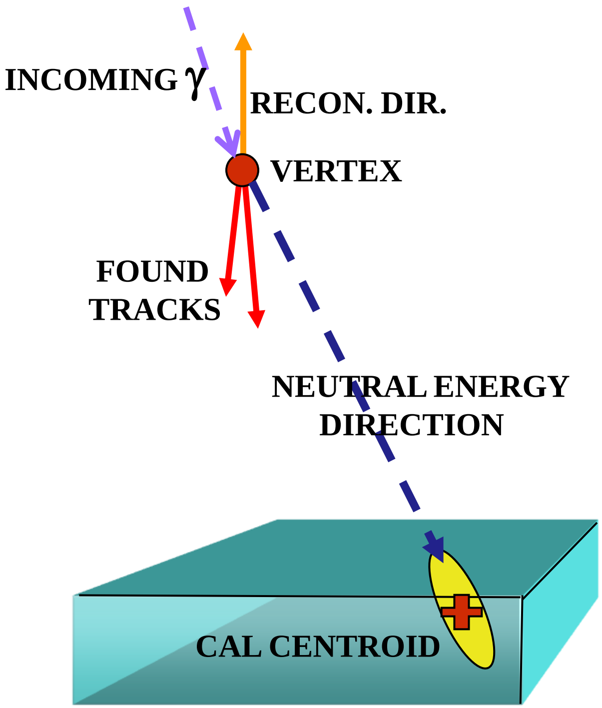
where

$$C_{\mathrm{Res}1,2} = C_{1,2} - C_{Pair}$$

The parameter vectors *P* are (x, S_x, y, S_y)

PSF Tails

Neutral Energy Concept



Sometimes at the start of the shower the charge pair does not well reflect the direction of the incoming photon.

Bremstrahlung can cause much (most) of the energy to windup in photons.

The Calorimeter centroid is a measure of where these photons impact the calorimeter.

A "Neutral Energy" direction can be inferred by connecting the found vertex with the Cal. Centroid.

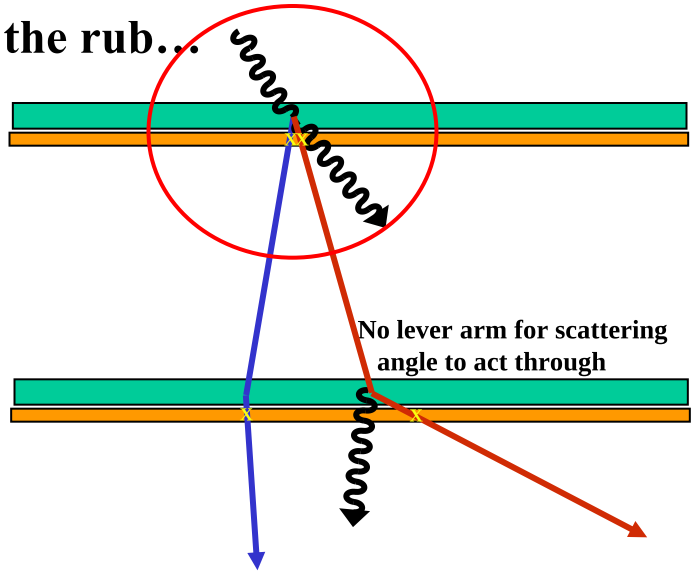
One can determine the covariant error matrix for this inferred direction by using the errors on the centroid location.

By having an imaging calorimeter, Fermi-LAT is the first Gamma Ray instrument able to do this!

Where does the Charged Solution go Wrong?

At energies < 1 GeV only the first 2 Tracker Hits determine the direction

Here's the rub…



Internal **and** ***External***

Brems. distort direction

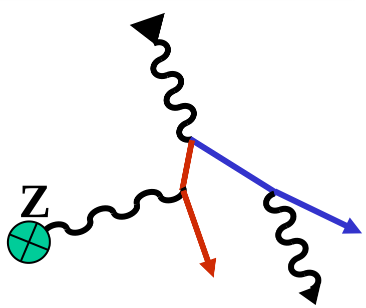
This is in addition to multiple scattering

Expect effect to be more sever in 18% radiators
(Thick - 18%, Thin - 3%)

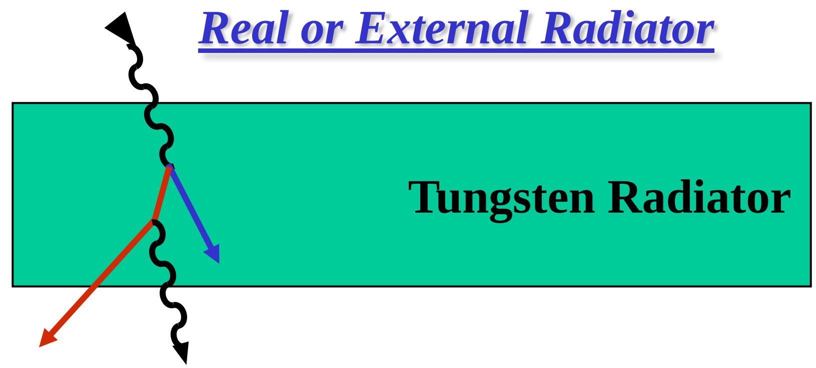
Brems. in 2nd and lower decks doesn't effect direction

Due to *Internal* Brems. ratio of effected events
(Thin : Thick Decks)
will be < ratio of Rad. Lens.

Internal Radiator

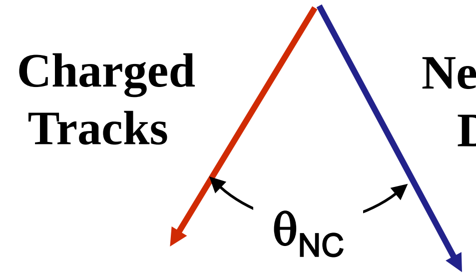


Real or External Radiator

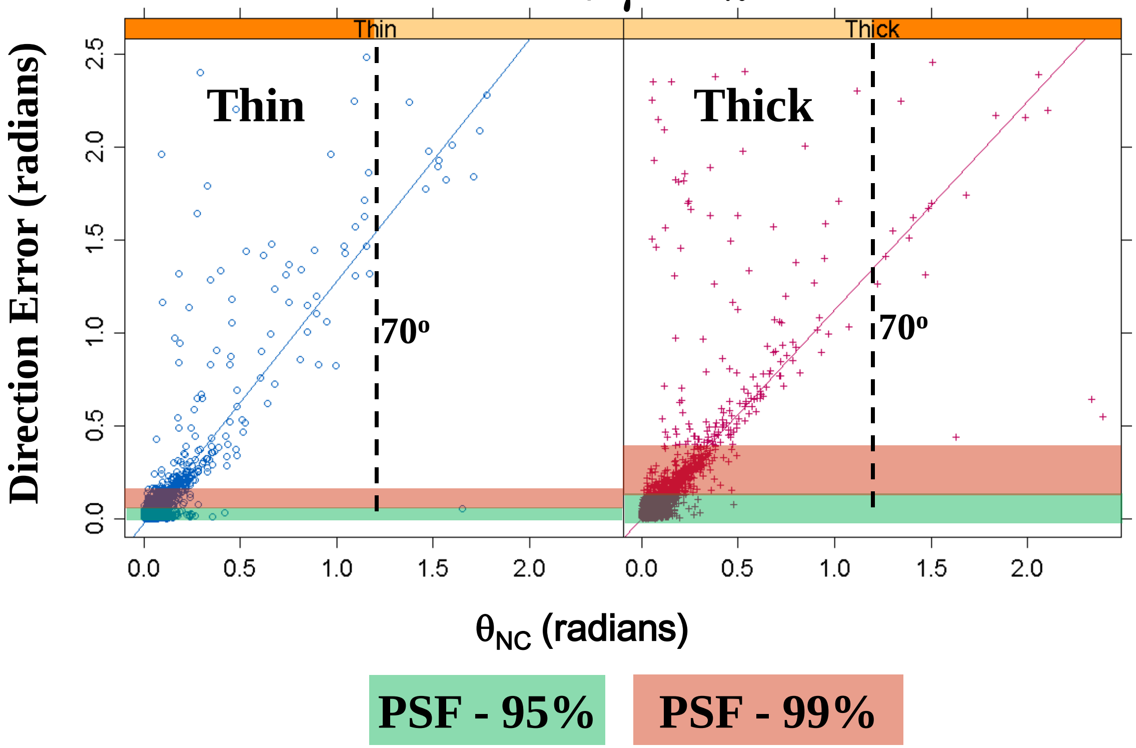


Is this Effect Present in the Monte Carlo?

Define θ_{NC} to be the angle between The VTX (Charged Tracks) Direction and the Neutral Energy Direction



500 MeV γs - 2π str



Note that the tail of the PSF is strongly correlated with θ_{NC} with unit slope!

Also this correlated tail is more pronounced in the Thick Radiators (as expected!)

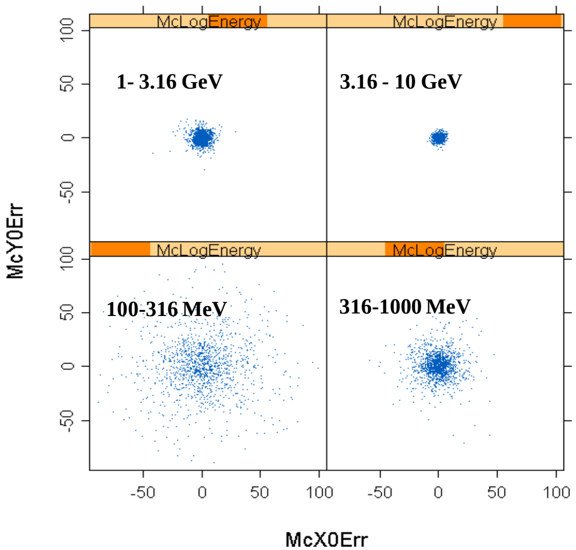
And... the tail extends from one side of the FoV to the other!

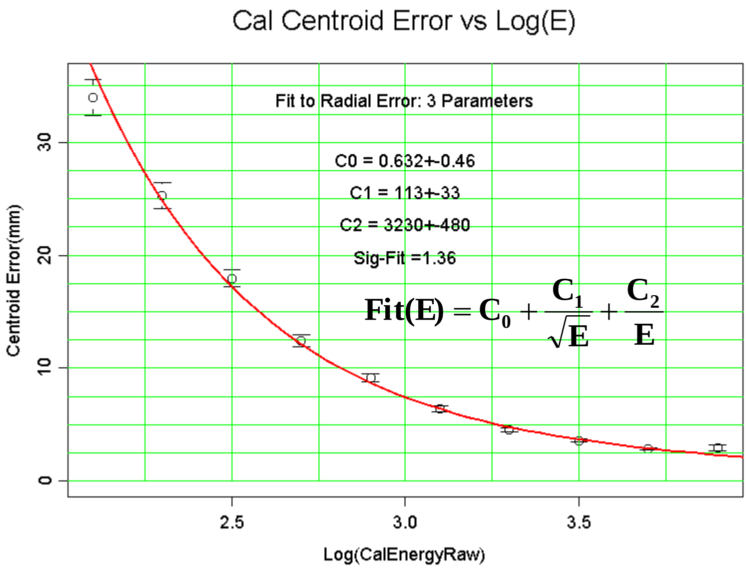
Neut. Energy Implementation

First Task: **Determine what the errors are as a function of energy**

Naively one expects the location error to ~ $R_{Moliere}/\sqrt{E}$

Run γs [100 MeV, 10 GeV] in a patch (x,y) = ([100,150], [100,150])
at normal incidence ($\hat{e}_z = -1$)





Second Task: **Combine Neutral energy direction covariently with Charged Soln.**

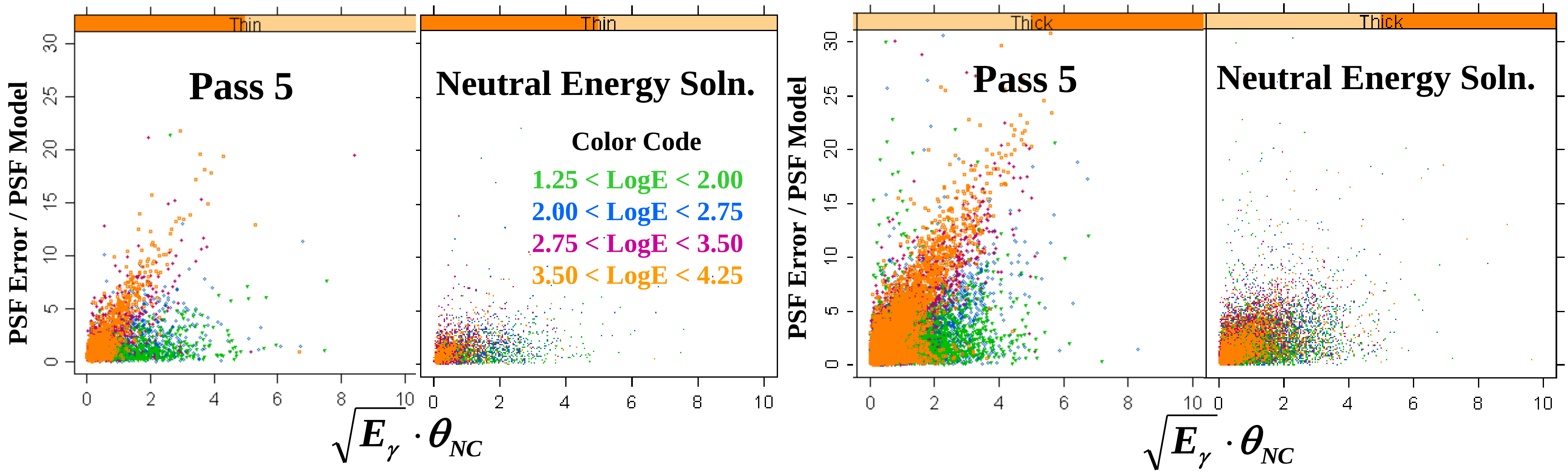
$$P_{SUM} = C_{SUM}(C^{-1}_{Charged}P_{Charged} + C^{-1}_{Neutral}P_{Neutral})$$

where P_i and C_i are the 4-element parameter vectors and the 4x4 *Covariance Matrices* and $C_{SUM} = (C^{-1}_{Charged} + C^{-1}_{Neutral})^{-1}$

All Gamma Results

Event Selection: CTBClassLevel > 0 & CTBCORE > .1

Events Using Neutral Energy Solution



Comments

- 44% events use the Neutral Energy Solution
- The effectiveness of using Neutral Energy increases with increasing energy
- The far tails on the PSF are reigned in

Making Covariance Usable

The Chicken and Egg problem

In this case the "*Chickens*" are the tracks and the "*Eggs*" are the energy determinations for the tracks.

Tracking needs an estimation of the event energy.

Energy reconstruction benefits greatly from knowing about the tracks.

Track Recon Logical Flow

Initial Energy Estimation

→ **Pattern Recognition**

→ **First Kalman Fits**

→ **Improved Energy Determination**

→ **Second Kalman Fits**

→ **Final Energy Determination**

Trouble: **Final Energy can be factors off from the energy used in the 2nd Kalman Fit**

Fixing the Energy Problem

Elements of the Cov. Matrix scales as $1/E^2$ below ~ 10 GeV ***(Mult. Scat. Dominated)***

Suggests $$C_{FINAL} \cong C_{FIT} \cdot \left(\frac{E_{FINAL}}{E_{FIT}} \right)^2$$

Best agreement found with

$$C_{FINAL} \cong C_{FIT} \cdot \left(\frac{E_{FINAL}}{E_{FIT}} \right)^{1.6}$$

The power 1.6 is consistent with PSF behavior ($1/E^{.8}$) ***(Tyrel Johnson, 2007)***

Correct Soln.: Iterate Fit with Final Energy

Find iteration ONLY affects Cov. Matrix, NOT the Parameters!

Covariant Errors on the Sky

Transformation of the Track Covariance Matrix to Sky Coordinates

R.P. Johnson September 19, 2007

The Kalmin fit gives us a covariance matrix in terms of the slope and intercept in each projection. Only the two slopes are relevant to the photon direction, so the first step is to reduce the error matrix from 4×4 to 2×2, simply by removing the rows and columns related to the intercepts. Let's call the two slopes s_1 and s_2. The associated 2×2 covariance matrix is σ_s^2.

Let $\hat{v}$ be the unit vector (direction cosines) denoting the *downward* (per LAT convention) photon direction in the LAT coordinate system and $\hat{w}$ the corresponding unit vector in the sky coordinates (galactic coordinates, for example). Finally, let $\mathbf{\Theta} = \{\ell, b\}$ be the photon longitude and latitude in the sky coordinates.

The transformation from $\hat{v}$ to $\hat{w}$ is linear and can be represented by a 3×3 orthogonal matrix $\mathbf{R}$ that includes a space inversion (i.e. negative determinant). The transformations from $\mathbf{s}$ to $\hat{v}$ and from $\hat{w}$ to $\mathbf{\Theta}$ are nonlinear, but for purposes of transforming the covariance matrix, all we need are the first-derivative matrices. Let $\mathbf{A}$ represent the 3×2 derivative matrix for the transformation $\mathbf{s} \to \hat{v}$ and $\mathbf{B}$ the 2×3 derivative matrix for $\hat{w} \to \mathbf{\Theta}$. The 2×2 matrix for the overall transformation $\mathbf{s} \to \mathbf{\Theta}$ then is $\mathbf{M} = \mathbf{BRA}$, and the covariance matrix transforms as

$$\sigma_\Theta^2 = \mathbf{M} \sigma_s^2 \mathbf{M}^T \, .$$

The transformation equations corresponding to $\mathbf{A}$ can be written

$$\hat{v}_z = -\left[1 + s_1^2 + s_2^2\right]^{-1/2}$$

$$\hat{v}_x = \hat{v}_z s_1$$

$$\hat{v}_y = \hat{v}_z s_2$$

and the matrix $\mathbf{A} = \partial \hat{v} / \partial \mathbf{s}$ can be written in terms of the direction cosines $\hat{v}$ as

$$\mathbf{A} = \begin{pmatrix} \hat{v}_z \left(1 - \hat{v}_x^2\right) & -\hat{v}_x \hat{v}_y \hat{v}_z \\ -\hat{v}_x \hat{v}_y \hat{v}_z & \hat{v}_z \left(1 - \hat{v}_y^2\right) \\ -\hat{v}_x \hat{v}_z^2 & -\hat{v}_y \hat{v}_z^2 \end{pmatrix} .$$

The transformation equations corresponding to $\mathbf{B}$ can be written

$$\ell = \tan^{-1}\left(\frac{\hat{w}_y}{\hat{w}_x}\right)$$

$$b = \sin^{-1} \hat{w}_z$$

and the matrix $\mathbf{B} = \partial \mathbf{\Theta} / \partial \hat{w}$ is

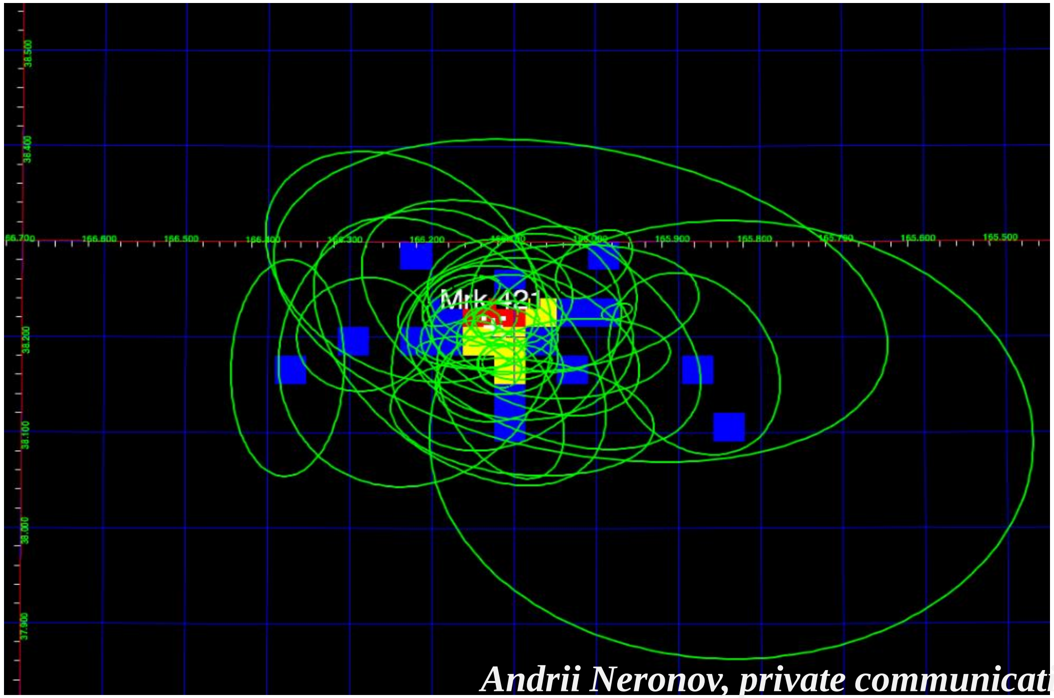
$$\mathbf{B} = \begin{pmatrix} \frac{-\hat{w}_y}{\hat{w}_x^2 + \hat{w}_y^2} & \frac{\hat{w}_x}{\hat{w}_x^2 + \hat{w}_y^2} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1 - \hat{w}_z^2}} \end{pmatrix} .$$

These equations are singular at the galactic pole ($\hat{w}_z = 1$), which has to be protected, and also at $\hat{v}_z = 0$, which should never occur, given that the Tracker cannot find horizontal tracks.

Fermi Gamma-ray Space Telescope

A First Example: Mrk 421

Neronov & Collaborators selected events around Mrk 421 with E > 100 GeV. This avoids multiple scattering issues. They applied Johnson's Inst.-2-Sky Transform.



Andrii Neronov, private communication

Results:

1) **Error Ellipses *(sort of)* point back towards Mrk 421**

2) **Smallest Ellipses consistent with limiting track resolution ($\approx \frac{\sigma_{\text{Pitch}} \cdot \sqrt{2}}{60cm} \cong .01\,\text{deg.}$)**

3) **Covariant Localization ~2 better then PSF Localization**

Summary

- **Fermi-LAT is a highly optimized HEP detector for detecting Gamma Rays in the Space Environment.**
- **The Reconstruction of the Directions on the Sky were realized through 2nd order (*covariance!*).**
- **A covariant analysis of source images will bring to bare the full power of the LAT.**
- **Pass 8 will provide the "correct" covariance matrix using the Final Energy.**
- **And… just perhaps we'll finally see Pair Halos**